Virial Evolution of Molecular Clouds

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Papers: FBK09,FBK11, FKB15, BFK15+ (in prog)

Orion GMC, Four frame mosaic, hydrogen alpha color composite FSQ106, STL11000, Total Exposure 50 Hours, Digitized Sky Survey; R. Gendler, R. Colombari, F Pelliccia, DSS; Image acquired at the Nighthawk Observatory

Molecular Clouds

- Relevant SFR/MC reviews (McKee & Ostriker 2007 ARAA; Dobbs et al. 2014 P; Krumholz 2014)
- Stars form in molecular clouds; about half Galactic gas in MC, lowest scale height of ISM gas, total mass~ 2 x $10^9 M_{sun}$
- Clouds require dust and FUV shielding to form H_2 and CO, and typically surrounded by layer of HI, (PDR)
- Clouds with supersonic velocity dispersions σ range in scales from 100pc (~10^7 M_{sun}) to 0.5 pc (~$100M_{sun}$);
- Giant Molecular Cloud (GMC) used to describe clouds > 10^4 M_{sun}, though not exact definition, and key feature for present discussion, is MC is $\sigma > c_s$ within factor of 5 of clump scale (Myers 1986; Quillen et al 05)
- Formation of GMC: largest clouds seem to need top down (e.g. Kim & Ostriker 02,06, Shetty and Ostriker 06; Dobbs et al 12); collisional formation also possible but makes intermediate clouds
- Within clouds, there is substructure down to small scales; clouds of given scale are part of a cloud complex,fractal like. ^{12}CO, ^{13}CO, used to probe
- If molecular clouds were self-gravitating and unfettered by kinetic support, star formation rate might be total mass in the largest molecular clouds divided by their free fall times. This gives a number ~100 times too large for the Galaxy (e.g. McKee & Ostriker 07; Krumholz & McKee 05; Krumholtz & Tan 07 and earlier)
- Either
    - clouds are sustained much longer than their free fall times by factor of 100 (not likely, not observed)
    - **clouds are intermittently catastrophically destroyed in bursts like SN, high mass stars : (likely)**
    - **and/or partial abatement occurs on characteristic scales (e.g. feedback from YSOs; mass dispersion on subparsec/parsec scales.)**
- Distinguish 1) abatement of SFR by feedback, 2) overall velocity dispersions in GMC on large scales 3) velocity dispersions on sub clump, or scales where outflows operate

Turbulent Driving vs. Fragmentation for Velocity Dispersion

- External driving, Internal Driving (rev. Dekel & Krumholz 07), or Fragmentation "driving"

- Large scale driving of supersonic turb (MacLow & Klessen 04; Federath & Klessen, Galtier & Banerjee 2011,Padoan et al. 2014)

- Federrath 2013 (very high res) scaling $\sigma \alpha R^{1/2}$ ($P(\sigma) \alpha k^{-2}$ Burgers) but $P(\rho^{1/3}\sigma) \alpha k^{-1.74}$ (solenoidal) $P(\rho^{1/3}\sigma) \alpha k^{-19/9}$ (compressive) whereas obs: $P(\rho^{1/3}\sigma) \alpha k^{-4/3}$ (need self gravity, cooling); momentum spectra don't quite scale as observed without self-gravity, or even with..

- Spectra depends on forcing e.g. outflows (Cunnngham et al. 06; Nakamura and Li 07; Wang et all 08) outflows vs. isotrropic Carroll et al. 2010, more later);

- ouflows and HII regions may abate SFR on sub GMC scales, but probably can't supply overall MC turbulence on GMC scales (more later)

- Fragmentation models (Hoyle 53; Liszt et al. 1974; Goldreich & Kwan 74; Vascquez-Semadeni et al 07; Ballesteros-Paredes 11; FBK 08, 11; FKB, BFK 15) One way or another velocity dispersions coming fully or largely from gravitational collapse
  - FBK 08, 11; FKB, BFK 15 **mass conserving** fragmentation cascade, **external pressure relevant**
  - Vasquez-Semandini et al. 2007; Ballesteros-Paredes et al. 2011: collisions lead to turbulence, cooling self-gravity, fragmentation—velocity dispersions perhaps ambiguous between infall and turbulence but result in equipartition between kinetic and gravitational energy. **Energy conserving cascade, external pressure irrelevant**

Key Observed Properties (Pre 2009)

- Larson 81; Solomon et al. 87, Heyer-Brunt 2004:
- size line-width relation $\sigma \propto R^{p_1}$
- density radius relation $\rho \propto R^{p_2}$
- Larson's "Laws" (1981): ,
  - p_1=0.38 p_2= 1.1
- Solomon 1987 (273 MC); p_1=0.5
- Heyer & Brunt (2004) p_1=0.6+/- 0.007 (24 MC)
- $V_0^2 = \sigma^2/R^{p_1}$ seemed consistent with with (1) constant p1 and (2) "simple viral equilibrium (SVE)"
- smaller clouds less consistent with SVE; HLC (Keto & Myers 1986; Bertoldi and McKee 1992 (Ophiucus) maybe ~ $40M_{sun}$ cutoff

Virial Equation

- Virial Equation is second time derivative of moment of inertia; and for a given mass provides a dynamical equation for R^2 :

Velocity Dispersion Pressure — Gravity — External pressure

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 3M\sigma^2 - \frac{\Gamma G M^2}{R} - 4\pi P_e R; \qquad I = \beta M R^2$$

(uniform sphere: $\Gamma = 3/5, \beta = 0.3$)

divide by M

$$\frac{\beta}{2R}\frac{d^2 R^2}{dt^2} = \frac{3\sigma^2}{R} - \frac{\Gamma G M}{R^2} - \frac{4\pi P_e}{M};$$

SVE: steady and no pressure

PVE: steady, but w/ pressure

$$\alpha \equiv \frac{3\sigma^2}{\frac{\Gamma G M}{R}} = \frac{5\sigma^2 R}{GM} \quad \text{(virial parameter);} \qquad V_0^2 \equiv \frac{\sigma^2}{R} \quad \text{(SLW parameter)}$$

$\sigma \propto R^{p_1}$, $p_1 = 0.5$ is better approximation above 0.5 pc

Author	Year	p_1	L(pc)	Comments
Blitz et al. 2007		0.5		Six galaxies
Brunt & Heyer 2002		0.6		Outer Galaxy
Caselli & Myers 1995		0.2	< 0.1	Massive cores
Casoli et al. 1984		0.2		
Dame et al. 1986		0.5	0–100	Large-scale survey
Fuller & Myers 1992		0.7±0.1		Dense cores
Goodman et al. 1998		0.2	0.1	Coherent cores
		0	< 0.1	”
Heithausen et al. 1996		0.5		HLC
Heyer et al. 2001		0.5	>7	
		0	< 7	
Heyer & Brunt 2004		0.5		
Heyer et al. 2006		0.7		Rosette
Keto & Myers 1986		~0.5		HLC
Larson 1981		0.4	0.1–100	From various authors
Myers 1983		0.5	0.05–3	
Snell 1981		0.5–1	~1	
Solomon et al. 1987		0.5	0.4–40	Large-scale survey

from FBK09

Author(s)	$\frac{5\sigma^2 R}{GM} = \alpha = M_V/M$	Mass ($M_\odot$) or L (pc)
Bertoldi & McKee 1992	$\gg 1$	$M < 100$–1000
Blitz 1987	$\cong 1$	Rosette ‘large’
	$\gg 1$	Rosette ‘small’
Carr 1987	> 1	$M < 30$
Dame et al. 1986	$\cong 1$	$10 < L < 100$
Heithausen 1996	$\gg 1$	HLC
Herbertz et al. 1991	$\gg 1$	$0.3 < L < 3$
Heyer et al. 2001	$\cong 1$	$M > 10000$
	> 1	$M < 1000$
Keto and Myers 1986	$\gg 1$	HLCs
Larson 1981	$\cong 1$	$0.1 < L < 100$
Leung et al. 1982	$\cong 1$	$0.3 < L < 30$
Loren 1989a,b	$\cong 1$	$M > 30$
	$\gg 1$	$M < 30$
McKee & Tan 2003	$\cong 1$	Giant molecular clouds
Myers 1983	$\cong 1$	$0.5 < L < 3$
Myers et al. 1983	$\gg 1$	$L \cong 0.3$, $M \cong 30$
Snell 1981	$\cong 1$	$L \cong 1$
Solomon et al. 1987	$\cong 1$	$0.4 < L < 40$
Strong & Mattox 1996	$\cong 1$	Giant molecular cloud
Williams et al. 1994	$\cong 1$	Rosette Nebula
	$\gg 1$	Maddelena Cloud
Williams et al. 1995	$\cong 1$	Rosette (15 per cent)
	$\gg 1$	Rosette (85 per cent)

from FBK09

Observed Properties (newer data)

- Heyer et al. 2009 observations challenged SVE using newer data from Galactic Ring Survey (GRS; Jackson et al. 2006):

  - MCs do not appear to be in SVE and $V_0 = \sigma^2/R$ varies in proportion to column density Σ

  - Earlier data that supported Solomon (S87) were consistent with this revised conclusion, but quality of the earlier data insufficient to make this evident

  - Both H2009 and S87 Used: 14-m Five College Radio Astronomy Observatory, but GRS was made with a 4 × 4 multi- pixel array camera (SEQUOIA) rather than a single-pixel detector. The improvement allowed GRS to sample the clouds at higher resolution (closer to limit spacing for **47 arcsec** beam).

  - S87 survey was under-sampled with **3 arcmin** spacing and missed high-density structures which tend to be of small scale.

  - Also: GRS used ^{13}CO rather than ^{12}CO of S87. This allowed measurement of higher column densities (> $100 M_{sun}/pc^2$) before line saturation.

