

Ch 29 - Electromagnetic Induction & Faraday's Law

1820-1821 electric current produces mag field.
1830 ?

Faraday (1791-1867) first thought magnetic field produces current: cut-thick sheet to Wed...

Concept of flux:

Magnetic flux

$$\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$$

(Concept of $\Phi_E = \vec{E} \cdot \vec{A}$)

if not uniform, $\Phi_B = \int \vec{B} \cdot d\vec{A}$

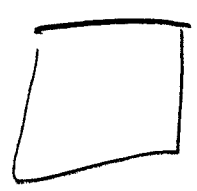
unit Weber: $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$

actually: changing flux creates a potential difference and that potl diff causes current to flow ($V = IR$)

induced potential diff = $\mathcal{E} = - \frac{d\Phi_B}{dt}$

Faraday's Law of Induction

Example: loop of wire in B field (ex 29-2)



$l = 5 \text{ cm}$
 $B = 0.16 \text{ T}$
 $R = 0.612 \Omega$

- a) Φ_B when $\vec{B} \perp$ to face of loop
- b) Φ_B when angle b/w \vec{B} and \vec{A} is 30°
- c) avg current when rotated $\omega = 5$
120.145

Soluk $\theta = 0!$

$$\begin{aligned} a) \quad \Phi_B &= \vec{B} \cdot \vec{A} = BA \cos \theta \\ &= (0.16 \text{ T}) (2.5 \times 10^{-3} \text{ m}^2) (1) \\ &= 4 \times 10^{-4} \text{ Wb} \end{aligned}$$

$$b) \quad \theta = 30 \quad \Phi_B = (4 \times 10^{-4}) \cos 30 = 3.5 \times 10^{-4} \text{ Wb}$$

← cos, as expected

$$\begin{aligned} c) \quad \mathcal{E} &= - \frac{d\Phi_B}{dt} \quad \|\mathcal{E}\| = \frac{\Delta\Phi}{\Delta t} = \frac{BA \cos \theta - BA \cos \theta}{\Delta t} \\ &= \frac{4 \times 10^{-4} \text{ Wb} - 3.5 \times 10^{-4} \text{ Wb}}{0.14 \text{ s}} \\ &= 3.6 \times 10^{-4} \text{ V} \end{aligned}$$

$$V = IR$$

$$I = \frac{V}{R} = \frac{\mathcal{E}}{R} = \frac{3.6 \times 10^{-4} \text{ V}}{0.012 \text{ } \Omega} = \boxed{0.030 \text{ A}}$$

Direction of Current

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

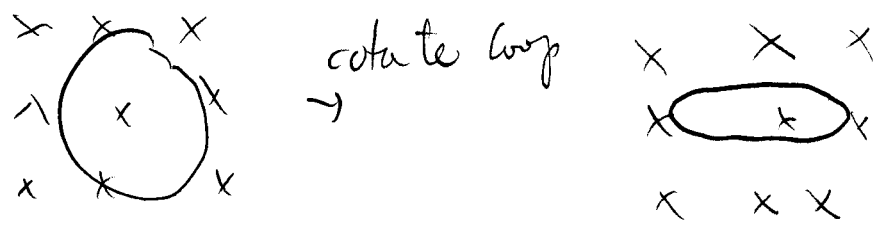
" The induced emf is always in a direction that opposes the original change in flux that caused it "

stated another way

" A current produced by an induced emf moves in a direction so that the mag field created by that current opposes the original change in flux "

↻ Lenz's law

example



is flux increasing or decreasing?

