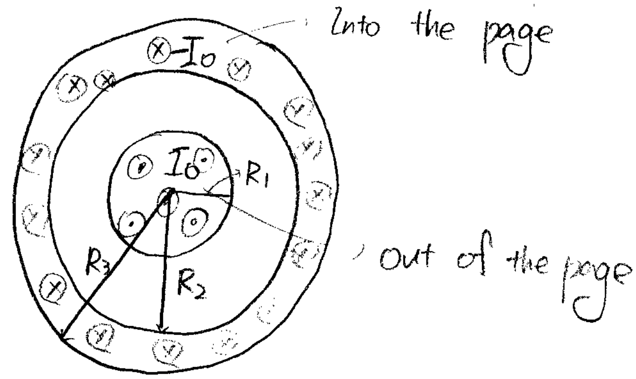
PHY 122 Workshop # 5

Ch 28

Prob 32



Let's find the constant C_1 and C_2 by integrating the currents over each cylinder and setting the integral equal to the total current.

For the inner conductor,

$$I_0 = \int \vec{j}_1 \cdot d\vec{A} = \int_0^{R_1} C_1 R \, 2\pi R \, dR = 2\pi C_1 \int_0^{R_1} R^2 dR = \frac{2\pi C_1}{3} R_1^3$$

↳ current density for the inner conductor

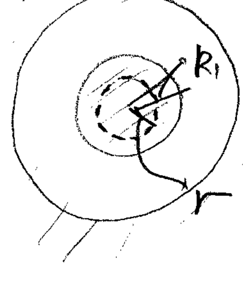
$$\Leftrightarrow \quad C_1 = \frac{3 I_0}{2\pi R_1^3}$$

For the outer conductor,

$$-I_0 = \int \vec{j}_2 \cdot d\vec{A} = \int_{R_2}^{R_3} C_2 R \, 2\pi R \, dR = 2\pi C_2 \int_{R_2}^{R_3} R^2 dR = \frac{2\pi}{3} C_2 (R_3^3 - R_2^3)$$

$$\Leftrightarrow \quad C_2 = -\frac{3 I_0}{2\pi (R_3^3 - R_2^3)}$$

(a) For $r < R_1$,



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$$

(Ampere's law)

$(LHS) = \oint \vec{B}\cdot d\vec{s} = B\,2\pi r$ (※ $\vec{B}$: counter-clockwise direction since I is flowing to out of page)

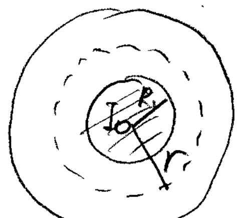
$$(RHS) = \mu_0 I_{encl} = \mu_0 \int \vec{j}_1 \cdot d\vec{A} = \mu_0 \int_0^r C_1 R \, 2\pi R \, dR = \mu_0 \, 2\pi C_1 \left.\frac{R^3}{3}\right|_0^r$$

$$= \frac{2\pi}{3}\mu_0 r^3 C_1 = \cancel{\frac{2\pi}{3}}\mu_0 r^3 \frac{\cancel{3} I_0}{\cancel{2\pi} R_1^3} = \frac{\mu_0 I_0}{R_1^3} r^3$$

$$\therefore \quad B\,2\pi r = \frac{\mu_0 I_0}{R_1^3} r^3$$

$$\Leftrightarrow \quad \boxed{B = \frac{\mu_0 I_0}{2\pi R_1^3} r^2} \quad (\text{Counterclockwise})$$

(b) For $R_1 < r < R_2$,

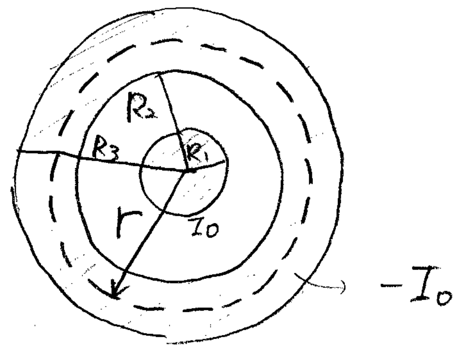


$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl} = \mu_0 I_0$$

$$\Rightarrow \quad B\,2\pi r = \mu_0 I_0$$

$$\Leftrightarrow \quad \boxed{B = \frac{\mu_0 I_0}{2\pi r}} \quad (\text{Counterclockwise direction})$$