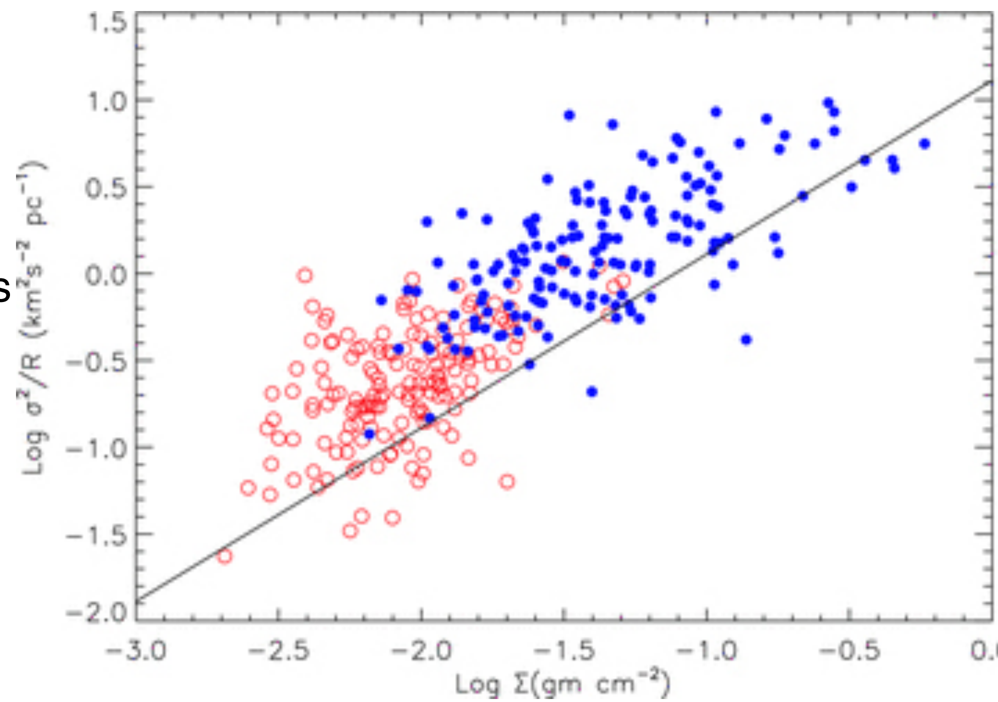
  - overall Heyer 09: better sampling and lower optical depth tracer enabled the GRS to extend the relationship between and the quantity V_0 to higher column densities (> 100 $M_\odot$ pc^2)

  - smaller scales < 0.5 pc seemingly $p_1 < 0.5$, and even less close to SVE

Observed Properties (how close to viral equipartition/equilibrium?)

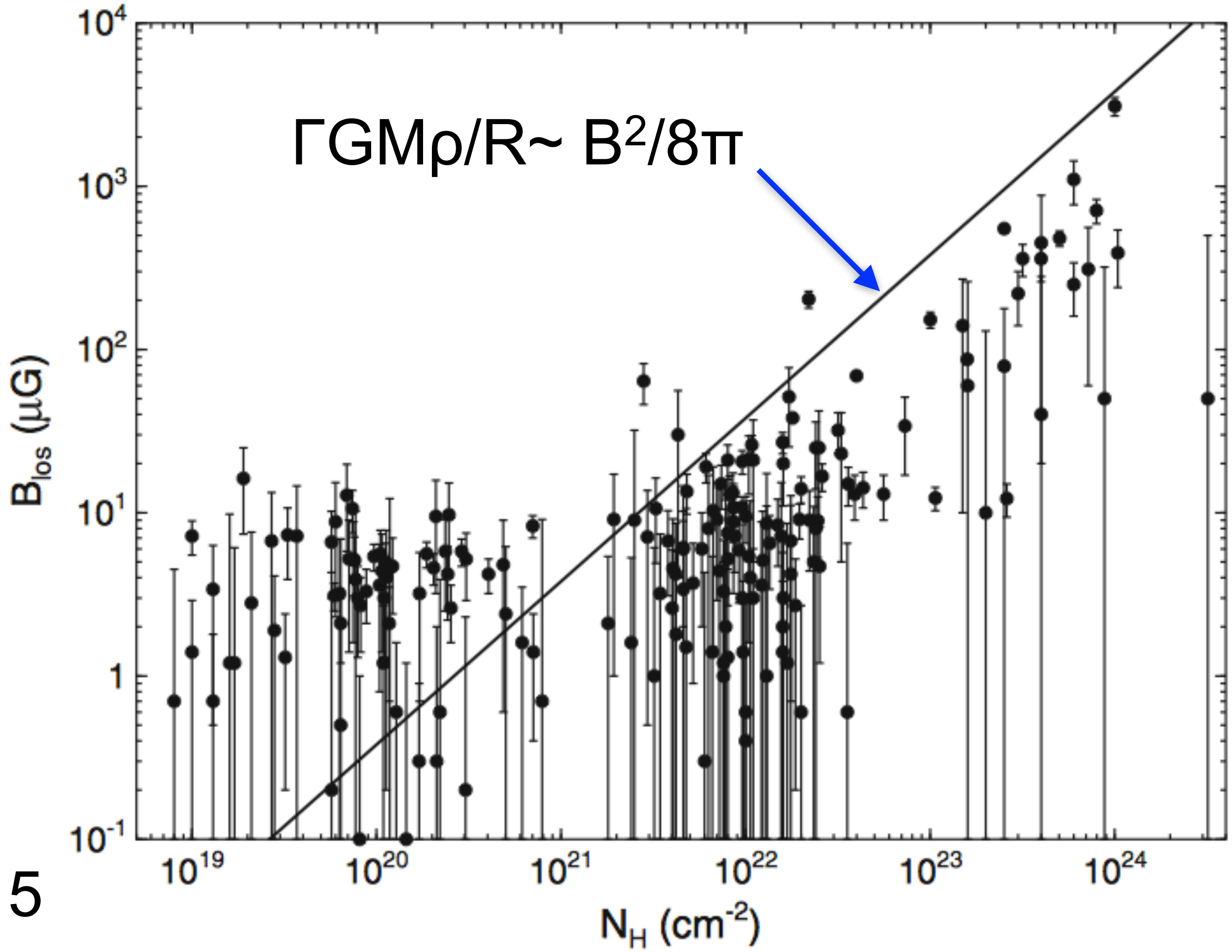
clouds of R > 0.4pc

- Simple virial equilibrium is line shown
- Constant surface density AND virial equilibrium would place all a points at single location
- Surface density not constant so points are distributed
- Parallel to virial line but above
- Either:
  - mass of clouds underestimated
  - clouds are unbound
  - P_e provides an additional confining force (Bonnor 1956; Keto & Myers 1986; Elmegreen 1989; Bertoldi & McKee 1992; Heyer & Brunt 2004; Lada et al. 2008; FBK 09; FKB 15; BKF 15).



Open-red and filled-blue symbols indicate same clouds in S87 survey and GRS, respectively. Better spatial sampling and use of a molecular tracer with lower optical depth detects clouds with higher column densities. From data of Heyer 09. (from FBK 2011)

Reasons for Ignoring B-fields for present purposes



Crutcher 2015

Fig. 15.2 H I, OH, and CN Zeeman measurements of the magnitude of B_{los} versus N_H. The *straight line* is for a critical $M/\Phi = 3.8 \times 10^{-21} N_H/B$. Clouds above this line are subcritical, those below are supercritical

More on B-fields ..

- tangled field “pressure” must be driven by the kinetic motions within clouds and so, by energy conservation, cannot be an extra source of pressure beyond what velocities supply

- larger scale fields amplified by energy source external to that of individual cloud could play a role; such fields would not be isotropic on the cloud scale

- energy density nearly constant with scale ($\rho v^2 \sim$ constant) so if not influential on largest scale, unlikely to be on smaller scales where fragmentation cascade is quasi-self similar (above clump scale)

- However, B-fields surely important on sub-core scales (accretion disks, outflows, jets…)

Virial Equilibrium Solutions

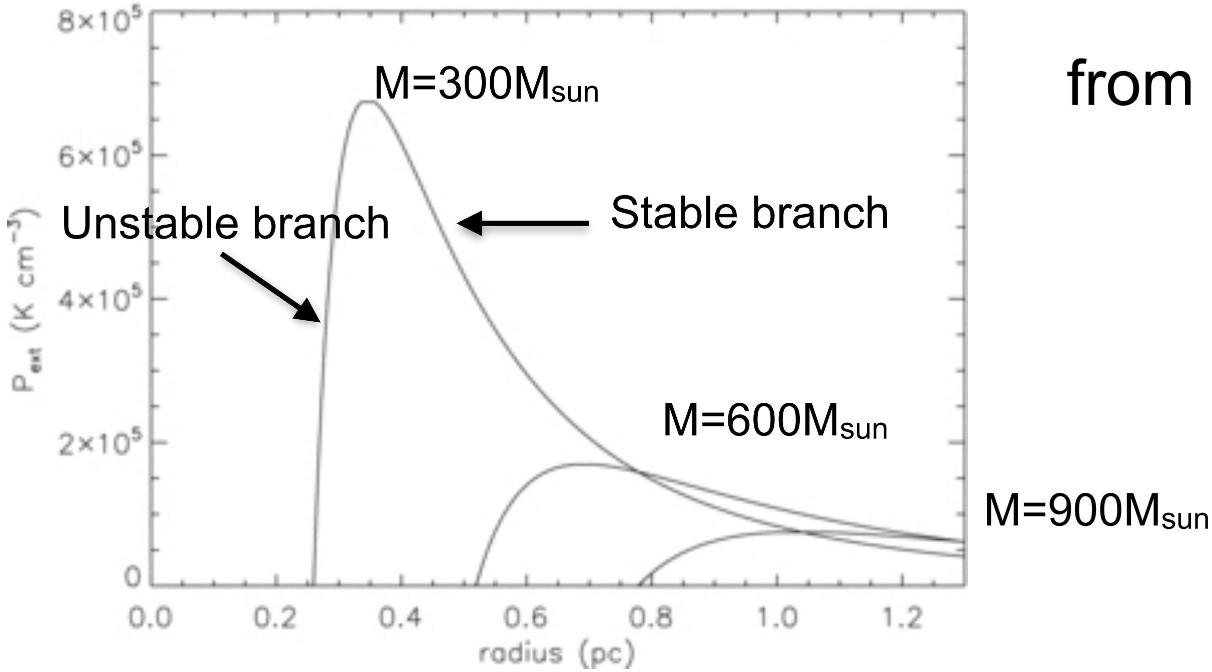
FBK11,FBK15

$$3\sigma^2 = \frac{\Gamma GM}{R} + \frac{4\pi P_e R^3}{M} - \frac{1}{2}\frac{d^2R^2}{dt^2} \text{, or}$$

$$P_e = \frac{3\sigma^2 M}{4\pi R^3} - \frac{\Gamma GM^2}{4\pi R^4}$$

- Balance between expansion “turbulent pressure” and contraction from gravity and external pressure
- For any combo of σ and P_e there exists critical radius R_c dividing equilibrium solutions from those with $d^2R^2/dt^2 \neq 0$
- Can find R_c in two ways: **minimize right side of above equation with respect to *R*** with either:
  - **(A) *M* and *σ* fixed (Boyle's Law,** Bonnor (56) Ebert (57)). Then at $dP_e/dR = 0$, $R = R_{c,M,\sigma} = \frac{4\Gamma GM}{9\sigma^2}$
    - for $R < R_c$ $dP_e/dR > 0$ and equilibrium is unstable because e.g. increasing P_e then implies increase of dR for equilibrium but there is no available outward restoring force.
    - *$R \geq R_c$ $dP_e/dR \leq 0$, and equilibrium is stable: increase in P_e compensated by increase in $\rho\sigma^2$*
    - substituting R_c into Eq (1) gives mass M_c for which $M>M_c$ collapses $M_c \simeq \frac{\sigma^2}{(\Gamma^3 G^3 P_e)^{1/2}}$
  - **(B) *M* and P_e fixed (Charles Law,** Chieze 87**)**: Then at $d\sigma^2/dR = 0$, $R = R_{c,M,P_e} = \left(\frac{\Gamma GM^2}{12\pi P_e}\right)^{1/4}$
    - *for $R < R_c$, $d\sigma^2/dR < 0$* and equilibrium is unstable increasing σ^2 requires decrease of dR for equilibrium but there is no available inward restoring force.
    - *for $R \geq R_c$, $d\sigma^2/dR \geq 0$* and equilibrium is stable $M_{c,M,P_e} \simeq \frac{\sigma^4}{(\Gamma G)^{3/2} P_e^{1/2}} \propto R_{c,M,P_e}^2$

