The usual space-time diagram for resolving the twin paradox is this:

In the diagram, the diagonal hatches are lines of simultaneity for each part of Alice's trip in her coordinates. (Alice moves with velocity \( +v \) out and \( -v \) back.)

If Alice travels for 10 years on her clock (\( t_{pA} = 10 \text{ yr} \)), her clock ticks away from Bob and returns with same speed, the diagram above applies when Bob's axes are taken as the rest axes. Note that the turnaround point \( Q \) for Alice is simultaneous with the halfway point as measured by Bob. On Alice's return trip, her axes rotate. Alice's world line is along her time axis \( t' \) as she is at rest in her frame. If Alice travels 10 yr out then she measures her age to be 20 yr. During each half of Alice's trip Bob ages \( t_{ps} = t_{pA} = \gamma (t_{pA} + \sqrt{\gamma} x_{pA}) = \gamma t_{pA} = 10 \gamma \)

So Bob ages a total of 20 yr.
The spacetime distance between spacetime points is invariant so. This result can also be derived by considering the spacetime separation between P and Q:

\[ S_{pq} = -c^2 t'_{pq} = -c^2 t_{pq} + x_{pq} \quad (A) \]

Alice\'s coords

\[ x_{pq} = \gamma (x'_{pq} + v t'_{pq}) \]

\[ x_{pq} = \gamma v t'_{pq} \]

plug into \#:

\[ t'_{pq} = t_{pq} - \frac{\gamma v^2 t_{pq}}{c^2} \]

\[ t'_{pq} = \frac{t_{pq}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_{pq}}{(1 - \frac{v^2}{c^2})^{\frac{1}{2}}} \]

so again

\[ t'_{pq} = \frac{t_{pq}}{\gamma} = \text{Alice age} \]

Bob is older by factor of \( \gamma \).

Now: we can see the same result from point 16 view of rest frame of Alice: \( \rightarrow \)
Calculation with Alice's coordinates perpendicular on page.
Now Bob is moving.

Q₁ and Q₂ are at same t' but cannot draw them at same location because lines of simultaneity rotate when Bob turns around and because lines SQ₁ or SQ₂ are not lines of simultaneity in Alice's coords.

\[ x_{pQ_1} = 0 \]
\[ t'_{pQ_1} = 10 \]
\[ t_{pQ_1} = t_{ps} = \gamma (t'_{pq} + \frac{Vx_{pQ_1}}{c^2}) = \gamma t'_{pq} \]
\[ \Rightarrow t_{ps} = \gamma t'_{pq} = \gamma 10 \text{yr} \]

Total age of Bob = \( \gamma \times 10 \text{yr} \)
= 20 \( \gamma \) years.
Same as approach when Bob's world line is drawn vertically.
so note that we get the same result independent of whether we draw a spacetime diagram with Bob's rest frame as or orthogonal axes, or Alice's axes as the orthogonal axes. The key point is that we state Alice travels for 10 years as measured on her clock and then turns around. So Alice's proper time of 10 years determines the turnaround point in Bob's frame. Alice just measures two trips of 10 years each. During each trip Bob measures the time to be 5/10. The awkwardness of the diagram in Alice's frame highlights the simplicity of presenting the diagram in a frame where Bob's axes are orthogonal because the turnaround point is simultaneous with the 1/2 way point on his world line in such a frame.