ASTC22 (GALAXIES) PROBLEM SET #6. HINTS AND SOLUTIONS

Points in the square brackets give the relative weight with which the problems count toward the final score. If you need any physical constants, you may find them in the textbook(s) or on the web.

1 [5p.] Tides on a stellar cluster

An open cluster is at a distance $d = 120$ pc) so you should have no trouble seeing it with a naked eye, since its diameter is 1.75 degrees of arc (three times the full moon). Its 800 stars have an estimated total mass of 300 solar masses.

What is the physical diameter D of the cluster in parsecs? What is the tidal (or Jacobi) radius r_J of that cluster in the Milky Way's potential? Assume a flat rotation curve of the Galaxy ($v_c = 220$ km/s) and a galactocentric distance of $R = 8$ kpc. How do r_J and D compare and what would happen if the relation were reversed?

SOLUTION

Physical diameter of the cluster is equal $D = (1.75 \cdot \pi/180) * 120$ pc, so the radius equals $R_c = D/2 = 1.83$ pc.

To compute the Jacobi radius, we need the mass ratio between the cluster and the Milky Way inward of the position of the cluster (which is near the sun's position). We are going to assume that the circular velocity can is given by $V_c^2 = GM(R_\odot)/R$ (where $R = 8$ kpc is the distance to the Galactic center) with sufficient precision for our calculation (in fact, the flatness of our Galaxy does not allow us to use the inherent spherical symmetry assumption, but we do know that we get V_c accurate to about 10% from this simple formula; besides, the outer parts of flat rotation curves are due to roughly spherical dark matter halos..).

$$r_J = R(m/2M)^{1/3}$$

, where $m = 300M_\odot$ and $M = M(R) = V_c^2 R_\odot$ is the mass of the Galaxy inside the solar position and $V_c = 220$ km/s. We get $r_J = (Gm/(2RV_c^2))^{1/3}R$. We can either plug in the constant G and crunch the numbers or rewrite this equation using known orbital speed of the Earth around the sun: $V_E^2 = GM_\odot/r_{AU}$, where $V_E = 30$ km/s and $r_{AU} = (1/206265)$ pc is the astronomical unit. The second apprach gives less chance for confusion about units, since it eliminates the physical constants like G:

$$r_J = R[(m/2M_\odot)(V_E/V_c)^2(r_{AU}/R)]^{1/3}$$

which evaluates to $0.0012\,R$, or $r_J = 9.5$ pc. Thus, the tidal radius is larger than the radius of the cluster (or cluster core) by a factor $\sim$5, which assures the stability of the cluster against a rapid destruction by galactic tides.

2 [5p.] Tidal danger

Tidal forces around black holes can be destructive. The closest and the furthest points of any object are at different distances from the center of the black hole, and thus have different gravitational accelerations (whether or not the body as a whole orbits the black hole or is falling into it). Estimate this difference of gravity for a body of size $L = 1.6$ m, which is a meant to approximate the length scale of a human body, at the distance of one Schwarzschild radius from the black hole. Express the result in units of Earth's gravity "g" ($g = GM_E/R_E^2$).

If you fall into a black hole, you pass through the event horizon at the Schwarzschild radius. Tidal forces would kill you (and, by the way, also destroy your spaceship) if the tidal differential force over the lengthscale L equals 40g or more (a typical accelerations pulling objects apart during a fatal car accident).

Calculate the mass of a black hole producing lethal tidal forces. What is safer: a flight into a black hole more or less massive than your derived mass? Recently, astronmers found that globular cluster Omega Cen harbors a medium-mass black hole (http://newswise.com/articles/view/539256/). According to your calculations, would you its tidal force at the event horizon?

SOLUTION

Has been discussed in tutorial.

3 [3.5p.] Describe differences

Describe briefly the main differences between dwarf ellipticals, dwarf spheroidals, and dwarf irregulars (Ch.4 of textbook). You may do it in a table with 3 columns for 3 types of galaxies and several rows for any physical quantities you want to discuss (e.g., color, size, density, number of stars).

4 [3p.] Surface brightness in a disk galaxy NGC540

Galaxy NGC540 has an exponential disk dominating the visible light image. Surface brightness is given by

$$I(R) = I_0 \exp(-R/R_d)$$

where I_0 is the central surface brightness, R distance from the center, and $R_d = 4$ kpc the exponential radial scale of the disk (e-folding distance) The total luminosity of the galaxy equals $L = 4 \cdot 10^{10} L_\odot$. Considering that the total luminosity is surface brightness $I(R)$ integrated over the *area* of the whole disk from $R = 0$ to $R = \infty$, compute I_0 (in units of $L_\odot/\mathrm{pc}^2$). What is the surface brightness at radius 8 kpc, $I(8kpc)$? Compare that value with the solar neighborhood surface brightness. Assuming a standard disk light-to-mass ratio $\Upsilon = 4M_\odot/L_\odot$ (there was a misprint in the original formulation of the problem!!) appropriate to many disk galaxies, convert your answer into the surface density of disk stars in units of $L_\odot/M_\odot$ and compare with your knowledge of the solar neighborhood of our Milky Way.