Boyle's Case (M,σ fixed)



from FBK11

Uniform density spheres in Pressure VE. The three curves show the equilib solutions for different masses. To the right of pressure maxima, d*P*/d*R* < 0, and the clouds are stable. To the left of the maxima, self-gravity dominates and the clouds are unstable

Boyle's Case (M,σ fixed)

- If instead of uniform sphere use outward decreasing density gradient as observed in some cases (Lada et al. 2009) can use Lane-Emden get (Bonnor 1956):

$$R_c = 0.5GM/\sigma^2$$

$$M_c = 1.2\sigma^4/(G^3 P_e)^{1/2}$$

Now construct critical Surface density

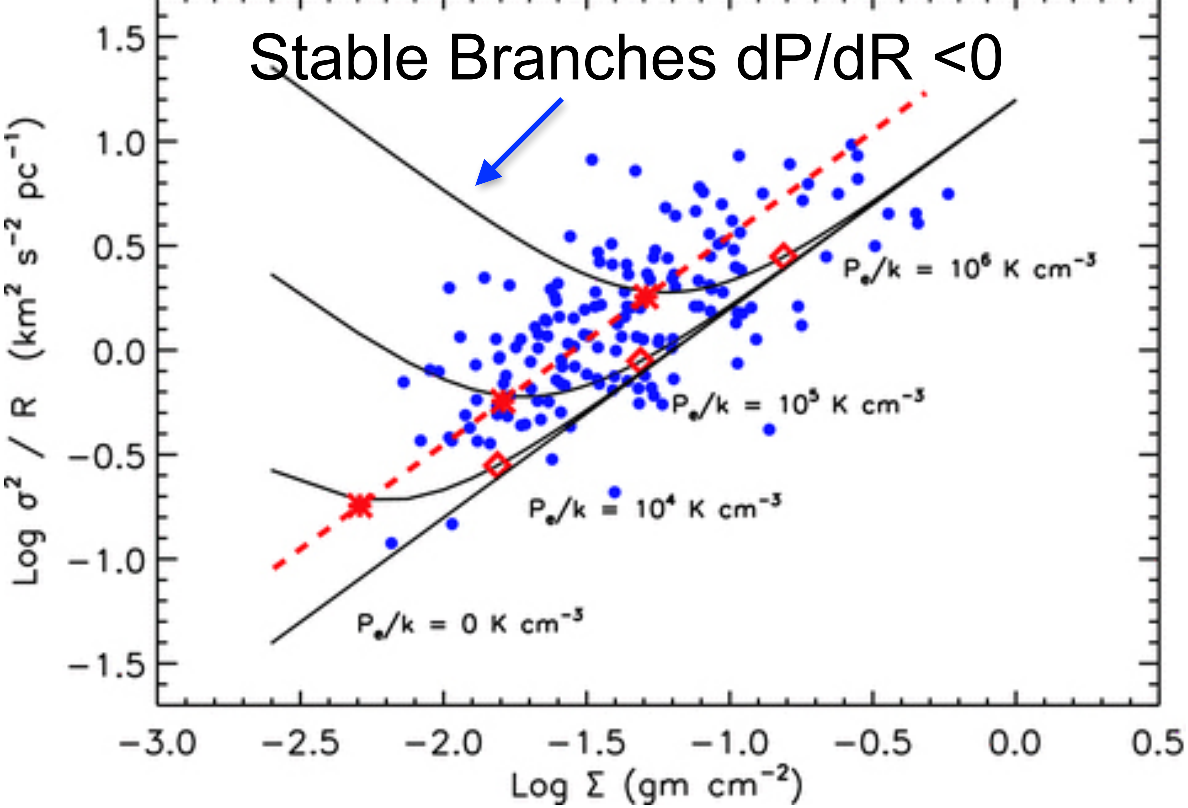
$$\Sigma_c = M_c/\pi R_c^2 = 0.3(P_e/k)^{1/2} M_\odot/\mathrm{pc}^2$$

....and rewrite equilibrium equation:

$$V_0^2 = \frac{\sigma^2}{R} = \frac{1}{3}\left(\pi\Gamma G\Sigma + \frac{4P_e}{\Sigma}\right), \text{ so}$$

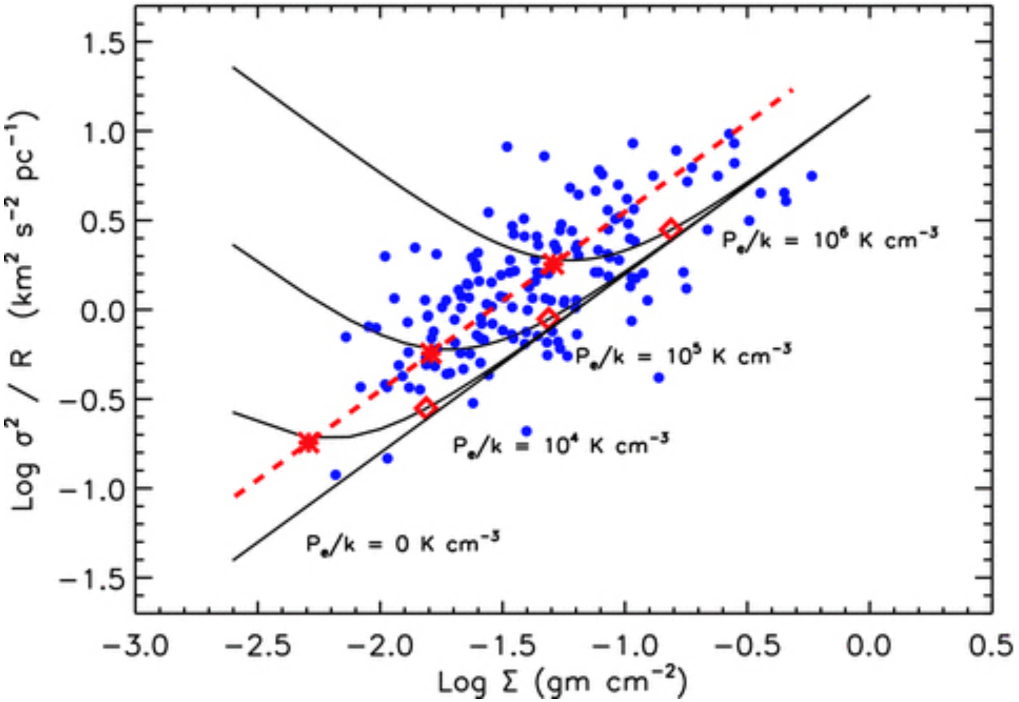
$$V_{0,c}^2 = \frac{\sigma^2}{R_c} = \frac{1}{3}\left(\pi\Gamma G\Sigma_c + \frac{4P_e}{\Sigma_c}\right)$$

FBK 2011



Comparing PVE theory versus observations

- Blue circles show Heyer09 data;
- V-shaped curves are solutions of PVE for different pressures
- Solid line shows SVE with no external pressure;
- Dashed line shows loci of clouds of critical mass for full range of pressures using LE;
- Difference between the asterisks is CD vs LE approximations



Clouds appear critical but why?

- Data not consistent with single external pressure
- clouds are not on stable equilibrium branch!
- clouds seem to lie along the line of criticality, which is also consistent with size-line width relations, and density size relations
- Clouds are critical but why?
- Consider dynamical evolution of virial equation…

Non-Dimensionalize (FKB15)

- Virial Equation again:

$$\frac{\beta}{2}\frac{d^2R^2}{dt^2} = 3\sigma^2 - \frac{\Gamma GM}{R} - \frac{4\pi P_e R}{M} \quad \text{(uniform sphere: } \beta = 0.3\text{)}$$

Non-dimensionalize using

$$r = \frac{R}{R_{c,M,P_e}}; \quad m = \frac{M}{M_{c,M,P_e}}, \quad \text{and} \quad d\tau = \left(\frac{2\beta}{3}\right)^{-1/2} dt / t_{x,c}$$