(c) For $R_2 < r < R_3$



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$\Leftrightarrow \quad B\,2\pi r = \mu_0 \left[I_0 + \int_{R_2}^{r} \underbrace{(C_2 R)}_{= j_2} \underbrace{2\pi R\,dR}_{= dA} \right]$$

$$= \mu_0 \left[I_0 + 2\pi C_2 \left.\frac{R^3}{3}\right|_{R_2}^{r} \right] = \mu_0 \left[I_0 + \frac{\cancel{2\pi}}{\cancel{3}}(r^3 - R_2^3)\left(-\frac{\cancel{3} I_0}{\cancel{2\pi}(R_3^3 - R_2^3)}\right) \right]$$

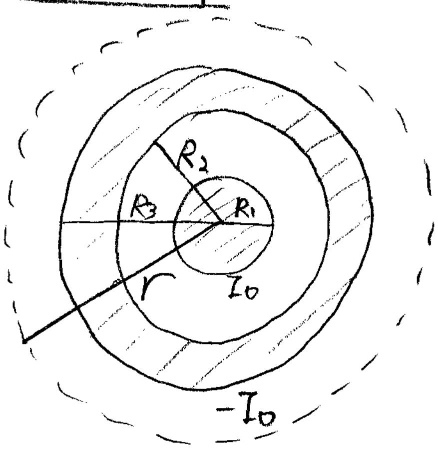
$$= \mu_0 I_0 \left[1 - \frac{r^3 - R_2^3}{R_3^3 - R_2^3} \right] = \mu_0 I_0 \frac{R_3^3 - \cancel{R_2^3} - r^3 + \cancel{R_2^3}}{R_3^3 - R_2^3}$$

$$= \mu_0 I_0 \frac{R_3^3 - r^3}{R_3^3 - R_2^3}$$

$$\Leftarrow) \quad B\, 2\pi r = \mu_0 I_0 \frac{R_3^3 - r^3}{R_3^3 - R_2^3}$$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r} \frac{R_3^3 - r^3}{R_3^3 - R_2^3}}$$ (Counterclockwise)

(d) $r > R_3$



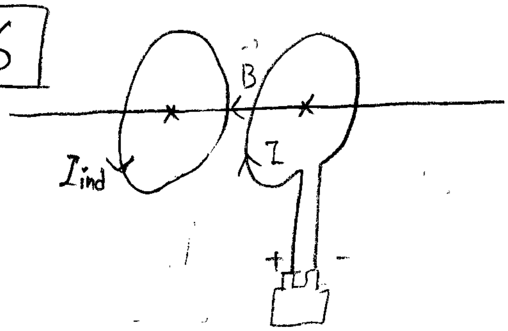
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$$

$$\Rightarrow \quad 2\pi r B = \mu_0 (I_0 - I_0)$$

$$\boxed{B = 0}$$

Ch29

Q6



(a) Will a current be induced in the second loop?

Yes! (by Lenz's law)

Ch29

(b) <u>If so, when does this current start?</u>

The current starts as soon as the battery is connected and current begins to flow in the first loop

(c) <u>When does it stop?</u>

The induced current stops as soon as the current in the first loop has reached its steady value.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (\mathcal{E} = 0 \text{ if there is no change in } \Phi_B)$$

(d) <u>In what direction is this current?</u>

The induced current in the 2nd loop will be counterclockwise, in order to oppose the change (Lenz's law)

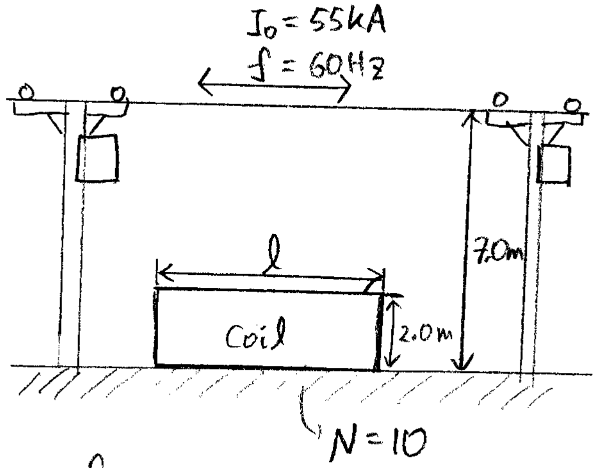
(e) <u>Is there a force btwn the two loops?</u>

Yes. While there is an induced current, there will be a force btwn the two loops.

