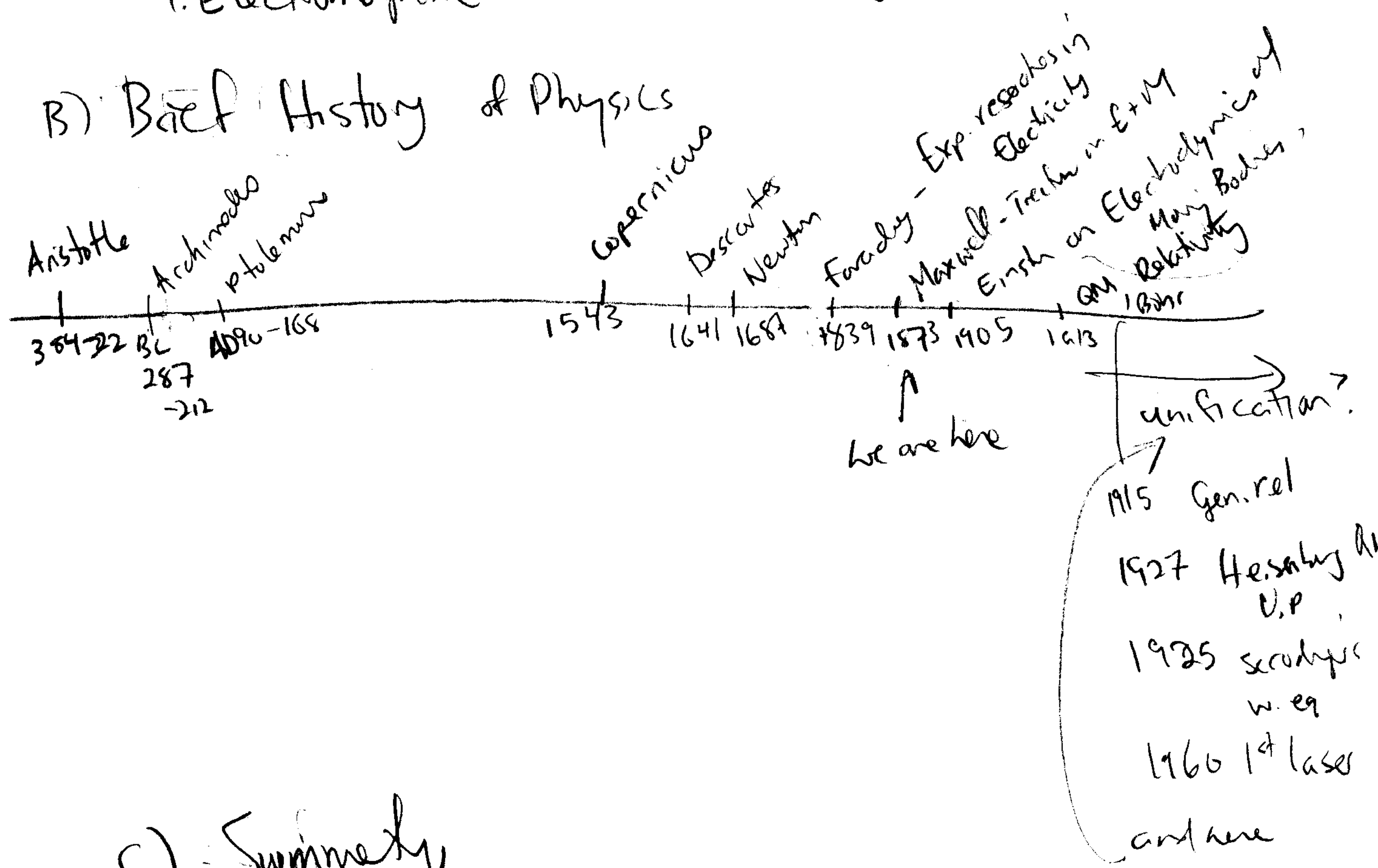
Electricity + Magnetism

I. Introduction

A) 4 forces of nature:

1. Strong — holds p and n tog., short range 100x more powerful than electrical forces
2. Weak — certain radioactive decay, weak
3. Gravitational — very weak (electrical 10^{42}x more) big mass = large effect
4. Electromagnetic — dominant force only one understood completely

B) Brief History of Physics



C) Symmetry

i. simplifies problems

ii. a lot in nature is symmetric

iii. symmetries in nature associated w/ physical laws (Noether's Theorem)

- if a system has a continuous symm. prop., then there are corresponding quantities whose values are conserved in time

Math Review

I Vectors

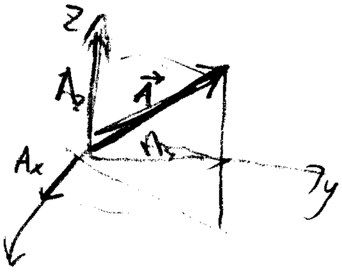
A Defn: vectors specify direction + magnitude
→ two vectors equal if " " " are the same

B. Can write vectors in terms of their components:

in rectangular: $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ or $\hat{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$

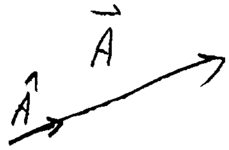
same

the length is given by $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

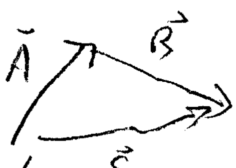


B. Unit vectors

$\hat{A} = \dfrac{\vec{A}}{|A|}$



D. Addition $\vec{A} + \vec{B} = \vec{C}$



using components: add components:

$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

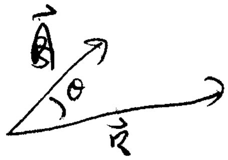
E. Multiplication

i) By scalar: multiply each component by scalar

eg $5\vec{A} = 5A_x\hat{i} + 5A_y\hat{j} + 5A_z\hat{k}$

ii Scalar (Dot) Product

$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$



proj of A onto B × B
" B onto A × A

components: multiply + add:

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

notice: if $A \parallel B$, $\vec{A}\cdot\vec{B} = |A||B|$ $(\cos 0 = 1)$

if $A \perp B$ $\vec{A}\cdot\vec{B} = 0$ $(\cos 90^\circ = 0)$

Properties: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

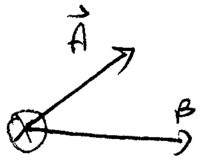
$\vec{A}\cdot(\vec{B}+C) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

$\vec{A}\cdot\vec{A} = |A|^2 \overset{\text{note}}{\Rightarrow} |A|^2 = A_x^2 + A_y^2 + A_z^2$

$|A| = \sqrt{\quad '' \quad}$

iii) Vector (cross) Product

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$ direction of $\hat{n}$ given by RHR



$\otimes$ = into board

$\odot$ = out of board

components:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_y & A_z \\ B_x & B_z \end{vmatrix}$$

$$= \hat{i}(A_yB_z - A_zB_y) + \ldots$$

property $(\vec{A}\times\vec{B}) = -(\vec{B}\times\vec{A})$ non-commutative

$A\times(B+C) = (A\times B) + (A\times C)$ distributive

Examples / Practice:

$\vec{A} = \hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{B} = 2\hat{i} + \hat{k}$

find a) $\vec{A}\cdot\vec{B}$ b) $\vec{B}\times\vec{A}$ c) the angle between $\vec{A}$ and $\vec{B}$

d) the magnitude of $\vec{A}$

Electostatics

I Electric Charge

A) Properties

1) Has two "varieties" +, −

like: repel
opposite: attract

2) Conserved — global + local conservation

3) Quantized ~~define~~ comes in units of charge
the unit is the electron: 1.6×10^{-19} C