original field: into the board

induced field: also into the board - why? opposes change in flux

direction of current? clockwise

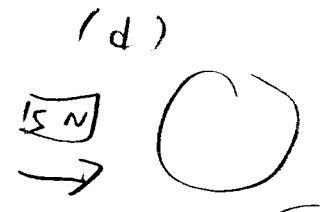
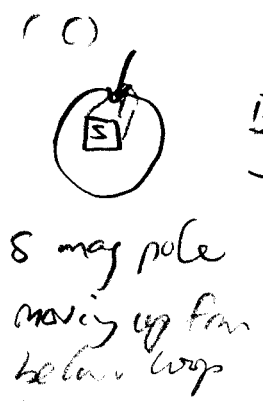
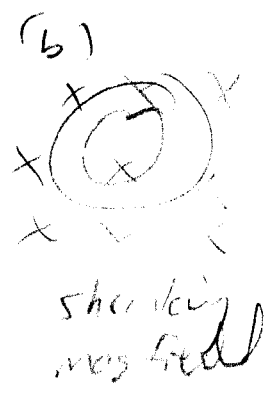
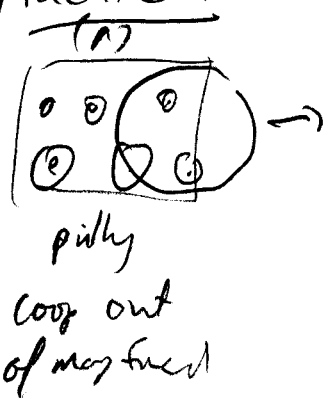
rotate back:



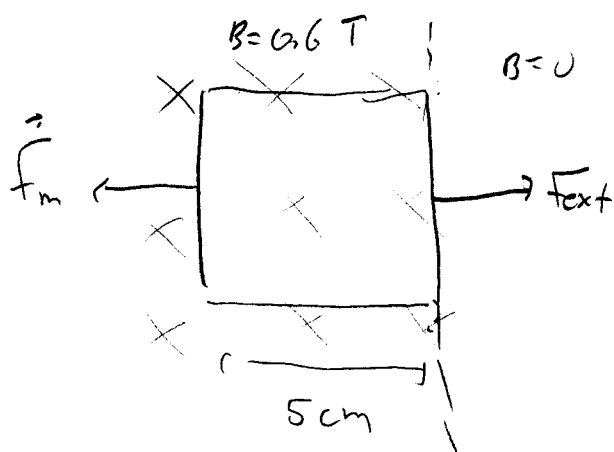
flux increasing
field into board
induced field: out of board

direction of current: ccw

Practice:



- a) ccw
 - b) cw
 - c) ccw
 - d) 0
- discuss solns

Example (29-5)

$$\text{and } R = 100 \Omega$$

A coil with 100 loops is pulled at a constant speed to the right.

At $t = 0$ shown

At $t = 0.15$ whole loop is in field free region

Find

- the rate of change in flux through the coil
- the emf and current induced.
- How much energy is dissipated in the coil?
- what was the avg. force required?

Solution

a) rate is constant

$$\frac{d\Phi_b}{dt} = \frac{\Delta\Phi_b}{\Delta t} = \frac{0 - BA \cos 90^\circ}{\Delta t}$$

$$= \frac{-(0.6 \text{ T})(0.05 \text{ m})^2}{0.15} = -1.5 \times 10^{-2} \frac{\text{Wb}}{\text{s}}$$

flux is decreasing ✓
(final smaller than initial)

b) $\mathcal{E} = \frac{d\Phi_b}{dt}$ for one coil

$$\mathcal{E} = -N \frac{\Delta\Phi_b}{\Delta t} \text{ for } N \text{ coils}$$

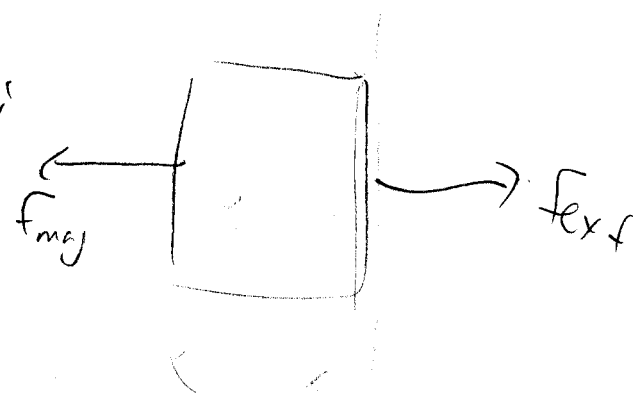
$$\mathcal{E} = -(100)(-1.5 \times 10^{-2} \text{ Wb/s}) = 1.5 \text{ V}$$