(f) <u>If so, in what direction?</u>

The force will be repulsive, since the currents are in opposite directions.

$\boxed{P16}$



The sinusoidal varying current in the power line creates a sinusoidal varying magnetic field encircling the power line, given by Eq. 28-1,

$$B = \frac{\mu_0 I}{2\pi r}.$$

Using Eq. 29-1b, $\Phi_B = \int \vec{B}\cdot d\vec{A}$, we integrate this field over the area of the rectangle to determine the flux through it. Differentiating the flux as in Eq. 29-2b, $\mathcal{E} = -N\frac{d\Phi_B}{dt}$, gives the emf around the rectange. Finally, by setting the maximum emf equal to 170V we can solve for the necessary length of the rectangle.

$$B(t) = \frac{\mu_0}{2\pi r} I = \frac{\mu_0}{2\pi r} I_0 \sin(2\pi f t)$$

$$\Phi_B(t) = \int \vec{B}\cdot d\vec{A} = \int_{5.0m}^{7.0m} B(t)\underbrace{l\,dr}_{=dA} = \int_{5.0m}^{7.0m} \frac{\mu_0 I_0}{2\pi r}\sin(2\pi f t)\, l\,dr$$

$$= \frac{\mu_0 I_0 l}{2\pi}\sin(2\pi f t)\int_{5.0m}^{7.0m}\frac{dr}{r} = \frac{\mu_0 I_0 l}{2\pi}\sin(2\pi f t)\ \ln r\Big|_{5.0m}^{7.0m}$$

$$= \frac{\mu_0 I_0 l \sin(2\pi f t)}{2\pi}\ln\left(\frac{7}{5}\right)$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -10\,\frac{d}{dt}\left[\frac{\mu_0 I_0 l \sin(2\pi f t)}{2\pi}\ln\left(\frac{7}{5}\right)\right]$$

$$= -10\ln\left(\frac{7}{5}\right)\frac{\mu_0 I_0 l}{\cancel{2\pi}}\cancel{2\pi} f\cos(2\pi f t)$$

$$= -10\ln\left(\frac{7}{5}\right)\mu_0 I_0 l\, f\cos(2\pi f t)$$

Since the peak value is given by $V_0 = 170\,V$,

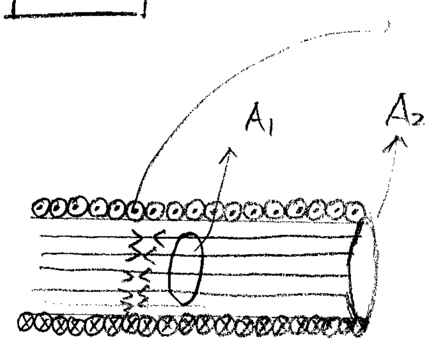
$$170V = \mathcal{E}_0 = 10\ln\left(\frac{7}{5}\right)\mu_0 I_0 l f$$

$$\Rightarrow l = \frac{170V}{10\ln\left(\frac{7}{5}\right)\mu_0 I_0 f} = \frac{170\,V}{10\ln(1.4)\,(4\pi\times10^{-7}\,T\cdot m/A)(55{,}000A)(60Hz)}$$

$$= \boxed{12m}$$

$B = \mu_0 n I$ (n = # of loops/length)

P22



The magnetic flux inside the solenoid is alternating since the electric current is alternating.

The magnetic field inside the solenoid is given by Eq. 28-4, $B = \mu_0 n I_0$.
Use Eq. 29-2a, $\mathcal{E} = -\frac{d\Phi_B}{dt}$, to calculate the induced emf. The flux causing the emf is the flux through the small loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA_1) = -A_1\frac{d}{dt}(\mu_0 n I_0 \cos\omega t) = -A_1\mu_0 n I_0\,\omega(-\sin\omega t)$$

$$= \boxed{A_1\mu_0 n\omega I_0 \sin\omega t}$$