SOLUTION

$$L = 2\pi \int_0^\infty I(R)\, R\, dR = 2\pi I_0 R_d^2 \int_0^\infty x e^{-x}\, dx = 2\pi I_0 R_d^2\,.$$

$$I_0 = L/(2\pi R_d^2) = 4\cdot 10^{10}/(2\pi 4000^2)\, L_\odot/\mathrm{pc}^2 \simeq 400\, L_\odot/\mathrm{pc}^2.$$

At 8 kpc the surface brightness amounts to $400\, e^{-8/4} = 54 L_\odot/\mathrm{pc}^2$. If $\Upsilon = 4M_\odot/L_\odot$ then $\Sigma(R = 8\mathrm{kpc}) = 216 M_\odot/\mathrm{pc}^2$. This is more than twice as large as the Milky Way's surface density.

5 [3p.] Dark matter in a disk galaxy NGC540

Using the surface density law

$$\Sigma(R) = \Upsilon\, I_0 \exp(-R/R_d)$$

suggested by the previous problem, and accepting that the formula $V_c^2(R) \approx GM(R)/R$, where $M(R)$ is mass inside cylinder of radius R, is accurate to within 15% (enough for this problem, but not always satisfactory!), please compute the rotation curve of NGC540 (galaxy described in the previous problem) and compare it with the observed rotation curve, which looks as follows: it rises from zero to about 205 km/s (at $R \sim R_d$) and becomes flat thereafter. Can you sketch how the amount of dark matter, expressed as a percentage of the total density, changes with radius?

SOLUTION

Υ used above refers to the stellar, luminous disk. Using it we can quantify our predicted velocity curve (V_c) for that disk. Comparing the computed and observed squared velocity values, we will find the percentage of the dark matter as a function of radius.

$$V_c^2(R) \approx GM(R)/R = (2\pi G\Upsilon/R) \int_0^R I(R)\, R\, dR.$$

$$V_c^2(R) \approx (2\pi G\Upsilon I_0 R_d^2/R) \int_0^{R/R_d} x \exp(-x)\, dx$$

Let's introduce a constant $V_*^2 = 2\pi G\Upsilon I_0 R_d$; then

$$V_c^2(R) \approx V_*^2 (R_d/R)(1 - e^{-R/R_d}(1 + R/R_d)).$$

Near $R \approx 0$, we have $V_c \sim V_*(R/R_d)$ (we get that when we approximate $\exp(-x)$ by $(1-x)$ and do the rest of the algebra dropping small x^2 term, where $x = R/R_d$.) This is a rising velocity curve part that is consistent with observed increase, so in the inner part of the system dark matter is non-existent or present in the amount proportional to the luminous matter.

However, at large distances we see that $V_c^2 \sim V_*(R_d/R)$, because $(1+x)\exp(-x) \to 0$ for $x \to 0$. The luminous disk produces a falling contribution to the observed rotation curve, which means the dark matter increases in its contribution, eventually totally dominating the rotation curve. The quantitative comparison of luminous and dark matter is left to the reader.

6 [1p.] The angular resolution of the MSAI

U of T considers building a hypothetical Maximum Size Antarctic Interferometer located in (you guessed it!) Antarctica.

What angular resolution at the wavelength of 21 cm could it theretically achieve? Express the answer in two ways: arcseconds, and equivalent size of a body (an object) on the Moon.

SOLUTION

$$\theta \approx \lambda/D$$

where λ is the wavelength (21 cm) and D the distance between the unit antennas from which the interferometer consists. The best resolving power results from maximum D.

Antarctica has parts reaching up to almost 65 degree south latitude.

http://www.gdargaud.net/Antarctica/MapSatellite/AntarcticStationsMap.gif

Each degree on a great circle counts for exactly 60 nautical miles (one arcminute = 1 nm), so the furthest points on Antarctica are 2*(90-65)*60 nm apart (3000 nm). $1nm = 1.85$ km, so $D \approx 5550$ km and $\theta \approx$ 3.78e-8. The resolution would be 0.0078 arcsec or a 15m-size object on the Moon.