Magnetoism

Imagine:

what happens?

they repel!

Discuss:

Electrostatic

Explain why charges repel - using - there are just as many resulting + charges, as many e- in any given segment - wire is neutral!

now consider:

What is happening? Magnetic fields + forces.

Source of E field: pt charges

Source of magnetic field: current (ie moving charges)

A compass measures a magnetic field. If you moved field near wire, what would you find?
magnetic field lines are curly!

recall: E field lines are guided by charge would move.
What if I put a charge on a mag. field line? nothing happens. What about opening example? Force was felt. Why?
there was moving charge

\[ \mathbf{F}_{\text{mag}} = q (\mathbf{V} \times \mathbf{B}) \]

in general, Lorentz force law
\[ \mathbf{F} = q \mathbf{E} + q (\mathbf{V} \times \mathbf{B}) \]

RHR: 1) point 4 fingers in direction of \( \mathbf{V} \)
2) curl to dir. of \( \mathbf{B} \)
3) \( \mathbf{F} \) points in dir. of thumb.

\[ \mathbf{D} = \mathbf{A} \times \mathbf{E} \]

direction of \( \mathbf{D} \)?
what is the force on a wire?

A current can be thought of as a line charge moving along with a velocity \( \vec{v} \)

\[
I = \frac{\Delta Q}{\Delta t} = \lambda \Delta t \nabla \\
\vec{E} = \nabla \phi \\
\vec{v} = \frac{\vec{E}}{\lambda}
\]

\[
\vec{F} = q \vec{E} \times \vec{B} = q \frac{\vec{E} \times \vec{B}}{\lambda}
\]

\[
\vec{F} = I \vec{L} \times \vec{B} \quad \text{(typically, used in books)}
\]

Example 1. A doubly charged helium ion, whose mass is 6.6 \( \times \) 10^-27 kg, is accel. by a volts of 2700 V. (a) what will be its radius of curvature if it move in plane 1 to a uniform 0.840 T field? (b) what is its speed of rotation in revolutions per second (c) potential \( \rightarrow \) kinetic energies

\[
\frac{q}{m} = \frac{1}{2} mv^2 + \phi v \\
v = \sqrt{\frac{2q\phi}{m}}
\]
\[ F_{mg} = q \vec{V} \times \vec{B} \quad \sin \theta = \sin 90^\circ = 1 \]

\[ |F| = qUV \]

Circular motion:
\[ F = \frac{\mu mV^2}{r} \]

\[ qUV = \frac{mv^2}{r} \]

\[ r = \frac{mv}{qB} = \frac{m}{qB} \sqrt{\frac{2qV}{m}} = \frac{1}{8} \sqrt{\frac{2mV}{q}} = \]

\[ = \frac{1}{0.344} \sqrt{\frac{2(6.6 \times 10^{-30} \text{ kg})(2700 \text{ V})}{2(1.6 \times 10^{-19} \text{ C})}} \]

\[ = 3.1 \times 10^2 \text{ m} \]

b) \[ v = \frac{2\pi r}{T} \]

\[ T = \frac{V}{B} = \frac{v}{2\pi r} \sqrt{\frac{2}{2mV}} = \frac{2\pi m}{qB} = \ldots = 3.8 \times 10^3 \]

\[ v = \sqrt{\frac{2qV}{m}} \]

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What is the magnetic field for a loop of wire?

"magnetic dipole"

Torque "next page"

What is torque on a loop of wire? (in an external field)"
\[ \mathbf{F}_1 = \hat{r} \times \hat{F} \]
\[ = \frac{b I A B}{2} \]
\[ \mathbf{F} = I \hat{I} \times \hat{B} \]
\[ \mathbf{F}_1 = I L B \]
\[ \mathbf{F} \cdot \mathbf{l} = I I L B \]
\[ l = a \]
\[ \mathbf{F} = I A B \]
\[ \mathbf{F}_2 = \hat{r} \times \hat{F} \]
\[ = M F L \sin \theta \]
\[ = \frac{b I A B \sin \theta}{2} + \frac{b I a B \sin \theta}{2} \]
\[ = b I A B \sin \theta \]
\[ \mathbf{l} = I A B \sin \theta \]
\[ \mathbf{F} = I A B \sin \theta \]

\[ \text{=} \] Magnetic field will tend to align loops of current
Wheel and magnets?