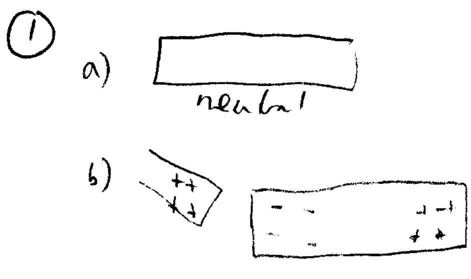
B) Behavior/Movement of Charge

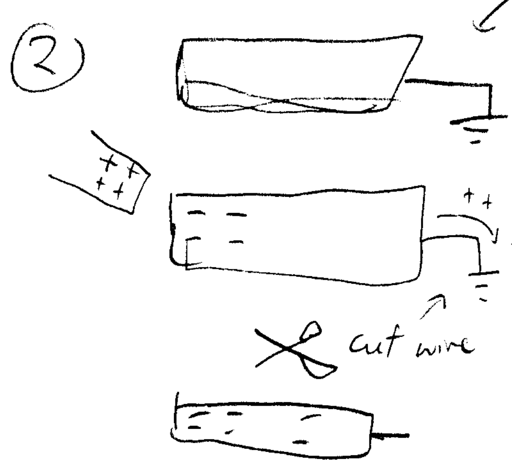
1) Materials:

- conductors - more "free electrons"
- insulators
- semiconductors

2) Induced Charge (as opposed to conducted charge)

2 scenarios:

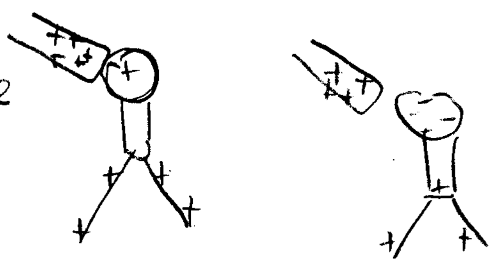
① 

② 

discuss ground

think of "sea of e^-"

Electroscope



conduction induction

Question: Is there a way you could use the electroscope to determine if an object is positively or negatively charged?

Discuss in groups of 2-3

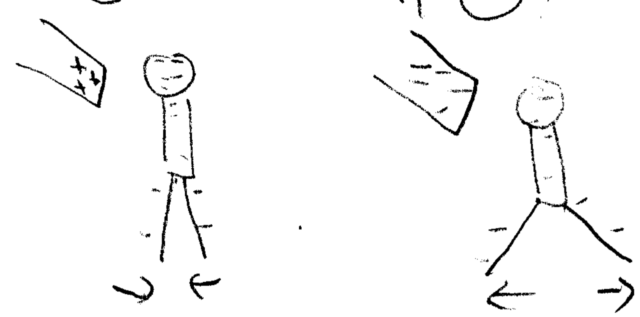
Solution

(know neg or pos)

<u>step 1</u> charge by conduction

<u>step 2</u> being close:

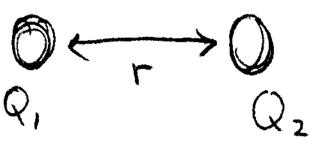
if (+): if (−):



(C) Coulomb's Law

how do we find exactly the magnitude of the force which attracts or repels?

Coulomb, 1780s, noticed force decreased with r^2



$$\boxed{F = k\frac{Q_1 Q_2}{r^2}}$$

where $k = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$ → add in later

units? vectors?

Units

unit of charge (SI) : Coulomb

r : meters

F: N

fill in

$\therefore F = k\frac{Q_1 Q_2}{r^2} \Rightarrow k = \frac{Fr^2}{Q_1 Q_2}$ $\left(\frac{Nm^2}{C^2}\right)$

how much is 1 coulomb?

let $Q_1 = Q_2 = 1C$

$r = 1m$

$F = \frac{(8.99 \times 10^9 \, N \cdot m^2 / C^2)(1C)^2}{1m^2} \approx \underline{9 \times 10^9 N}$

compare to how big something would be to weigh this much on earth:

$F = mg$

$9 \times 10^9 N = m \, 9.8 \, m/s^2$

$m \approx 10^9 kg$!

