II Electric field

Discuss - what is a field?

A) defn Electric field is the force per unit charge

$$\vec{F} = \sum_i \frac{k q q_i}{r_i^2} \hat{r}_i$$

$$\vec{E} = \frac{\vec{F}}{q} = \sum_i \frac{k q_i}{r_i} \hat{r}_i$$

$$\boxed{\vec{F} = q\vec{E}}$$

what are the units of E? F is ---

E is measured in N/C

(21-6)
Example: find mag + dir of field for pt charge at pt P which is 30cm to the rt of a pt chg $Q = -3 \times 10^{-6}\,C$

$$E = \frac{kQ}{r^2} = \frac{(9\times10^{9}\,N\,m^2/C^2)(3\times10^{-6}\,C)}{(0.3m)} = 3.0\times10^{5}\,N/C$$

Have class do. ex 21-8 (like eg for force)

B) Continuous charge distr.

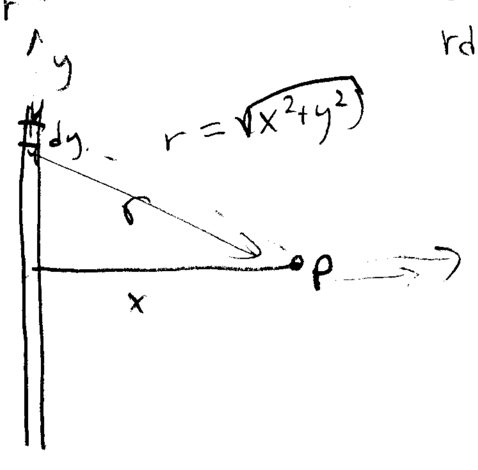
$$\vec{E} = \int d\vec{E} \qquad dE = \frac{k\,dQ}{r^2}$$

$r^2 = x^2 + y^2$

$2r\,dr = 0 + 2y\,dy$

$r\,dr = y\,dy$

Example (21-11)



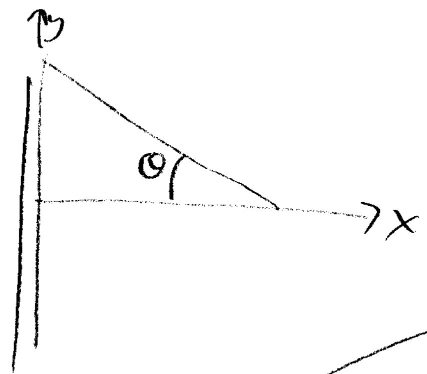
$$\vec{E} = \int d\vec{E} = k\int \frac{dQ}{r^2}\hat{r} = k\int \frac{\lambda\, dl}{(x^2+y^2)}\hat{r}$$

note $\hat{r}$ changes

- consider x, y components ($\hat{i}$, $\hat{j}$ don't change dir

$$E_y = \int dE \sin\theta$$

$$E_x = \int dE \cos\theta$$



$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath}$$

notice symmetry? $E_y = 0$

notice: x is treated as a constant here

if desired:

$$E_y = k\int \frac{\sin\theta\, dy}{x^2+y^2}$$

$$\sin\theta = \frac{y}{(x^2+y^2)^{1/2}} \rightarrow E_y = \int \frac{y\, dy}{(x^2+y^2)^{3/2}}$$

see appendix B-4

$$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = -\frac{1}{\sqrt{x^2+y^2}}\Big|_{\infty}^{\infty} = 0 - 0 = 0 \checkmark$$

$$E_x = \int \frac{k\lambda\cos\theta\, dy}{x^2+y^2}$$

notice $\frac{y}{x} = \tan\theta \rightarrow y = x\tan\theta$

$$dy = \frac{x\, d\theta}{\cos^2\theta}$$

also $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

$$\frac{\cos^2\theta}{x^2} = \frac{1}{(x^2+y^2)}$$

so $E_x = \int \frac{k\lambda \cos\theta \, x \, d\theta \, \cos^2\theta}{\cos^2\theta \; x^2}$

$= \frac{k\lambda}{x}\int \cos\theta \, d\theta$

limits? $-\pi/2$ $\pi/2$

$E_x = \frac{\lambda k}{x} \sin\theta \Big|_{-\pi/2}^{\pi/2} = \frac{2\lambda k}{x}$ if sub $k = \frac{1}{4\pi\epsilon_0}$ $E_x = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x}$

C) Field Lines

remember page of arrows?