Imaginary model of loops of current:

\[ B = \vec{\mu} \times \vec{H} \]

- is all the same way, thus magnetic field.

Since electrons orbit, can think of them as loops of current. Then why are not all metals magnets?

Reality - much more complicated.

Materials exhibit 3 characteristics:

1. Paramagnetism - dipoles associated with spins of unpaired electrons aligning to lie along the external field.

2. Dia magnetism - the orbital speed of the \( e^- \) is altered in such a way as to charge the orbital dipoles to oppose the field.

3. Ferromagnetism - favorable for spins to align. Domains are nearly aligned.

Put in field

\[ = \text{torque on loops} \]

Take away field, and still have magnet.
Magnetic Monopoles?

If you break up a magnet, you just get more:

\[ S \rightarrow S N N S N N S N \]

not

\[ S N \rightarrow S N \]

Major difference: the Electricity + magnetism

Whey do magnets pick up things? 
- small opposite field induced in material by strong field from magnet.

\[ S \rightarrow W \]

Do Demo'

Magnetic field lines from closed loop

\[ \Rightarrow fields \]
Ex 27-1

\[ \vec{F} = I \vec{L} \times \vec{B} \]

\[ |\vec{F}| = ILB \sin \theta \]

\[ = (30 \, \text{A})(0.12 \, \text{m})(0.9 \, \text{T}) \sin (60^\circ) = 2.8 \, \text{N} \]

Ex 27-3

\[ \vec{F} = \sum d\vec{F} = \sum dF_x \hat{i} + \sum dF_y \hat{j} \]

\[ dF_x = dF \cos \phi \]

\[ dF_y = dF \sin \phi \]

\[ \vec{F} = \int I dl \vec{B} \cos \phi \hat{i} + \int I dl \vec{B} \sin \phi \hat{j} \]

\[ dl = Rd\phi \]

\[ = IB \left[ \int_0^\phi Rd\phi \cos \phi \hat{i} + \int_0^\phi Rd\phi \sin \phi \hat{j} \right] \]

\[ = IBR \left[ 0 - \cos \phi \right] \hat{i} + \int_0^\phi \hat{j} \]

\[ \vec{F} = IBR (2) \hat{j} \]
CRT - Cathode Ray Tube:

- What would $e^-$ do if plates and coils off?
  \[ F = qE = \frac{eE}{q} \]

- What if $+t$ turn on a potential difference? (e$^-$ go up)

- What if $+t$ turn off and turn on coils?

Use CRT to measure charge-to-mass ratio of $e^-$ from above, \[ \frac{e}{m} = \frac{V}{Br} \]

Use CRT to measure charge-to-mass ratio of $e^-$ from above, \[ \frac{e}{m} = \frac{V}{Br} \]

turn on both fields, so that beam is undeflected

\[ \Sigma F = 0 \]

\[ F_{mag} - F_{ed} = 0 \]

\[ F_{mag} = F_{ed} \]

\[ \vec{v} \times \vec{B} = \vec{E} \]

\[ \vec{v} = \vec{E} \times \vec{B} \]

so

\[ \frac{e}{m} = \frac{1}{Br} \left( \frac{E}{B} \right) = \frac{E}{B^2 r} \]

\[ \frac{e}{m} = 1.76 \times 10^{11} \text{ C/kg} \]
The Hall Effect (37-8)

- Put current-carrying conductor in B field
- B force on e-
- Creates a potential drift.

\[ F = q \vec{v} \times \vec{B} \]

\( \vec{E} \) = electric field
\( \vec{F} \) = force
\( q \) = charge
\( \vec{v} \) = velocity
\( \vec{B} \) = magnetic field

What if we consider current to be the flow of +ve charges?

Free is down.

Goes opposite potential drop.

\( \vec{E} \) = electric field
\( \vec{F} \) = force
\( q \) = charge
\( \vec{v} \) = velocity
\( \vec{B} \) = magnetic field

This first revealed that it is neg charge moving inside conductor.