$\Rightarrow 1C$ = large amount of charge

more typical amount:

charge of electron $e = 1.602 \times 10^{-19} C$

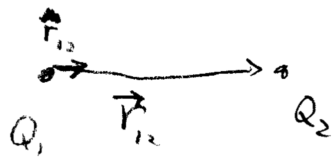
Note: consider gravity: $F = \frac{Gmm}{r^2}$ $\quad F \propto \frac{1}{r^2}$

may help to make analogies

~~Examples / Practice:~~

Vectors:

$$\vec{F} = \frac{k Q_1 Q_2}{r_{12}^2} \hat{r}_{12}$$ which directn?



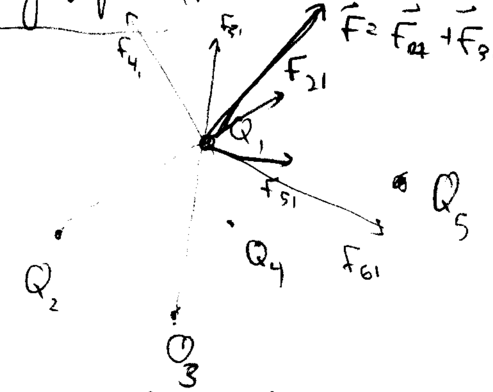
$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

r points from charge to test charge

D) Principle of Superposition (in other words, common logic)

↳ Forces add

$\vec{F} = \vec{F}_{41} + \vec{F}_{31} + \vec{F}_{41} + \dots$



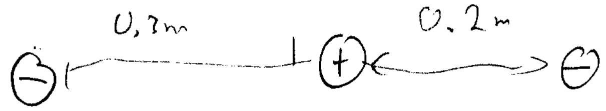
add like you would usually add vectors

So $\vec{F}_{net} = \sum_i \frac{k Q Q_i}{r_i^2} \hat{r}_i$

where ..

Example/Practice Questions:

(1)

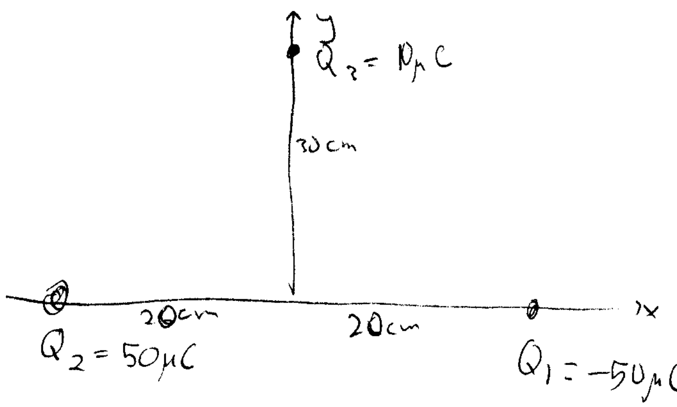


$Q_1 = -8.0 \mu C$ $Q_2 = 30 \mu C$ $Q_3 = -4.0 \mu C$

Find net force on Q_2, and Q_3

have class do / try

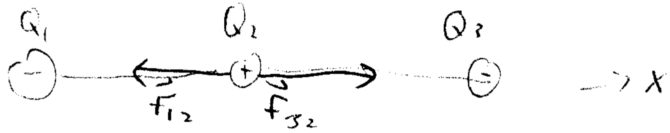
(2)



find net force on Q_3

Solutions.