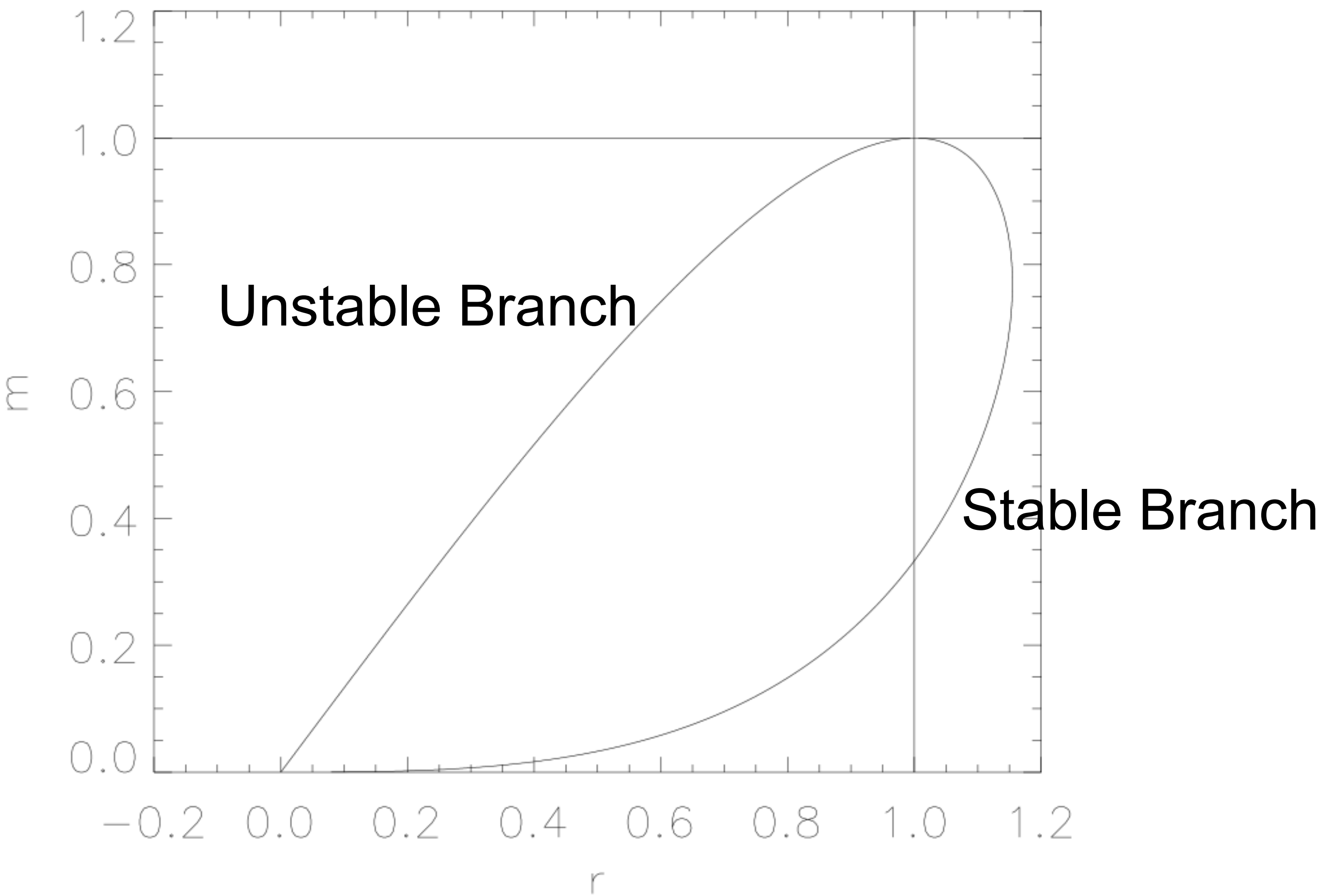
$$\text{where } R_{c,M,P_e} = \left(\frac{\Gamma GM^2}{12\pi P_e}\right)^{1/4}; \quad M_{c,M,P_e} \simeq \frac{\sigma^4}{(\Gamma G)^{3/2} P_e^{1/2}}; \quad t_{x,c} = R_c / \sigma$$

Crossing time associated with R_c

$$\Rightarrow \qquad \frac{d^2 r}{d\tau^2} = 4 - 3\frac{m}{r} - \frac{r^3}{m}$$

Equilibrium solutions (steady state)

FKB 15



Importance of Dissipation

- Something supplies the kinetic energy that supports the clouds, there may be multiple sources from top down (ambient: SN; top down: gravity, bottom up: outflows, other feedback)

- but for a given structure supported by velocity dispersion, its collapse and/or fragmentation is facilitated by dissipation of internal kinetic energy

- the time dependent equation above does not include this so we must supplement

- consider dissipation as exponential decay process:

Dissipation and Time Dependence

$$\frac{d\sigma^2}{dt} = -\frac{\sigma}{t_L}$$

where t_L is the loss time of the turbulence, (possibly influenced by feedback)

Can be replaced by equation time evolution of m because:

$$m = \frac{M}{M_{c,M,P_e}} \simeq \frac{(\Gamma G)^{3/2} P_e^{1/2}}{\sigma^4}$$

$$\frac{dm}{m} = -\frac{2d\sigma^2}{\sigma} = 2\left(\frac{2\beta}{3}\right)^{1/2} \frac{\gamma d\tau}{r} \; ; \quad \text{where } \gamma \equiv t_x / t_L \quad \text{so,}$$

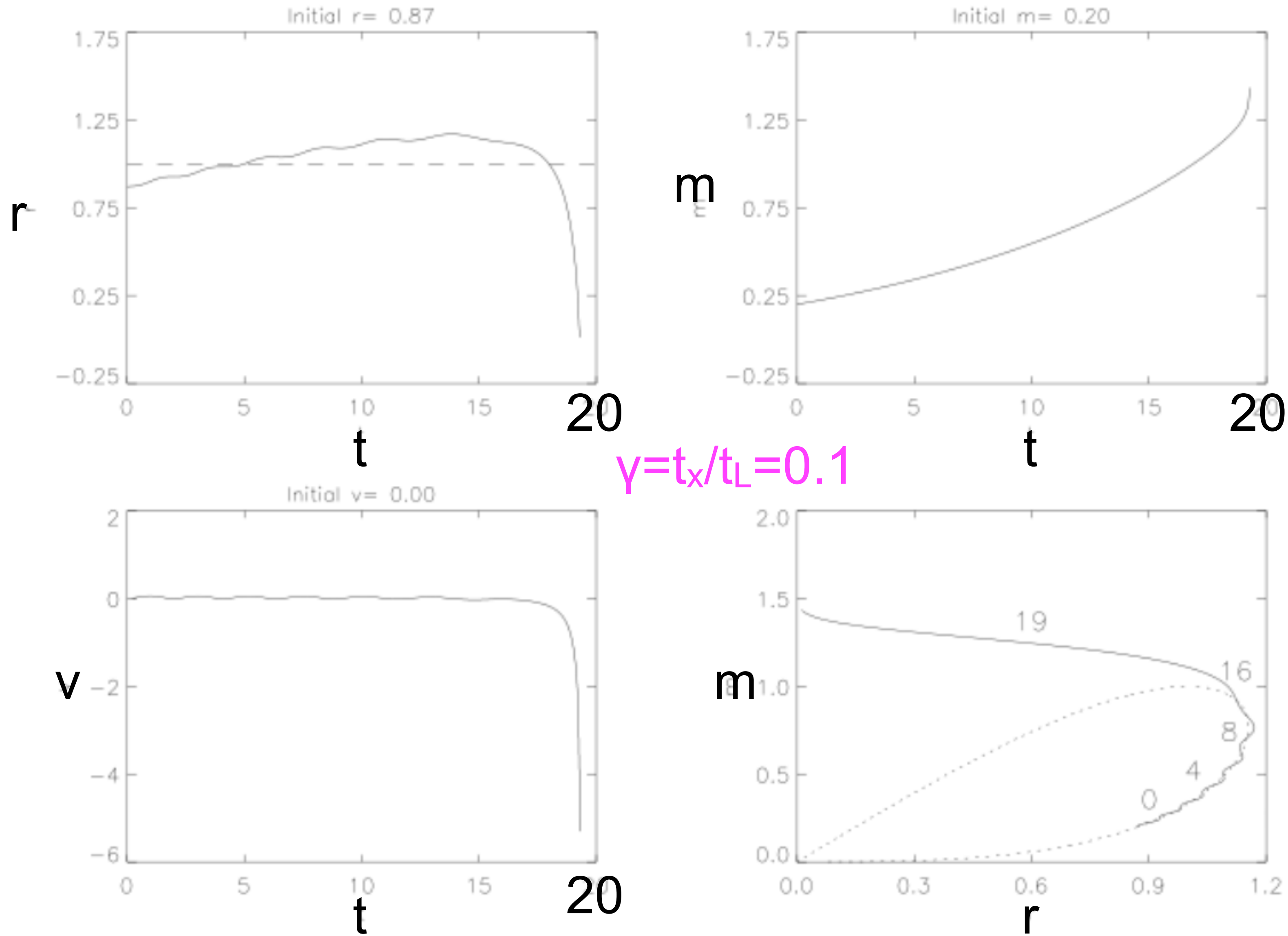
$$\Rightarrow \frac{dm}{d\tau} = -2\left(\frac{2\beta}{3}\right)^{1/2} \frac{\gamma m}{r}$$

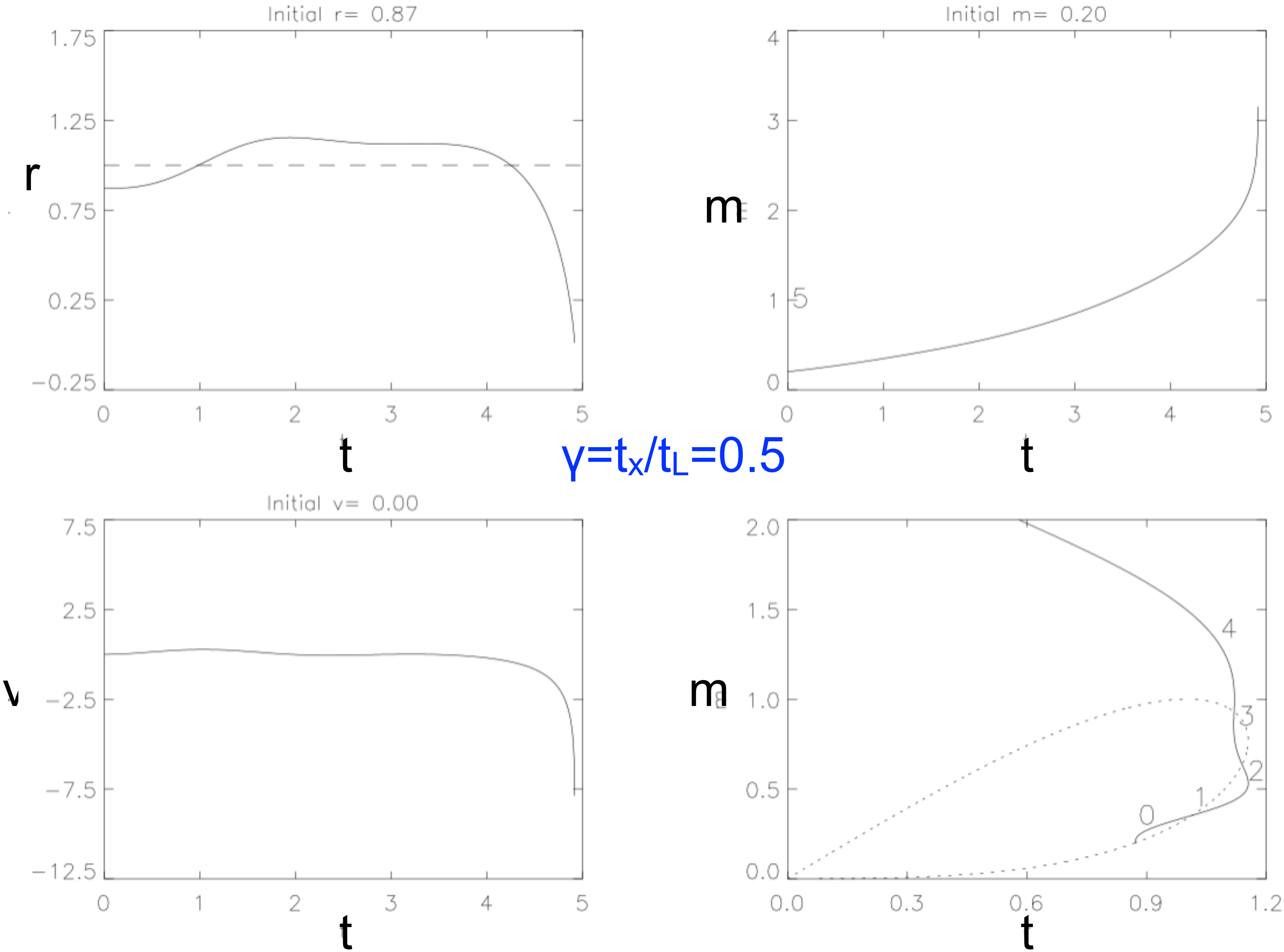
$$\frac{d^2 r}{d\tau^2} = 4 - 3\frac{m}{r} - \frac{r^3}{m}$$

