More on the onset of core collapse (Type II SN)

Call that from H burning ($T_{\text{ign, H}} = 2 \times 10^7 \, \text{K}$)

successive burning in the cores of massive stars proceeds through $\alpha$-particles (He, $^{12}$C, $^{16}$O, and $^{28}$Si ($T_{\text{ign, He}} = 3 \times 10^9 \, \text{K}$) terminating at $^{56}$Fe (nucleus with maximum binding energy per nucleon). Most all 1-0 cases show that the cores reach mass $M_c = 1.5 \, M_\odot$ for initial stellar masses $8 M_\odot \leq M \leq 20 M_\odot$.

Arnett (1979) offered a nice explanation for the tendency of all the models to converge in this final state:

In the late stages of evolution, 'core is'...core is relatively uniform. Now consider the equation of state, which is dominated by electrons:

\[
\frac{P}{\rho} = \frac{Y_e Y_t \Gamma}{M_B} + K_F Y_e \Gamma^{-1} \quad (19)
\]

$M_B = M_p$

$Y_e = \frac{\text{# electrons}}{\text{baryon}} = \frac{N_e}{N_B} = \frac{1}{M_e}$

$K_F = \text{constant, depends on polytropic index } \Gamma$, which varies from \( \frac{4}{3} \) (relativistic) to \( \frac{5}{3} \) (non-relativistic).
For spherical configuration of mass $M$ and radius $R$, equation of hydrostatic equilibrium ($\nabla \Phi = -\nabla P$) implies roughly

$$\frac{P_c}{\rho_c} = \frac{C-M}{R} \leq f GM^{2/3} \rho_c^{1/3}$$

where "c" indicates central values and $f = f(R)$ is a numerical factor. Using $\theta^2 = \frac{\rho}{\rho_c}$ in (19) and combining with (20) gives:

$$\frac{T_c}{k_e} = \frac{f GM^{2/3} \rho_c^{1/3}}{\kappa_T Y_e^\frac{5}{2}}$$

Now consider the maximum $T_c$ a given configuration can achieve. For large $M$, first term on right of (21) dominates, and then $T_c \propto \rho_c^{1/3}$. Thus continued contraction would lead to higher temps, and "any" fuel will eventually ignite. However, for small $M$ and $\Gamma > \frac{4}{3}$, $T_c \to 0$ from (21) when

$$s \to \text{Scrit} = \left( \frac{f GM^{2/3}}{\kappa_T Y_e} \right)^{1/3}$$

$s$, when $s < \text{Scrit}$, we have $T_c \propto \rho_c^{1/3} M^{3/2}$ and when $s \to \text{Scrit}$ we have $T_c \to 0$, and $s > \text{Scrit}$ is unphysical.
This implies that the core actually passes through a maximum $T_c$ during contraction, and then cools as it approaches the final state when degenerate electrons dominate the pressure.

Roughly, the mass marking the transition is between the $T_c \propto \rho c^2 M^{2/3}$ dependence and $T_c \rightarrow 0$ can be estimated by the Chandrasekhar mass, for which $T_c=0, \Gamma=3/4$. Using (21), we then have:

$$M_{\text{crit}} = \left( \frac{K \rho_{13}}{f G} \right)^{3/2} Y_e^2 \approx 5.8 Y_e^2 M_0, \quad (23)$$

$5.8 \approx$ for $n=3$ polytrope

where $p \approx \rho + n \ln$.

Silicon burning, which is the final core burning stage, requires very high temps $kT \gtrsim 0.6m_e c^2$, and thus one expects the cores to approach the limiting mass near the onset of Si burning. If the core mass $M_c < M_{\text{crit}}$, then system must sit until more ash from pre-Si burning phase is dumped into core for Si burning to start. Thus condition for Si burning go to completion.
in core is \[ M_{\text{core}} > M_{\text{Si, Ig}} \geq M_{\text{Si, Ig}} < 1.2 M_\odot \] (Note that very low mass stars never get there).

However, note also that if Moore were much larger than \( M_{\text{Si, Ig}} \) at some earlier phase of core evolution, then contraction would rapidly proceed to high central temperatures and between burning stages neutrino energy release would reduce the entropy gradient, reducing convection, and thus reducing mixing, resulting in the now central core being a smaller region of the initial core, and a smaller mass. In this way, the system evolves so that the core is driven toward a mass \( M_{\text{enr}} \) prior to Si burning. (confirmed by simulations)

The ash from Si burning is primarily \( ^{56}\text{Fe}, ^{58}\text{Fe}, ^{60}\text{Fe}, ^{63}\text{Ni} \).
In short, an $8M_\odot \leq M \leq 20M_\odot$ will have a core mass just slightly larger than the Merit of $1.2M_\odot$ (typically simulations show $M_{core} \approx 1.5M_\odot$) before the core becomes neutron degenerate and collapses ensues. More specifically, just before the onset of collapse:

\[ T_c = 8 \times 10^9 \text{K}, \quad \rho_c = 3.7 \times 10^{12} \text{g/cm}^3 \]
\[ Y_e \approx 0.42, \quad M_{core} = 1.5M_\odot. \]

The dissociation of Fe and neutronization of the core which subsequently ensue are endothermic, as discussed above, and lead to collapse.