$$I = \frac{\mathcal{E}}{R} = \frac{1.5 \text{ V}}{100 \Omega} = 15 \text{ mA} \quad \text{CW}$$

$$c) E = Pt = I^2 R t = (15 \times 10^{-3} \text{ A})^2 (100 \Omega) (0.15) = 2.25 \times 10^{-3} \text{ J}$$

$$d) W = Fd \Rightarrow F = \frac{W}{d} = \frac{2.25 \times 10^{-3} \text{ J}}{0.05 \text{ m}} = 0.045 \text{ N}$$

Alternatively :

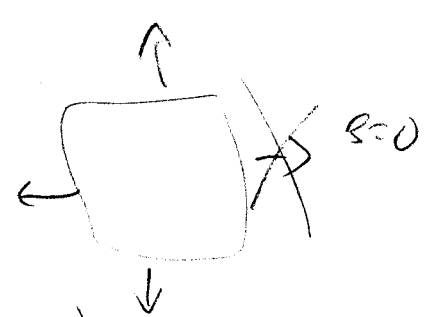


Just!

$$F_{mg} < F_{ext} \rightarrow F_{mg} \approx F_{ext}$$

$$F_{mg} = N \pm \vec{l} \times \vec{B}$$

↑
l = 5 cm

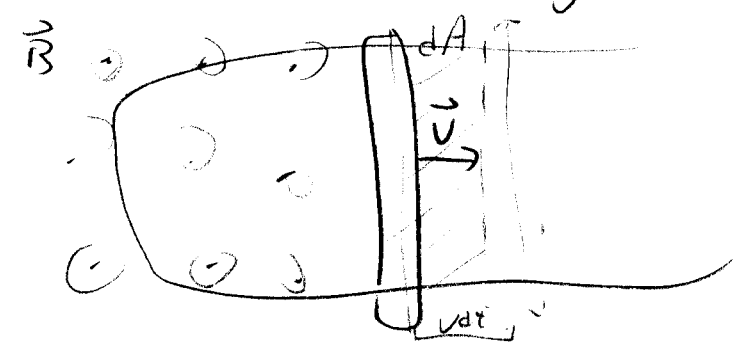


$$F = (100)(0.015A)(0.05m)(0.06T)$$

$$= 0.045N$$

July 27

Consider the following



$$dA = l v dt$$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{B dA}{dt} = \frac{B l v dt}{dt} = Blv$$

"motional emf"

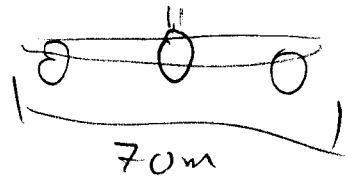
Do Demo

29-6

Ex 29-8 What about airplanes?

$$v = 1000 \frac{\text{km}}{\text{h}}$$

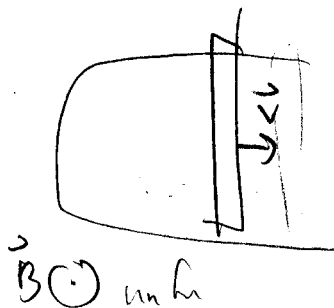
earth's mag field $\sim 5 \times 10^{-5} \text{ T}$ vertical



$$\mathcal{E} = Blv = (5 \times 10^{-5} \text{ T})(70 \text{ m}) \left(\frac{1000 \text{ km}}{\text{hr}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \approx 1 \text{ V}$$

okay !!