Coupled equations to solve for Virial Evolution (FKB15)

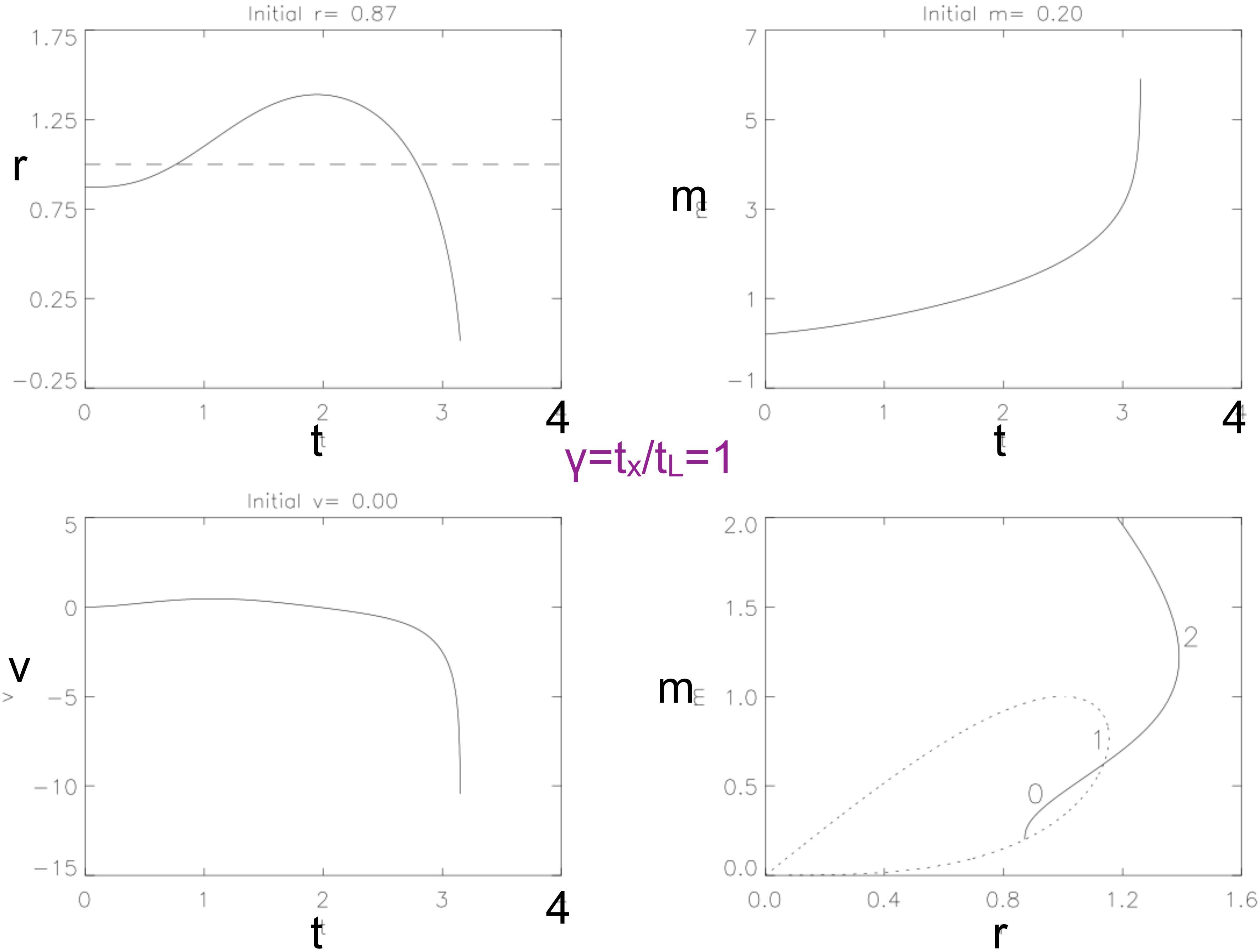
Solutions

- look at solutions for different values of the crossing time to dissipation time $\gamma = t_x/t_L$.