**Neutrino luminosity**

As the opacity increases during core collapse, the neutrinos find escape more and more difficult. At densities above

\[ \rho_{trap} = 3 \times 10^{11} \text{g/cm}^3 \]

neutrinos are trapped; they co-move with the
matter and actually build up a degenerate sea. At $s \approx \text{Strap}$, the scale for neutrinos to diffuse from core becomes comparable to collapse time. If we use this to define $\text{Strap}$, we can see how it is derived:

Hydrodynamical collapse timescale is of order the free-fall time scale:

$$t_{101} = \frac{1}{(G\cdot g)^{1/2}} = 4 \times 10^{-3} \sqrt{s_{12}} \text{ sec}$$  \hspace{1cm} (58)

where $s = \frac{4}{3} \pi R^3 M_c$ is the mean density $s_{12} \equiv \left(\frac{9}{10}\text{ g cm}^{-3}\right)$ of the collapsing core, with $M_c \approx 0.5 M_\odot$ as above.

The neutrino diffusion time scale can be estimated by assuming that coherent scattering is the dominant opacity source:

$$t_{\text{diff}} = \frac{\lambda_{\text{coh}} N_{\text{sc}}}{c}$$  \hspace{1cm} (26)

where $\lambda_{\text{coh}}$ is the mean free path of a neutrino amidst heavy nuclei from coherent scattering; $N_{\text{sc}} \gg 1$ is # of scatterings before escape. (e.g. $\nu + (Z,A) \rightarrow \nu + (Z,A)$)
Coherent scattering induces a random walk trajectory for the neutrino, but without changing its energy. We then have, for a random walk,

\[ l_A^{coh} \sim R \text{ or } N_{sc} \]

which is the condition for escape, similar to protons incurring elastic Thomson scattering. The mean free path can be estimated from the cross section for coherent scattering \( \sigma_A^{coh} \):

\[ \frac{1}{l_A^{coh}} = N_A \sigma_A^{coh} = \left( \frac{\rho}{A m_{1B}} \right) \sigma_A^{coh} \]

Nuclear physics calculations give

\( \sigma_A^{coh} = \sigma_A^{coh} (E_V) = 10^{-45} \left( \frac{E_V}{m_{e \text{eV}}} \right)^2 \text{ cm}^2 \) for characteristic

\( E_V = 33 \, \text{MeV} \) for electron capture onto protons.

Then, correct numerical constants in \( \sigma_A^{coh} \) are included for A = 56:

\[ (l_A^{coh})^{-1} = 4 \times 10^{-5} \text{ cm}^{-1} \, \text{ for A = 56} \]
Using (29), (26) and (27), we have:

\[ t_{\text{diff}} = \frac{\lambda_{\text{coh}} N_{\text{sc}}}{C} = \frac{R^2}{\lambda_{\text{coh}} C} = \frac{R^2}{C} \times 4 \times 10^{-5} \, s_{12} \, \text{sec} \]

since \( R^2 = \left( \frac{M}{4/3 \pi \rho} \right)^{2/3} \), we get:

\[ t_{\text{diff}} = \left( \frac{2 \times 10^{33}}{4/3 \pi (10^{12})^{2/3}} \right) \left( \frac{4 \times 10^{-5}}{C} \right) \left( \frac{M}{M_{\odot}} \right)^{2/3} \, s_{12} \]

\[ = 0.1 \, s_{12} \, \text{sec} \]  \( (30) \)

Eqn (30) and (25) show that

\[ t_{\text{diff}} = t_{\text{ff}} \text{ when} \]

\[ g = g_{\text{trap}} = 1.4 \times 10^{11} \text{ g/cm}^3 \]  \( (31) \)

Thus, as advertised we have derived the critical density for neutrino trapping!

This result (31) is important because neutrino luminosities are greatly reduced from trapping: Once center of core reaches \( 3 \times 10^{14} \text{ g/cm}^3 \), neutron degeneracy prevails.
Further collapse and most of the core luminosity is emitted via neutrinos. In the absence of trapping, the total binding energy released in the core (which is most of its gravitational energy) would be released on a collapse time scale (e.g. a free fall time at $R_{\text{cc}} = 12$ km for $M = M_{\odot}$). This would produce a neutrino luminosity of $L_{\nu, \text{max}} = \frac{G M^2}{R_{\text{cc}}} = 10^{57} \text{ erg/sec}$ \( \text{(2)} \) from \( \text{(25)} \)

Instead, because neutrinos escape only on a time scale $t_{\text{dif}} = 0.1 s_{12}$ sec from \( \text{(30)} \), the actual neutrino luminosity is

$L_{\nu, \text{act}} = \frac{G M^2}{R_{\nu\nu} c} = 10^{53} \text{ erg/s}$ \( \text{(33)} \)

$\Rightarrow$ most of gravitational binding energy is not immediately released as neutrinos, but first into heating and nuclear excitation, and core bounce kinetic energy. The late stages of core collapse proceed adiabatically. But can neutrinos ultimately drive the explosion?
More on understanding Difficulty of Driving Type II SN with Neutrons