(1)



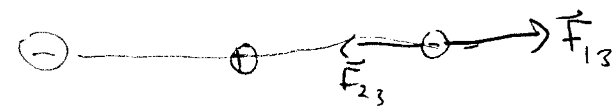
Q_2 : $\vec{F} = \vec{F}_{12} + \vec{F}_{32} = (F_{12} + F_{32})\hat{i}$

$\vec{F} = \left(-k\dfrac{Q_1 Q_2}{r_{12}^2} + k\dfrac{Q_3 Q_2}{r_{23}^2}\right)\hat{i}$

$= k\left(\dfrac{-(8\mu c)(3\mu c)}{(0.3m)^2} + \dfrac{(3\mu c)(4\mu c)}{(0.2m)^2}\right)$

$= 0.3N$

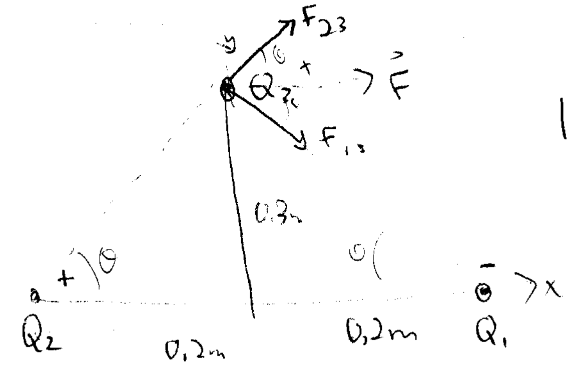
Q_3



$\vec{F} = \left[\dfrac{k(-8\mu c)(-4\mu c)}{(0.3m+0.2m)^2} + \dfrac{k(3\mu c)(-4\mu c)}{(0.2m)^2}\right]\hat{i}$

$= 0.477$

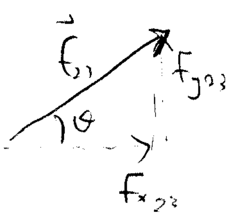
(2)



$|r_{23}| = |r_{13}| = \sqrt{0.2^2 + 0.3^2}$

notice!
only x components should add.

Zoom in on F_{23}



$F_{x_{23}} = |F_{23}|\cos\theta\,\hat{i}$

$F_{y\,23} = |F_{23}|\sin\theta\,\hat{j}$

θ?

$\tan\theta = \dfrac{0.3\cancel{m}}{0.2\cancel{m}}$

F_{13}

$\vec{F}_{x13} = |F_{13}|\cos\theta\,\hat{i}$

$F_{y13} = -|F_{13}|\sin\theta\,\hat{j}$

net: $\vec{F}_x = |F_{23}|\cos\theta\,\hat{i} + |F_{13}|\cos\theta\,\hat{i}$

$\vec{F}_y = |F_{23}|\sin\theta\,\hat{j} - |F_{23}|\sin\theta\,\hat{j}$

but $|F_{23}| = |F_{13}| = \dfrac{k(50\mu C)(10\mu C)}{(\sqrt{0.2^2+0.3^2})^2}$

so $\vec{F}_x = \dfrac{2k(50\mu C)(10\mu C)}{(0.2^2+0.3^2)}\,\hat{i}$

$\vec{F}_y = 0\,\hat{j}$ ✓

<u>Steps</u>

1) Make drawing. Notice symmetries (if any)
2) Draw components
3) Identify angles (if needed)
4) Solve with variables
5) Put in numbers

E) Continuous Charge Distributions

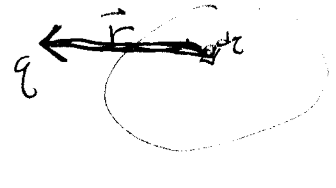
recall. the force on a charge q by charges q_i

is $\vec{F} = \sum_i \dfrac{k q q_i}{r_i^2}\hat{r}_i$ discrete charges

for continuous charges

$\vec{F} = \int \dfrac{k q\, dq}{r}\hat{r}$

Caution! $\vec{r}$ is the vector between q and each part dq — can change direction.



Charge distributions:

volume charge density : $\rho = \frac{q}{V}$ = $\frac{\text{charge}}{\text{volume}}$

surface " " $\sigma = \frac{q}{S}$ =

linear " " $\lambda = q/l$ =

$dq = ?$

$dq = \rho \, dV$

$dq = \sigma \, ds$

$dq = \lambda \, dl$