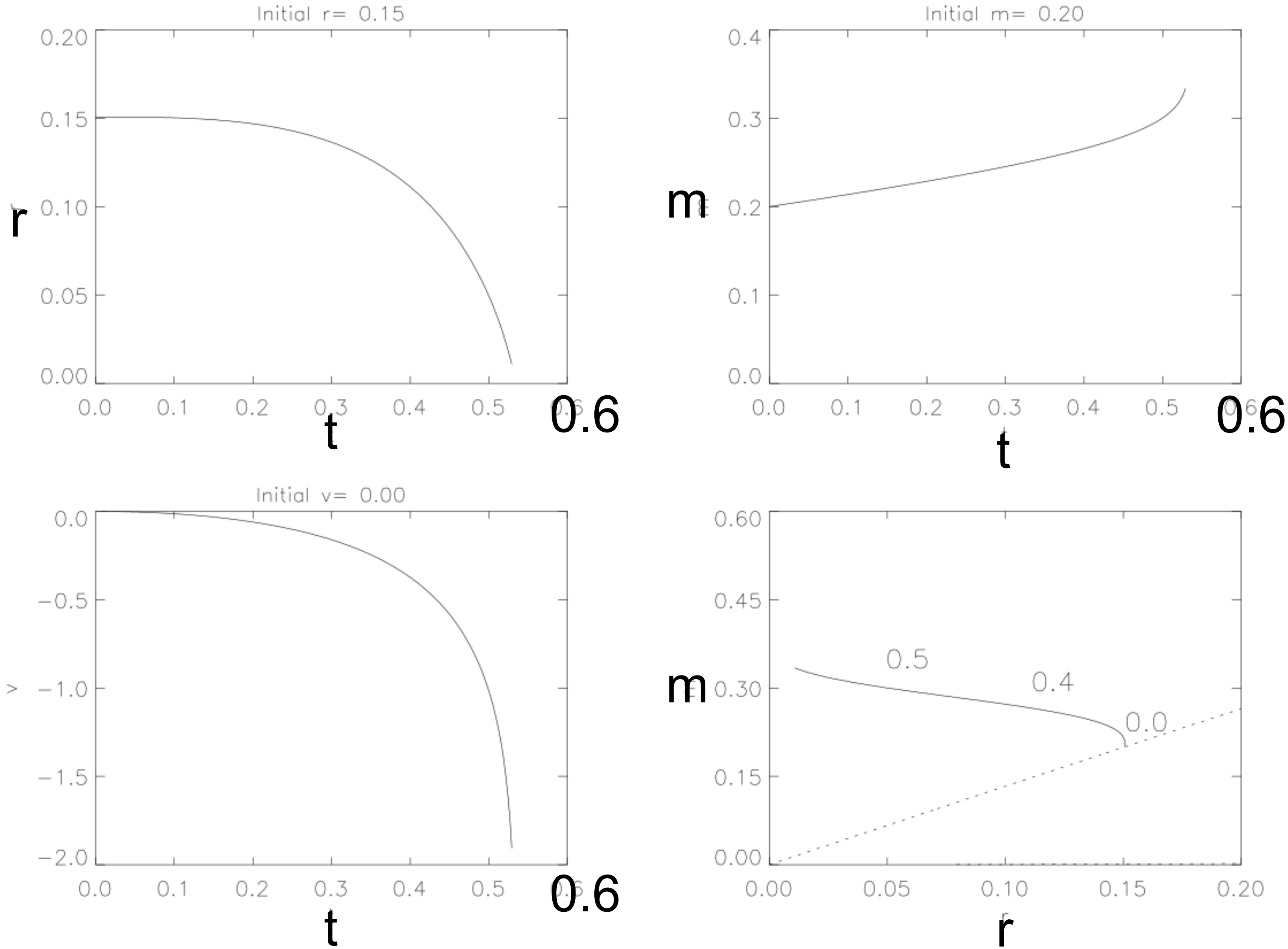


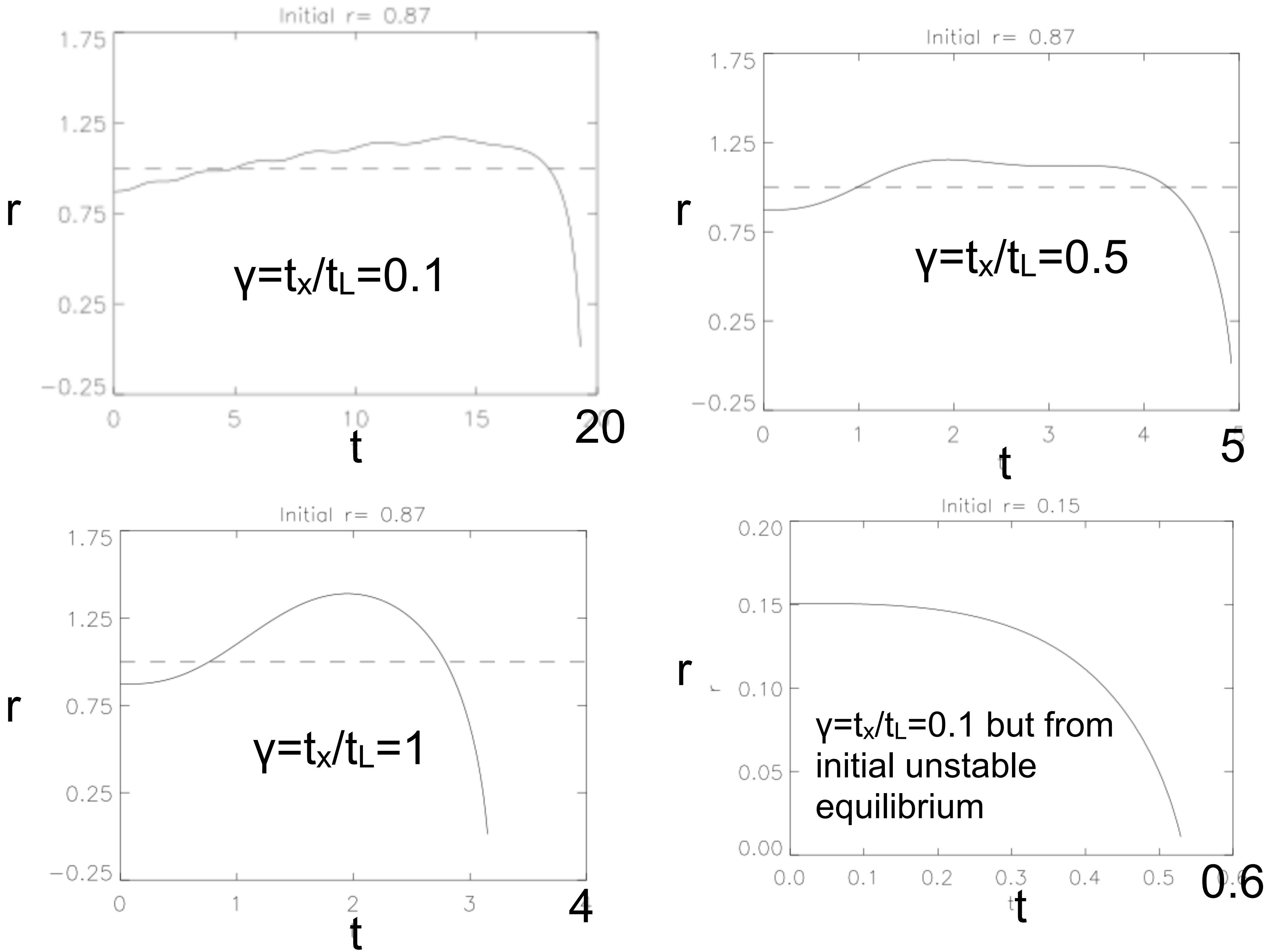
Initial r= 0.87
r
t
Initial m= 0.20
m
t
γ=tx/tL=0.5
Initial v= 0.00
t
m
t

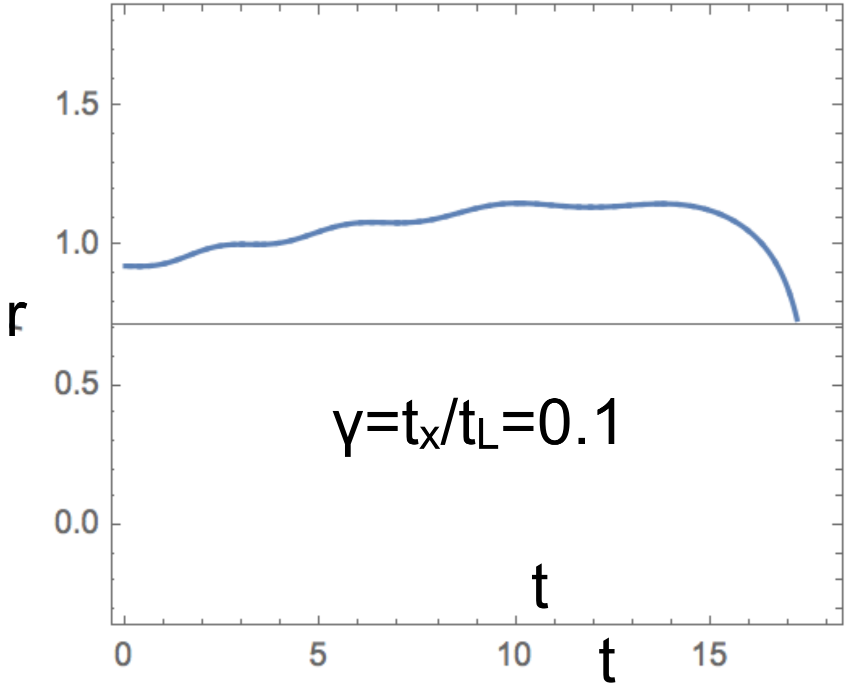


Initial r= 0.87
r
t
Initial m= 0.20
m
t
γ=tx/tL=1
Initial v= 0.00
v
t
m
r
0
1
2

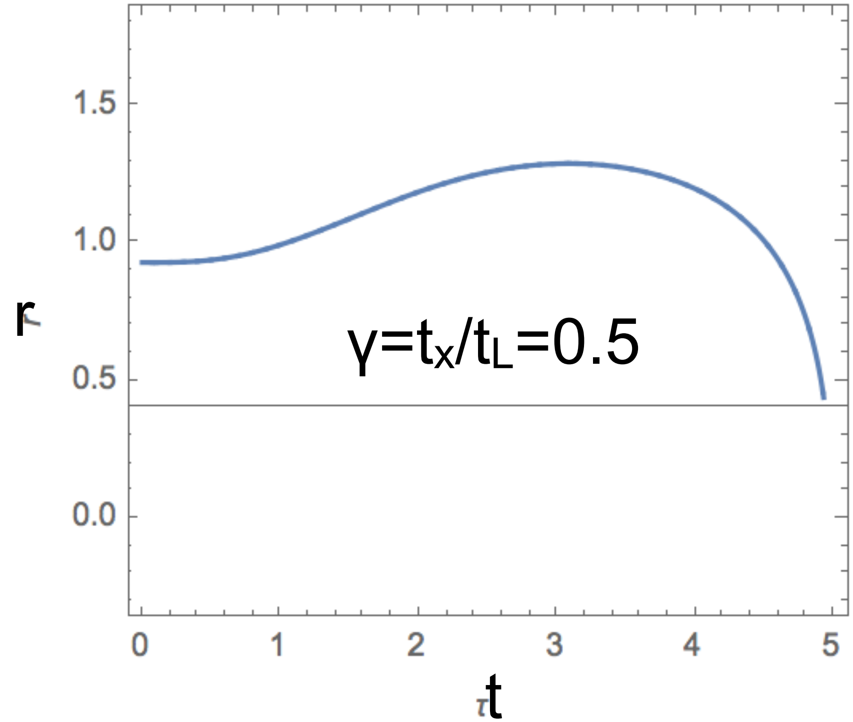
$\gamma = t_x/t_L = 0.1$ but starting in unstable equilibrium



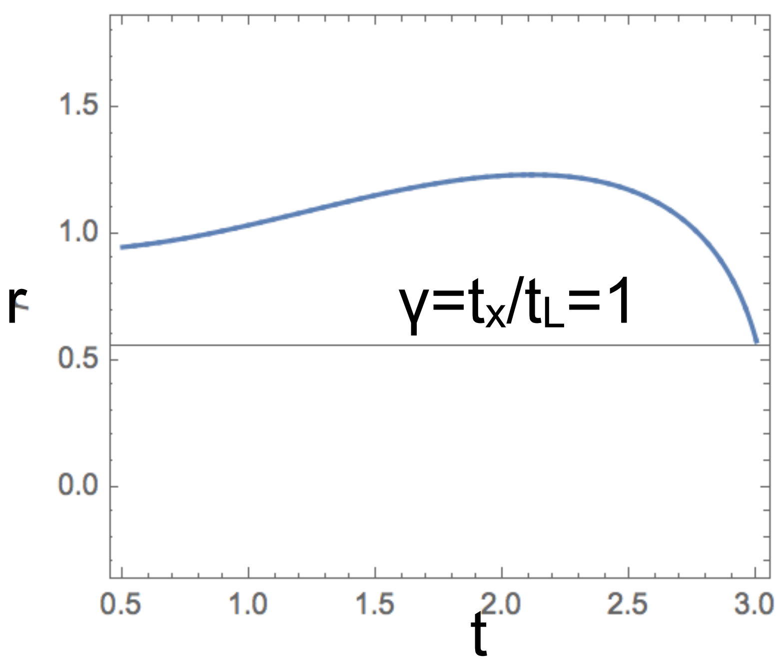

Initial r= 0.87
r
t
γ=tx/tL=0.1
Initial r= 0.87
r
t
γ=tx/tL=0.5
Initial r= 0.87
r
t
γ=tx/tL=1
Initial r= 0.15
r
t
γ=tx/tL=0.1 but from initial unstable equilibrium



r
1.5
1.0
0.5
0.0
γ=tx/tL=0.1
t
0
5
10
15
t



r
1.5
1.0
0.5
0.0
γ=tx/tL=0.5
0
1
2
3
4
5
t



r
1.5
1.0
0.5
0.0
γ=tx/tL=1
0.5
1.0
1.5
2.0
2.5
3.0
t

Interpretation of Virial Evolution

- Individual clouds evolve: velocity dispersion decreases but if dissipation time scale $t_L \geq 2R/\sigma$, then clouds evolve on stable equilibrium paths close to critical
- Implies that size vs. line-width relation maintained, objects appear to be near R_c and M_c as a population even though none are actually in strict virial equilibrium
- after enough dissipation, clouds do collapse and, in present of density fluctuations, participate in fragmentation cascade

Fragmentation

- MCs likely have substructure from density fluctuations imposed during their formation, either from background ISM in spiral arms in top down formation (e.g. Kim & Ostriker 02,06, Shetty and Ostriker 06; Dobbs et al 12), for largest clouds or from collisional formation; some initial source of turbulence likely seeded therein in addition to whatever feeds back from below (Heitsch et al. 2005 09; Vásquez-Semadeni et al. 07 Toalá et al 2015 etc.)

- Virial equation used can be thought of as averaged over substructure of MC and also applying to each sub-MC therein, which itself is an averaged structure

- Dissipation time scale, scales as $R^{1/2}$ so that smaller structures lose their energy faster and kinetic energy support is lost faster for smaller structures

- The latter exacerbates, but won't ensure fragmentation without initial substructure because the free-fall time depends inversely on density and so unless fluctuations grow fast enough with density large enough to have a faster free fall time than the ambient structure in which it is embedded, the whole structure can collapse before fragmentation ensues (e.g. Layzer 63).