- When core density reaches several times $\rho_{\text{nu}} = 3 \times 10^{14} \text{g/cm}^3$ core is stiff enough to halt the collapse.
  (As stated earlier, the core would be neutrinized iron at this stage.)
- Outer collapsing core is in free fall and upon crashing into the stiff core can rebound somewhat.
  Thus rebounding material acts as a piston that drives a shock into the layers above.
- The outward-moving material carries off of order $10^{51}$ erg, some of which is thermalized at the shock, via neutrino absorption and nuclear dissociation (fission).
- $10^{53}$ erg released directly as neutrons.
  The randomization of the bulk motion, or possibly the draw of binding energy into quark-gluon plasma, could be able to drive the SN explosion. The neutrinos could act as radiation pressure to drive off the outer layers of the star. But can they?
- To understand this, let us first look at how radiation pressure works for plasmas
Eddington luminosity for photons

Consider the force exerted on ionized plasma via radiation. Assume that the dominant opacity is Thompson scattering (photon scattering off of free electrons) with cross section \( \sigma_T \approx 6.6 \times 10^{-25} \text{ cm}^2 \).

Force must involve the cross section because the latter measures how effectively the matter and radiation couple.

A photon of momentum \( p \) deposits, on average, a momentum \( p \) to electron per scattering. Energy of photon is \( pc \), so if all the photons are moving radially in a spherical system, the number of photons crossing a unit area per time is

\[
\text{Flux} \rightarrow F = \frac{L}{4\pi r^2 c} = \frac{L}{4\pi r^2 pc}
\]

The number of scatterings per electron \( @ \) per time is determined by multiplying by the cross section

\[
\Rightarrow N = \frac{L\sigma_T}{4\pi r^2 pc}
\]
the Force per electron is the rate at which momentum is deposited per unit time, since each photon transfers momentum \( p \), the force is then

\[
f = Np = \frac{L\sigma_{\text{e}}}{4\pi r^2 c}
\]

(36)

Now if this force is competing against gravity, we note that both (36) and grav force vary as \( 1/r^2 \). Because electrons are coupled by Coulomb collisions to protons for dense enough systems, the gravitational force on electrons is communicated through the protons. Thus force balance is

\[
\frac{L\sigma_{\text{e}}}{4\pi r^2 c} = \frac{GMmp}{r^2}
\]

(37)

\( \Rightarrow \)

\[
L_{\text{crit}} = \frac{4\pi cGGMmp}{\sigma_{\text{e}}} = 1.3 \times 10^{38} \left( \frac{M_i}{M_{\odot}} \right) \text{ erg} \text{ s}^{-1}
\]

(38)

for \( L \geq L_{\text{crit}} \), radiation force can blow off outer layers of object.

Eq (38) is the standard Eddington luminosity note its dependence on the mass and \( \sigma_{\text{e}} \).
Now since neutrinos are also relativistic ($E_\nu = 25\,\text{MeV}$) in environments of neutronized stellar cores we can, by direct analogy to the photon case, derive an "Eddington neutrino luminosity"

\[
L_{\text{edd}, \nu} = \frac{4\pi 6\,\text{MeV}}{\kappa_\nu} \tag{39}
\]

where $\kappa_\nu = \frac{\sigma_{\text{coh}}}{A_{\text{M}_\nu}}$

$\kappa_\nu$ is the dominant neutrino opacity (which has units of cross section per mass, and here $A_{\text{M}_\nu}$ plays the role of $M_\odot$ in (38))

* atomic mass unit

Assuming that opacity in the mantle above the outward propagating shock is dominated by coherent scattering with neutrinos, from the equation below, and using $A = 56$ ($Z = 26$)

\[
\kappa_\nu = 2 \times 10^{-17}\,\text{cm}^2/\text{g}
\]

Using this in (39) =

\[
L_{\text{edd}, \nu} = 2 \times 10^{54}\,\text{erg/sec} \left(\frac{M}{M_\odot}\right) \tag{40}
\]

But this is larger than eqn (33)!
From the comparison, one can see that even if one could increase \((33)\) by an order of magnitude and reduce \((41)\) by an order of magnitude by changing assumptions, the values would still be close and given the complexity of the problem the debate rages on! What assumptions might one change?

- The calculation above assumes spherical symmetry. If the infall of material were not spherically symmetric, the neutrino luminosity could be larger in some directions if the $E^2$ term in \((33)\) could be made shorter, and the mass required to be ejected along that direction reduced. (See also discussion on page 45)
- Radiation pressure could help to remove some of the pre-existing envelope, reducing the burden on neutrinos.
- Neutrino driven convection can reduce $E^2$ in \((31)\)
- Change the neutrino spectrum to take advantage of $\sigma_\nu \approx E^2$ (e.g. Ramirez-Ruiz & Socrates 2004, 2005)
- Rotation: if one allows rotation one has natural asymmetries, additional energy sources from both differential rotation and total rotation. $\Rightarrow$ mag. fields.

In general, core collapse requires understanding neutrino transport and magnetic fields.