Fall 2008: WELCOME TO A231 EXAM 1

Do you best to answer each question. You may use calculators and any course materials other than classmates.

Question 1 (40 points):
A group of AST231 students is ready to take an exam. The professor gives permission for students to take any world line they choose during the exam but they can work on the exam only during a time interval $T_0 = 2$ hours as measured on the room clock by “student 0” who stays in the classroom for the entire exam. For each of the students below calculate the the amount of time each student would have for the exam relative to $T_0$. Note that $GM_e/R_e^2 \simeq 2 \times 10^{-10}$ where $M_e = 6 \times 10^{27}$g and $R_e = 6.4 \times 10^8$cm are the mass and radius of Earth respectively.

(a) Student 1: takes rocket up with velocity $V_{u1} = c/3$ away from Earth and returns with velocity $V_{d1} = -2c/3$. (Assume that the accelerations and decelerations at the leaving, turning, and return points are instantaneous).

(b) Student 2: takes the exam in slow elevator that moves up at constant velocity $V_{u2} = 10^{-10}c$ and returns at velocity $V_{d2} = -V_{u2}$. (Assume that the accelerations at the leaving, turning, and return points are instantaneous.)

(c) Student 3: sits on a platform (along with an oxygen tank) at height $H = 10^4$ meters above the main classroom to take the exam.

(d) Which student (0,1,2,3) has taken the world line that allowed her/him the most time to complete the exam? (this can be determined even without completing parts a,b,c)

Question 2 (20 points):
The fission (breaking apart) of the element uranium results in particles whose total rest mass is 99.9% of the original rest mass of the initial rest mass. If all the kinetic energy produced by fissioning 500g of uranium could be converted into electrical power and sold by a power company for 20 cents per kilowatt-hour what would the total cost be for all the power produced by this amount of uranium?

Question 3 (20 points):
A star is moving forward with speed $v > 0$ along the +x-axis and Lorentz factor $\gamma$, as measured by a stationary laboratory observer in Minkowski space.
The star emits isotropically in its rest frame. Derive the solid angle measured in the lab frame into which photons from the star’s entire forward hemisphere are beamed for $\gamma >> 1$.

Question 4 (20 points):
Consider two four-vectors in Minkowski space with components $J^\mu = (1, 1, 2, 0)$ and $S^\mu = (1, 1, 0, 0)$.

(a) Determine whether $J$ and $S$ are null, spacelike, or time like (show your choice of metric).

(b) Calculate the scalar product $J \cdot S$

(c) If you transform both 4-vectors to a new frame moving with speed $v$ along the $+x$ axis, what is the scalar product of these vectors in the new frame?