- Carroll et al. (unpub) simulations to test this with BE sphere; indeed required significant fluctuations in velocity to produce the density fluctuations for fragmentation to occur.

- Latter is also consistent with Toalá et al. 2015 (next slide) seems that 10% fluctuations might be the minimum needed, not extreme, but important

Fluctuations and Fragmentation

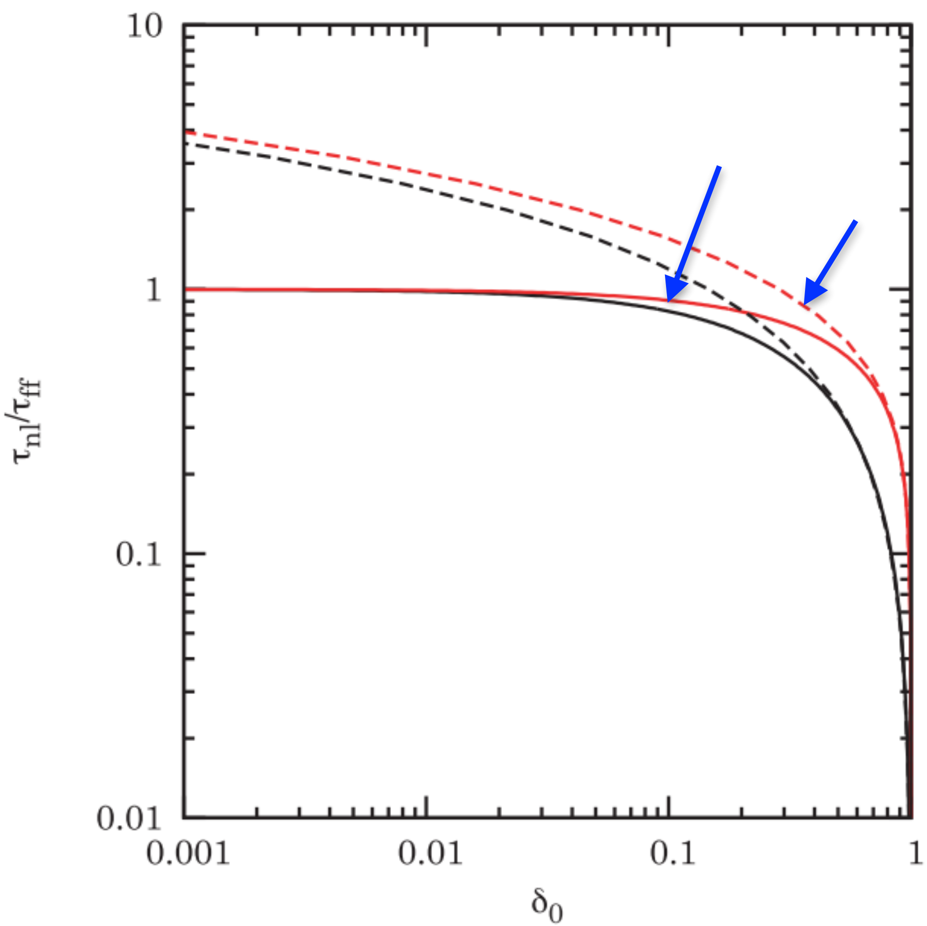


Figure 3. Time τ_{nl} for the perturbation to grow to $\delta = 1$ in both the Jeans (dashed lines) and the IHF (solid lines) cases, versus the initial fluctuation amplitude, δ_0. As in Fig. 2, the black lines denote the cosmological case, and the red lines denote the MC case.

- Toalá et al. 2015: Relative time scale for density fluctuations to grow to over order 1 in grow in collapsing background (Inverse-Hubble) flow;
- Although faster than in non-contracting flow, still need at least 10% fluctuations to get to of order unity by free fall time.

Number Spectrum

- If dissipation time is greater or equal twice free crossing, we have statistical circumstance of clouds appearing critical as described
- assume that the requisite conditions can be met for fragmentation as discussed
- Estimate the number-size spectrum by assuming a fragmentation cascade that **conserves mass**

Predicted Number Spectrum

$$\sigma \propto R^{0.5} \quad (\text{since } R \sim R_c)$$

$$M \propto \sigma^4 \propto R^2 \quad (\text{since } M \sim M_c)$$

$$N = \int_{M_S}^{M_L} \frac{dN}{dM} dM$$

Mass Conservation

$$M\sigma \frac{dN}{dR} = M\sigma \frac{dN}{dM}\frac{dM}{dR} = const$$

so

$$\frac{dN}{dR} \propto \frac{1}{M\sigma} \propto R^{-2.5}$$

$$\frac{dN}{dM} \propto M^{-1.75}$$

- Consistent with observations (Lada et al. 1991; Kramer et al. 1998; Heyer et al. 2001 Roman Duval et al. 2014..)

Origin of External "Pressure"

- Elmegreen(89) estimated neutral ISM ~10^4 K/cm^3 and adding to this gravity giving 5 x 10^4 K/cm^3 of HI halo, something like Pe

$$P_e = \int \rho g dr \sim GM\mu_h / R^2$$

$$\text{column density: } \mu_h = 3\times10^{-3} a_v \text{mag}^{-1}\text{g.cm}^{-2} = Ka_v$$

$$\frac{4\pi P_e R^3 / M}{\Gamma GM / R} = \frac{4\pi K a_v R^2}{\Gamma M} = \frac{4Ka_v}{\Gamma\Sigma} \sim a_v$$

- Also, possible recoil pressure of order 10^5K/cm^3 coming from momentum conservation via release of HI atoms dissociated by FUV radiation (FBK09)
- Observations suggest 0.5-5 x 10^5K/cm^3 e.g. (Bertoldi & McKee 92 in Ophiuchus 10^5K/cm^3. Lada et al. 08 in Pipe Nebula 7 x 10^4K/cm^3; Belloche et al. 11: 5 x 10^5K/cm^3 around 60 starless cores)
- turbulent pressure (but only if scale of turbulence is less than cloud scale); Ambient winds, other…
- plausible that there is dependence on environment

Cascades with and without radiative loss

Kolomogorov, energy conserving:

$$\dot{\varepsilon} \simeq \frac{\sigma^3}{R} \propto R^s = \text{constant, s} = 0$$

$$t_{\dot{\varepsilon}} = \frac{\dot{\varepsilon}}{d\dot{\varepsilon}/dt} = \infty$$

Fragmentation MC, mass conserving but not energy conserving:

$$\dot{\varepsilon} \simeq \frac{\sigma^3}{R}\gamma \propto R^s = R^{1/2}\text{, s=1/2}$$

$\gamma=(R/\sigma)/t_L$

$$t_{\dot{\varepsilon}} = \frac{\dot{\varepsilon}}{d\dot{\varepsilon}/dt} = \frac{2R}{3\sigma}\gamma = \frac{2}{3}t_L \propto R^{1/2}$$

Predict scale independence of $\varepsilon_{ff}=t_{ff}/t_{dep}$

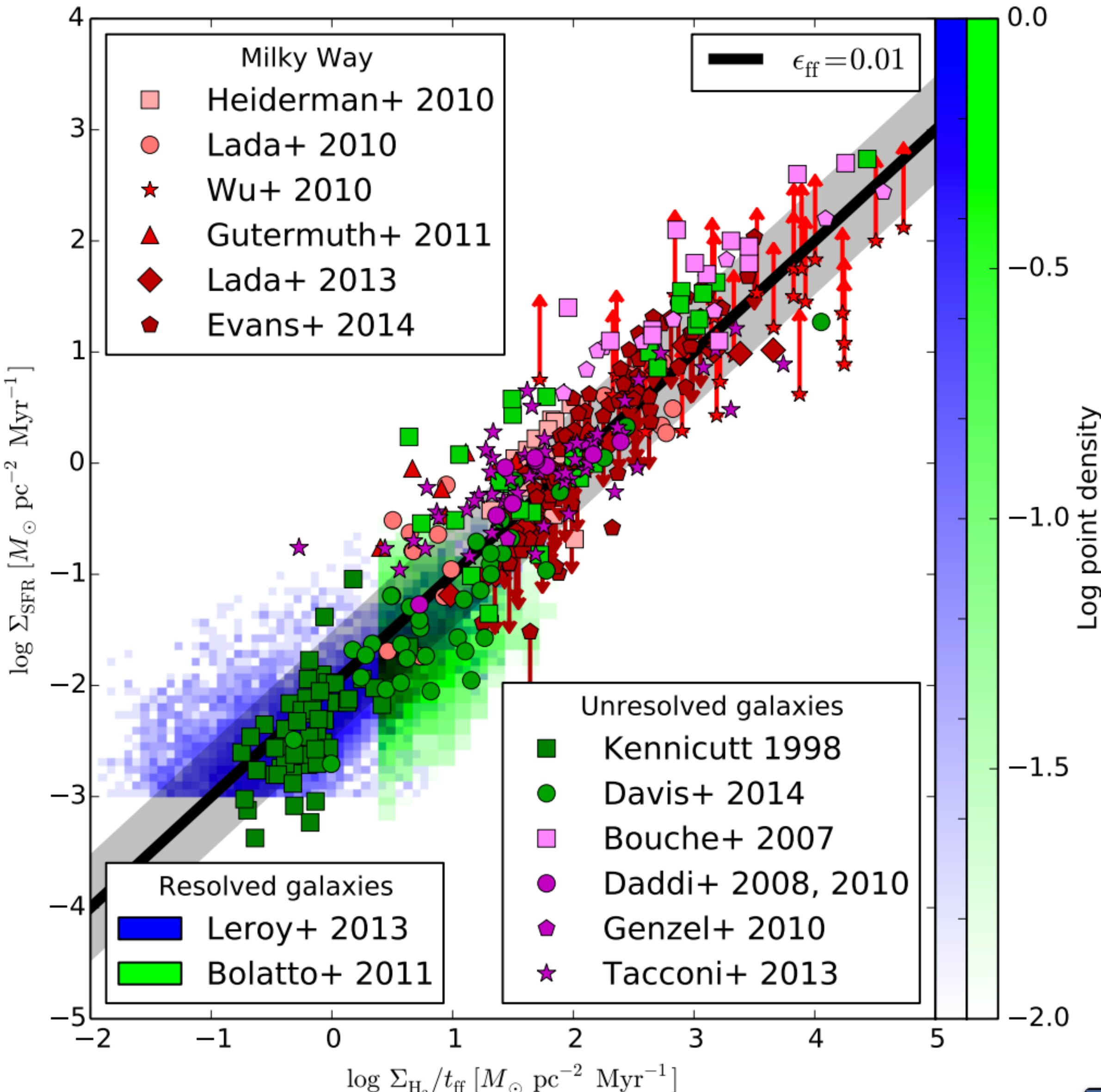
- As a function of cloud size one can estimate the volume and area filling fraction scalings

$$f_V(R) \sim \frac{R^3}{R_1^3}\frac{dN}{dM}\frac{dM}{dR}R \sim 0.5\left(\frac{R}{R_1}\right)^{1.5}$$