Ex 29-8 what is the force required to steadily move rod?



$$\vec{F} = I \vec{l} \times \vec{B}$$

$$|F| = I l B$$

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$

$$F = \frac{Blv}{R} l B = \frac{B^2 l^2 v}{R}$$

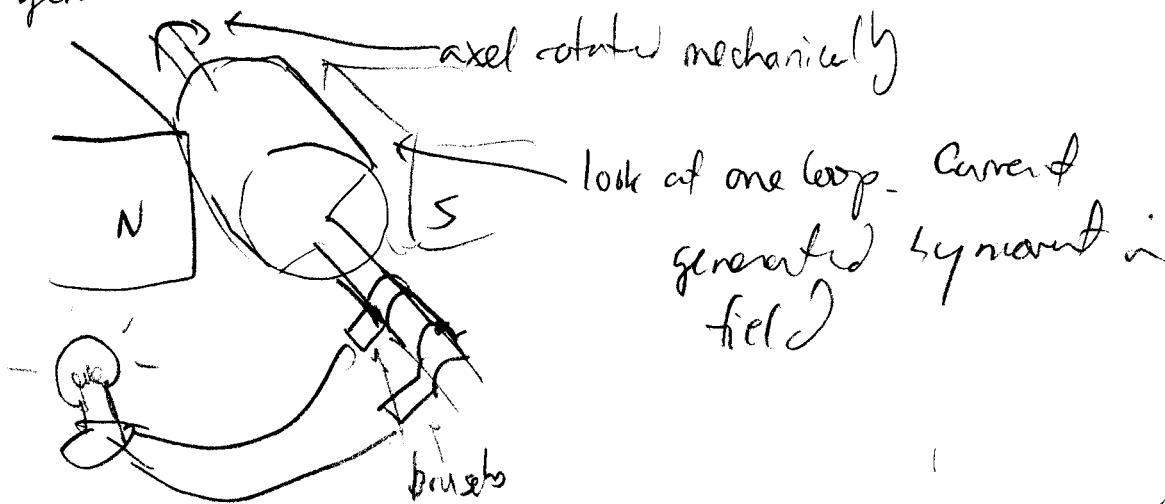
is this a constant force? (yes if B & R constant)

what if R not constant?

R increases, so force decreases

Electric Generators

motor: electrical \rightarrow mechanical energy
 gen: mechanical \rightarrow electric energy



uniform field:

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{A} = - \frac{d}{dt} B \cdot A = - \frac{d}{dt} (BA \cos \theta)$$

$$= -BA \frac{d}{dt} (\cos \theta)$$

$$= +BA \sin \theta \frac{d\theta}{dt}$$

$$\equiv +BA \sin \omega t (\omega)$$

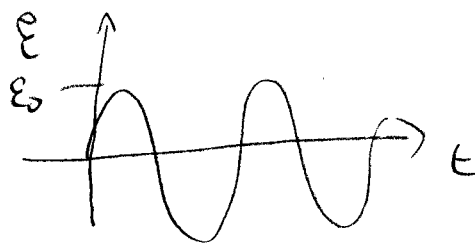
$$\frac{d\theta}{dt} = \omega \quad \text{let } \theta = 0$$

$$\theta = \omega_0 + \omega t$$

for N loops of wire

$$\mathcal{E} = NAB\omega \sin \omega t$$

$$= \mathcal{E}_0 \sin \omega t$$

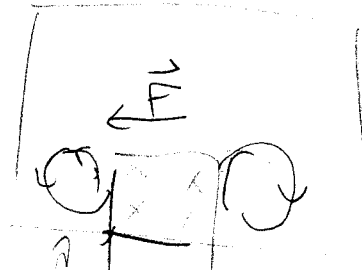
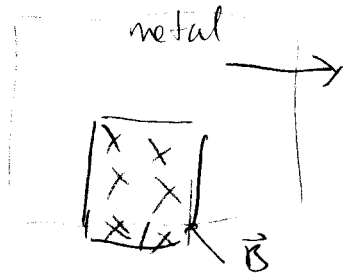


