Angular momentum:

Lessons from stellar binary systems

- Binaries, (in particular X-ray binaries) are where we have learned a lot about accretion: why binaries?

- Orbiting system, tidal forces
  \( \Rightarrow \) Shearing of material into a disk; to accrete, 4 momentum must be shed (or equivalently, transported outward)

Two reasons for binary mass transfer via accretion:

1. One of the stars may increase in size during evolution; companion can rip off outer layers

2. Ejection of mass by stellar wind

Important concept is Roche lobe overflow
19th century Edouard Roche studied destruction of planetary satellites (moons, etc.)

Basic idea was to consider orbit of test particle in grav potential of two orbiting, massive bodies

Assume two stars orbit each other in keplerian, circular orbits, and consider test particle gas motion in the potential (also called "restricted 3-body problem", because gas is assumed not to influence the binary orbit)

\[ \begin{align*}
&\text{Gas flow between stars governed by Euler equation. In rotating frame:} \\
&\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\nabla \phi_R - 2\vec{\omega} \times \vec{V} - \frac{1}{\rho} \vec{F} \\
&\text{Grav + Cent. force} \\
&\frac{\text{Coriolis force}}{\text{mass}}
\end{align*} \]
From Kepler's law, \( \vec{\omega} = \omega \hat{z} \):

\[
\vec{\omega} = \left[ \frac{G (M_1 + M_2)}{a^3} \right]^{1/2} \hat{z}
\]

normal to the orbit plane, \( a \) = binary separation

\[
\phi_r = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{1}{2} (\vec{\omega} \times \vec{r})^2
\]

Roche potential.

Set \( \phi_r \) = constant and plot:

L4, L5 maxima, (but coriolis force stabilizes)
L1, saddle

material

Can overflow if e.g. \( M_3 \) fills lobe, it can accrete onto \( M_1 \)

Note: distance

\[
d_{L1-M1} = \frac{1}{2} a - 0.23 a \log \frac{M_2}{M_1}
\]
agine evolution:

1. Assume both stars are smaller than Roche lobe and are in circular orbit, and tidally locked

   \[ \Rightarrow \text{surface of each star corresponds to circular equipotential surface. (follows from momentum equation with } \dot{V} = 0 \text{ and } \nabla p = 0 \text{ on surface of star)} \]

2. Binary is fully detached

3. If one star then swells & fills Roche lobe (usually called secondary star) then primary can accrete

   This is a semi-detached binary

3. Can you guess what a contact binary is?

   (Both stars fill Roche lobes)
Formation of Disk in Binary

Note that material is pushed through Roche lobe at \( \sim C_S \) but because of orbit, \( V_{\|} \ll V_\perp \) for typical systems.

\[
M_1 \quad \downarrow \quad V_{\|} \quad \rightarrow \quad V_\perp
\]

\( \rightarrow \) remember, previous equations are in rotating frame about c.o.m; as soon as material edges over \( L_1 \), \( M_1 \) sees it in orbit.

\[
V_{\|} \sim C_S \quad \text{and} \quad V_\perp \approx V_K
\]

typically \( V_{\|} \ll V_\perp \), \[
\frac{V_\perp}{V_{\|}} \approx 10^2 M_1 / (1 + M_2 / M_1) \text{ km/s}
\]

\[
\rightarrow \quad \text{gas has} \quad \times \text{momentum which it needs to shed to accrete}
\]

\[
\Rightarrow
\]
Gas will first orbit in circle at
at \( V_\phi(R_{circ}) = \left(\frac{G M_1}{R_{circ}}\right)^{1/2} \)
with \( V_\phi(R_{circ}) R_{circ} = d_{L1-M1}^2 \) orbit velocity
momentum conservation

Using kepler: \( \frac{2}{3} \pi a^3 = G (M_1 + M_2) \frac{p^2}{a} \)
and \( p = \frac{2\pi}{\omega} \) and \( V_\phi(R_{circ}) = \left(\frac{G M_1}{R_{circ}}\right)^{1/2} \)

and formula for \( d_{L1-M1} \) on page 134:

\[ R_{circ} / a = \left(\frac{M_2}{G M_1 p^2}\right) a^3 \left(d_{L1-M1} / a\right)^2 \]

\[ = R_{circ} / a = (1 + \frac{M_2}{M_1}) \left(\frac{1}{5} - 0.23 \log \frac{M_2}{10 M_1}\right) \]

(it is possible to have \( R_{circ} < R_{SB} \), but never true for NS, BH, or WD systems)

So we have gas orbiting at
\( R = R_{circ} \) but that's not the end of the story!
Internal dissipation will lead to radiation $\rightarrow$ loss of radiation $\rightarrow$ loss of kinetic energy $\rightarrow$ material sinks deeper into grav. potential $\rightarrow$ accretion $\rightarrow$ loss of x momentum.

Now $t_{\text{cool}}$ (cooling time) is usually $<< t_{\text{acc}}$ and $t_{\text{dyn}}$ $<< t_{\text{acc}}$ so that material spirals in slowly.

but if material loses x momentum what carries it? some material actually goes outward, so "initial ring" turns into disk.

usually for compact objects disk is not self gravitating ($g << \frac{M}{R^3}$)

$\Rightarrow \mathcal{L} = \mathcal{L}_K = \left( \frac{G M_1}{R^3} \right)^{1/2}$ keplerian orbits
kinetic energy of gas element $\Delta m$ in keplerian orbit is

$$\int \frac{G M \Delta m}{R^*} \Rightarrow \text{luminosity lost during accretion is}$$

$$L_{\text{disc}} = \int \frac{G M \Delta m}{R^*}, \quad \text{but grav pot energy is} \frac{G M \Delta m}{R^*}, \text{so } \frac{1}{2} \text{is radiated or dissipated in disk, other } \frac{1}{2} \text{released at surface of star}$$

compare fraction to $\mathcal{A}$ momentum: $R^2 \mathcal{A}(R) \propto R^{1/2}$

now since $R_{\text{circ}} \gg R^*$ in general

nearly all $\mathcal{A}$ momentum must be lost. $\Rightarrow$ dissipation processes which cause conversion of kinetic energy to heat must also transport $\mathcal{A}$ momentum