\( Q \) is it possible to measure \( B \) using the Hall effect?
Calculating Magnetic Field

Experimentsally, \( \bar{B} \propto \frac{I}{r} \) \( \Rightarrow \bar{B} = \frac{C}{r} \)

\[ C = \frac{\mu_0}{2\pi} \Rightarrow \bar{B} = \frac{\mu_0 I}{2\pi r} \]

Similar to Coulomb's law, a point charge gives only an axial \( \vec{d} \vec{B} \)

more generally, \( \bar{B} = \int \vec{d} \vec{B} \)

\[ \frac{d\vec{B}}{d\tau} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{d} \vec{r} \times \vec{r}}{r^2} \]

\[ \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{d} \vec{r} \times \vec{r}}{r^2} \]

\[ \text{also note: } \hat{r} = \frac{\vec{r}}{r} = \frac{\vec{r}}{r^2} \]

Notice: Vector sum

Example:

1. Draw good picture, w/ vectors
2. Consider direction ( & consider \(\theta \))

\[ \bar{B} = \mu_0 I \int \frac{d\vec{y} \times \vec{r}}{r^2} \]

\[ \bar{B} = \frac{\mu_0 I}{4\pi} \int dy \int dx \sin \theta \]

\[ r \text{ changing, } y \text{ changing, } \theta \text{ changing} \]

\[ r^2 = R^2 + y^2 \]
2 ways:

1) \[ \sin \theta = \frac{R}{r} \]
\[ r^2 = R^2 + y^2 \]
\[ r = (R^2 + y^2)^{1/2} \]
\[ B = \frac{\mu I}{4\pi} \left( \int \frac{dy}{(R^2 + y^2)^{3/2}} \right) \]
\[ = \frac{\mu I}{4\pi} \left[ \frac{y}{(R^2 + y^2)^{3/2}} \right]^{\infty}_{0} \]
\[ = \frac{\mu I}{4\pi R} \left[ 1 - (-1) \right] \]

Another example:

\[ B = \frac{\mu I}{4\pi} \left( \int \frac{dx}{r^2} \right) \]
\[ = \frac{\mu I}{4\pi} \left( \int \frac{R}{r^2} \right) \]
\[ = \frac{\mu I}{4\pi} \int \frac{dx}{r^2} \]
\[ = \frac{\mu I}{4\pi} \int \frac{R}{r^2} \]

\[ y = -\frac{R}{\tan \theta} \]
\[ dy = R \cos \theta \, d\theta = R \frac{db}{\sin \theta} \]
\[ \sin \theta = \frac{R}{r} \]
\[ dy = \frac{R \cos \theta}{(R^2 + y^2)^{3/2}} \]
\[ B = \frac{\mu I}{4\pi} \left( \int \frac{R \cos \theta}{(R^2 + y^2)^{3/2}} \right) \]
\[ = \frac{\mu I}{4\pi} \left( \frac{1}{R^2} \right) \]
\[ = \frac{\mu I}{4\pi} \left( \frac{R}{R^2} \right) \]
\[ \vec{B} = \frac{\mu_0 I}{4\pi} \frac{dl \times \hat{R}}{(R^2 + x^2)^{3/2}} \]

\[ = \frac{\mu_0 I \rho}{4\pi (R^2 + x^2)^{3/2}} \int dl \hat{R} \]

\[ = \frac{\mu_0 I \rho}{4\pi (R^2 + x^2)^{3/2}} 2\pi R \hat{R} \]

\[ \vec{B} = \frac{\mu_0 J R^2}{2 (R^2 + x^2)^{3/2}} \hat{R} \]

Where is \( \vec{B} \) max? At \( x=0 \), the center of the loop.

Compare to E-field - \( \vec{E} = 0 \) @ center.

At \( x=0 \)

\[ \vec{B} = \frac{\mu_0 I}{2\pi R} \]

Example 28-13

\[ \vec{B} = \frac{\mu_0 I}{2\pi} \int \frac{dl \times \hat{R}}{R^2} = \int d\vec{B} \]

\[ \hat{B} \]

\[ \pi \]

\[ \hat{R} \]

\[ \hat{B} \]

\[ \vec{B} = \frac{\mu_0 I}{2\pi} \int \frac{dl (1)}{R^2} \]

\[ = \frac{\mu_0 I}{2\pi} \left( \frac{2\pi R}{4} \right) \]

\[ = \frac{\mu_0 I}{4\pi} \left( \frac{2\pi R}{4} \right) \]

\[ \vec{B} = \frac{\mu_0 I}{8\pi} \]