DO DEMO

AC current $(I = \frac{V}{R})$

29-8

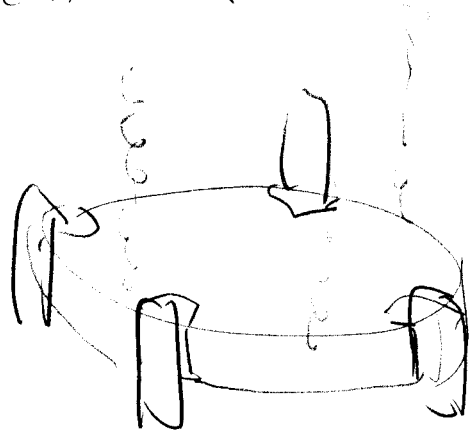
Eddy Currents



increasing field

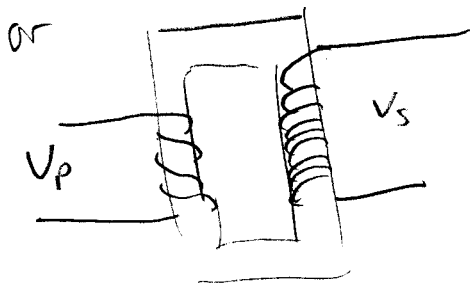
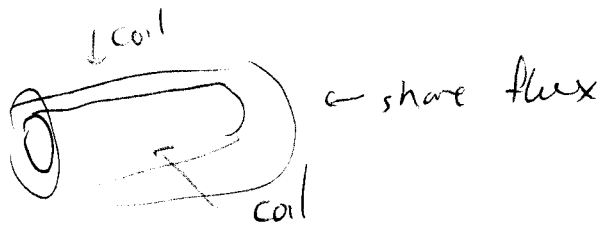
decreasing field

resists motion



damps vibrations!

Transformers



current from primary source creates magnetic field

secondary voltage induced in secondary

Coils is $V_s = -N_s \frac{d\Phi_B}{dt}$

while $V_p = N_p \frac{d\Phi_B}{dt}$

The rate of change of flux is equal

$$\frac{V_s}{N_s} = \frac{V_p}{N_p}$$

$$\Rightarrow \boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

can transform one voltage to another.
note V_s will have the same frequency as V_p .

$N_s > N_p$ "step-up transformer"

$N_s < N_p$ "step-down transformer"

| commonly 79% efficient

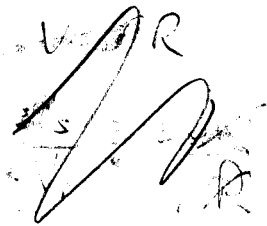
Example: cell phone charger

$$120 \text{ V}_{ac} \rightarrow 5 \text{ V}_{ac}$$

a) find N_p suppose $N_s = 30$ charger supplies 70 mA

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \rightarrow N_p = N_s \frac{V_p}{V_s} = \frac{(30)(120 \text{ V})}{(5 \text{ V})} = 720 \text{ turns}$$

b) find the current in the primary coil



$$P = IV \sim 100\% \text{ efficient so}$$

$$I_p V_p = I_s V_s$$

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$I_p = I_s \frac{N_s}{N_p} = (0.070 \text{ A}) \left(\frac{30}{720} \right) = 29 \text{ mA}$$

c) what is the power?

$$P = I_s V_s = (0.070 \text{ A})(5.0 \text{ V}) = 3.5 \text{ W}$$

29-7 Changing Mag field Produces an Electric field

Faraday $\mathcal{E} = - \frac{d\Phi_B}{dt}$
↑
potential difference

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l}$$

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$$

↑
path encloses area through which mag field is changing

notice: forces due to changing \vec{B} are nonconservative

$$\oint \vec{E} \cdot d\vec{l} \neq 0$$