to avoid confusing pages covered in arrows, there is a standard convention

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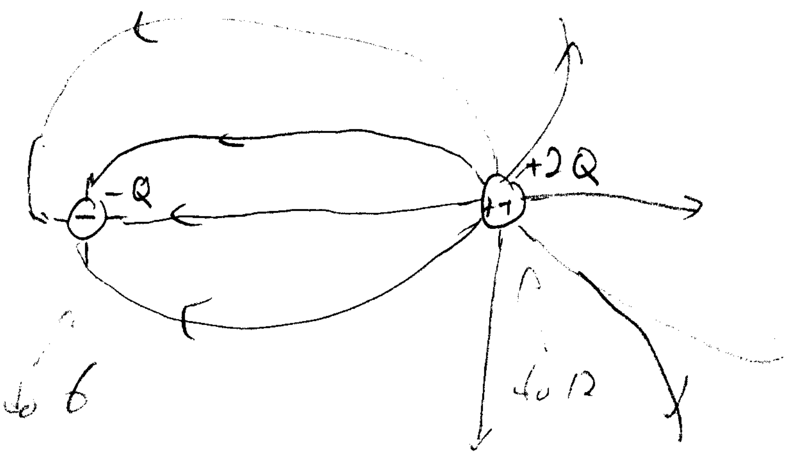
1. field lines point in direction of E field.
2. Lines are drawn so that the magnitude of the electric field is prop. to the number of lines crossing unit area ⊥ to the lines
3. field lines start on positive charges + end on neg charges.

eg.

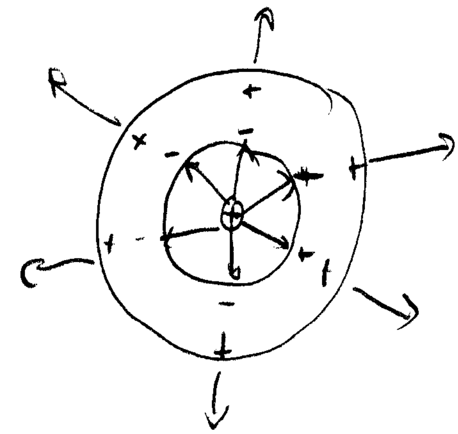


remember: the pt in the dir. a ⊕ive charge would go

D) Fields + Conductors

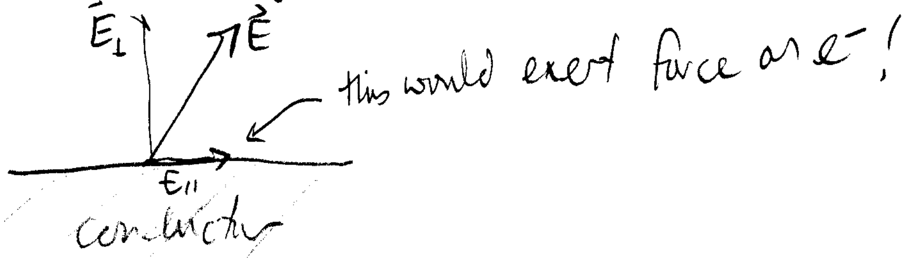
① Electrons move until in equilibrium. This corresponds to the situation where there is no force + thus no electric field inside the conductor. *

eg



② The electric field is always ⊥ to the surface of a conductor.

Consider:



so it must be that $\vec{E}_\parallel = 0$ for conductors

E) Motion of a Charged Particle in an Electric Field.

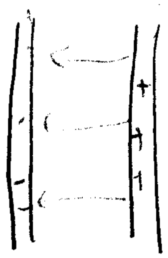
$$\vec{F} = q\vec{E}$$

Newton? $\vec{F} = m\vec{a}$

set equal: $q\vec{E} = m\vec{a}$

$$\vec{a} = \frac{q}{m}\vec{E}$$

Example An electron drifts very slowly into the space btwn 2 charged plates ~~separated by a distance~~ which have a field $|\vec{E}| = 2\times10^4\,N/C$ between them. How fast is the electron moving after 1.5 cm?



$$F = ma = qE$$

$$a = \frac{q}{m}E$$

$$v^2 = v_0^2 + 2a\Delta d \quad (v_0 = 0)$$

$$v = \sqrt{2a\Delta d} = \left(\frac{2q}{m}E\Delta d\right)^{1/2}$$

$$= \left(2\,\frac{(1.6\times10^{-19}\,C)}{(9.1\times10^{-31}\,kg)}(2\times10^4\,N/C)(1.5\times10^{-2}\,m)\right)^{1/2}$$

$$v = 1\times10^7\,m/s \qquad (\text{compare } c = 3\times10^8\,m/s)$$