\[ \vec{B} = \frac{\mu_0 I}{8\pi} \text{ into the board} \]
\[ B = \frac{\mu_0 I}{4\pi} \int \frac{\text{d}x \times \hat{F}}{r^2} \]

\[ \hat{F} = x^\perp \hat{y} + y^\perp \hat{x} \]

\[ \text{d}x = R d\varepsilon = \text{r} \]

\[ \text{d}x = R d\varepsilon \cos \theta - R d\varepsilon \sin \theta \]

\[ \hat{x}^\perp = 0 \]

\[ \hat{y}^\perp = 0 \]

\[ \hat{j} \times \hat{r} = \hat{z} \Rightarrow \epsilon \mathbf{r} \]

\[ \hat{k} \times \hat{r} = \hat{z} \]

\[ \text{d}x \hat{x} = R d\theta \left[ -x \sin \theta \hat{x} + x \cos \theta \hat{y} + y \sin \theta \hat{x} + y \cos \theta \hat{y} \right] \]

\[ = R d\theta \left[ -x \sin \theta \hat{x} + x \cos \theta \hat{y} - y \cos \theta \hat{x} \right] \]

\[ \hat{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} R d\theta \left[ x \sin \theta \hat{z} + x \cos \theta \hat{c} + y \cos \theta \hat{c} \right] \frac{1}{(x^2 + y^2)^{3/2}} \]

This is too complicated.
Steps to remember:
- Draw a careful diagram, labeling vectors
- Consider separate segments, and symmetry
- Figure out cross product
- Figure out |r| and it together. Integrate, be careful of limits.
  
Is there an easier way? Remember Gauss' law...

\[
\oint \mathbf{B} \cdot d\mathbf{e} = \mu_0 I \text{ Encircled} \]

Strategy: Pick loop (closed path) for which \( \mathbf{B} \cdot d\mathbf{e} = B \, dl \, (\mathbf{e}_1) \) or \( \cos \theta \).

\[
B \, dl \, (\mathbf{e}_1) \cos (\mathbf{B}_1 \cdot \mathbf{e}_1)
\]

For example: Log straight wire.
Symmetry tells us B points \( \mathbf{e}_1 \).
Pick loop in circle + draw \( \mathbf{H} \) and \( \mathbf{r} \).

\[
\mathbf{B} \cdot d\mathbf{e} = B \cdot d\mathbf{l}
\]

\[
\oint \mathbf{B} \cdot d\mathbf{e} = \mu_0 \int \mathbf{I} \cdot d\mathbf{l}
\]

\[
B = \frac{\mu_0 I}{2\pi r}
\]

Guess \( |B| \) only gives the magnitude.
Circle $(x-2)^2 + (y-3)^2 = 4$

Symmetry: ...

Outside

\[ \int B \cdot dS = \mu_0 I \text{ and } B \cdot dS = \mu_0 I \]

Inside \: \text{and} = ?

Same form: circular, uniform density, so take area ratio

\[ T_{enc} = \frac{I}{\pi R^2} \]

\[ B \cdot 2\pi r = \mu_0 \left( \frac{I}{2\pi R^2} \right) \]

\[ B = \frac{\mu_0 I}{2\pi R^2} \]

Solenoid (demo?)

Cut away \( 20 \times 10 \text{mm} \)

Field really straight

Field really zero

In simplified case, land up...
\[ \mathbf{B} \cdot d\mathbf{e} = \mathbf{B} \cdot (\mathbf{v} + \mathbf{v}' + \mathbf{v}'' + \mathbf{v}''') \]

\[ = B \mathbf{v} = \mu_0 I \text{end} \]

\[ I \text{end} = NT \]

\[ B = \mu_0 NT \]

\[ v = \frac{v}{\text{cm}} \]

\[ B = \mu_0 N I \]

toroid: find \( B \) in and outside toroid, home class try

under low \( I \) end = 0

\[ B \text{int} = \mu_0 N I \]

\[ B = \frac{\mu_0 N I}{2\pi r} \]

concluded ch 8
For a 5th wire

\[ B_1 = \frac{\mu_0 I_1}{2\pi d} \]

\[ F_2 = I_2 B_1 l_2 \]

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\[ F_2 = I_2 \left( \frac{\mu_0 I_1}{2\pi d} \right) l_2 \]

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\[ F_2 = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l_2 \]

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Attractive

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Repulsive

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