- If the fraction of star-formation from clouds on scale R, satisfies $f_R \propto f_V$ then the following quantity (e.g. Krumholz McKee 05; Krumholz Tan 07) is scale independent in agreement with observations:

$$\varepsilon_{ff} = \frac{\dot{M}_{obs} f_V t_{ff}(R)}{M(R)} = \frac{\dot{\Sigma}_{obs} t_{eff}}{\Sigma_{H_2}} = \frac{t_{ff}}{t_{dep}} \propto R^0 = \text{constant}$$

- A consequence of mass conservation in on every scale, mass flow rate is same, SFR occurs on small scales
- Observations say $\varepsilon_{ff} \sim 0.01$
- Impulsive feedback on clumpish scales and below would determine specific value of constant



rface density of star formation versus surface density of molecular gas normalized by estimated free-fall time Σ/t_{ff}. The free-fall time
s has been estimated following the method of Krumholz et al. (2012a). The black thick line shows $\epsilon_{ff} = 0.01$; the gray band indicates
scatter about this value. The data shown in the plot are as follows: individual molecular clouds in the Milky Way (red-hued points) are
nan et al. (2010, red squares), Lada et al. (2010, red circles), Wu et al. (2010, red stars, upward arrows indicate lower limits), Lada et al.

Krumholz 2014

More on Feedback

$$\varepsilon_{ff} = \frac{\dot{M}_{obs} f_V t_{ff}(R)}{M(R)} = \frac{\dot{\Sigma}_{obs} t_{eff}}{\Sigma_{H_2}} = \frac{t_{ff}}{t_{dep}} \propto R^0 = \text{constant}$$

- distinguish constant from value of the constant
- source of fluctuations and feedback is hidden in $\gamma=t_c/t_L$, the crossing time over loss time for kinetic energy
- γ could be scale dependent; e.g. smaller on small < pc scales via e.g. outflows; and intermittent feedbacks could change “average” γ
- Total energy output rate of net sum of stellar feedbacks is orders of magnitude more than needed to abate cascade (adding up SN, outflows, ionizing radiation, etc..) but much couples inefficiently to the gas, so momentum is what matters and its effect depends how and on what scales coupling occurs
- abating star formation is less demanding on small scales than on large scales in mass conserving fragmentation cascade ->

Demands for Abating SFR Are Scale Dependent

$$\frac{dP_g}{dt}(R) = \left(\frac{G^{1/2} M^{1/2}(R)}{R^{1/2}} \frac{\gamma M(R) R dN}{t_{ff} dR} \right) \qquad \frac{dE_g}{dt}(R) = \left(\frac{GM(R)}{R} \frac{\gamma M(R) R dN}{t_{ff} dR} \right) \propto \gamma R$$

$$\sim 3 \times 10^{34} \left(\frac{R}{50\mathrm{pc}} \right)^{1/2} \mathrm{g.cm/s^2} \propto \gamma R^{1/2} \qquad 3 \times 10^{40} \left(\frac{R}{50\mathrm{pc}} \right) \mathrm{g.cm/s^2} \propto \gamma R$$

$$(M_L = 10^6 M_\odot, R_L = 50\mathrm{pc}, \mathrm{N}_L = 2000)$$

- Energy and momentum deposition rate associated with free fall for mass conserving cascade; smaller scales require less feedback energy and momenta to abate the cascade and reduce SFR
- highlights difference between slowing SFR versus supplying turbulence to entire cloud complex
- For 2000 clouds of $10^6 M_{sun}$,at R=50pc, momentum per unit mass is ~ 10 km/s and ~1 km/s at 0.5pc scale; (might be partly abated by outflows, e.g. Quillen et al 05; Matzner 07;Heyer et al 09; Nakamura et al 10 Carroll et al. 2010, maybe within order of magnitude of what is needed, B2015)
- Clumps may convert 30% mass to stars (Tan et al. 06) maybe surviving 4 t_{ff} but not more. So still, the big reduction may have to happen by influence of high mass stars SN/SW more catastrophically/intermittently

More on Feedback from outflows

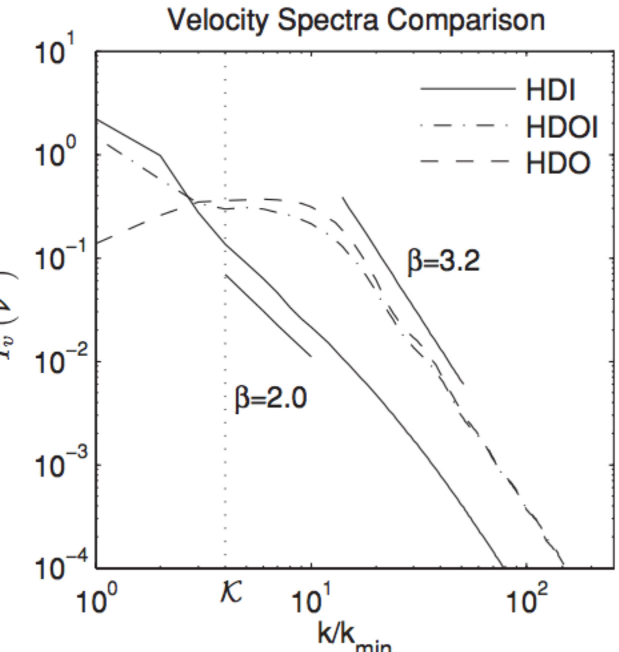
- Outflows from low mass stars may delay clump collapse for a few free fall times but likely not the entire story of mass dispersal. But outflows or other or "bottom up forcing" from winds is distinct from top down forcing

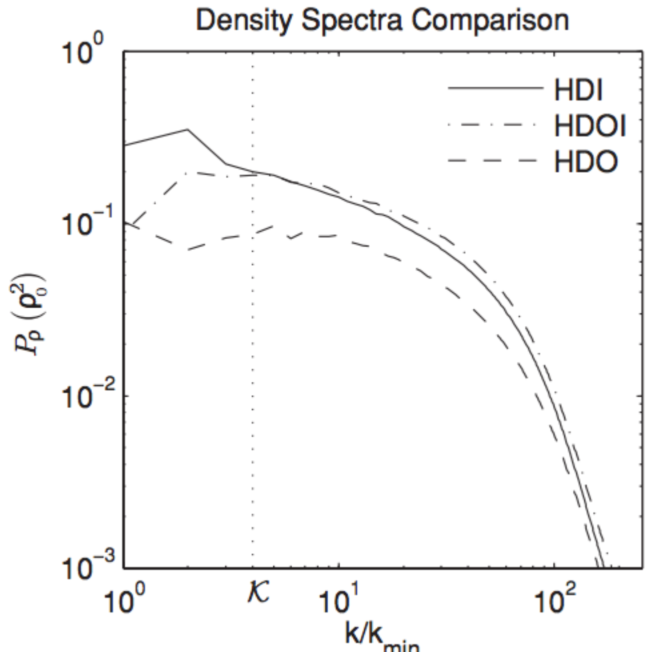
- Carroll et al. 2010,we compared at outflow driving vs. isotropic magic paddle forcing (Others did outflows Nakamura & Li 07; Wang etal. 2010, and separately magic paddles, Kritsuk, Federath etc.)

- 1.5pc cloud, 2.5 x 10^{-20}g/cm^3; used Matzner (2007) scalings, turbulence 0.2km/s sound speed, dP/dt ~10^{29}g.cm^2/s, 1km/s turbulent speed average comes out for both case but spectra are different

- but spectra are quite different in velocity and density for the two cases

Outflow vs. Isotropic (solenoidal) driving





HDI = Isotropic (box scale,
HDO= Outflow (length 1/4
scale, width 3500AU)
HDOI= both

- Outflows have less velocity on larger scales, steeper spectra on small scales; outflows sweep up small scale structures
- flatter density profiles
- also did detailed principal component analysis (PCA) of synthetic velocity channel maps to assess whether method can identify source of dominant momentum injection and distinguish outflows vs. isotropic forcing (e.g. Brunt & Heyer 2009)
- upshot was that method was sensitive to dominant scale of the velocity but not the dominant scale of the momentum because of biases associated with density-velocity scaling relations intrinsic to isotropically forced cases that are not self-consistently justified a posteriori in outflow driven cases.
- other methods VCS (Lazarian 09; Padoan et al. 09) should be analyzed (likely similar ambiguities)

Carroll et al. 2010

Table 1
Table of Runs

Run	Isotropic Forcing	Outflow Forcing	Initial Resolution	Final Resolution	Regrid Time in $\mathcal{T}$ 128	256	512
HDO	No	Yes	128^3	512^3	...	5	6
HDI	Yes	No	512^3	512^3	...	...	...
HDOI	Yes	Yes	64^3	512^3	3	5	6

Table 2
Outflow Scales

Outflow Scale	cgs Units	Astronomical Units	Isotropic Scale
ρ_0	2.51×10^{-20} g cm^{-3}	371 $M_\odot$ pc^{-3}	ρ_0
$\mathcal{P}$	3.98×10^{39} g cm s^{-1}	20.0 $M_\odot$ km s^{-1}	$128\mathcal{P}$
$\mathcal{S}$	6.31×10^{-68} cm^{-3} s^{-1}	58.4 pc^{-3} Myr^{-1}	$\mathcal{S}/128$
$\mathcal{M}$	3.73×10^{34} g	18.7 $M_\odot$	$64\mathcal{M}$
$\mathcal{L}$	1.14×10^{18} cm	0.370 pc	$4\mathcal{L}$
$\mathcal{T}$	1.07×10^{13} s	0.338 Myr	$2\mathcal{T}$
$\mathcal{V}$	1.07×10^{5} cm s^{-1}	1.07 km s^{-1}	$2\mathcal{V}$
$\mathcal{A}$	1.00×10^{-8} cm s^{-2}	3.23 pc Myr^{-2}	$\mathcal{A}$

Outflow Parameters

Description	Symbol	Value	Comment
Density	ρ_0	2.5×10^{-20} g cm^{-3}	...
Velocity	V_o	65.5 km s^{-1}	...
Duration	t_o	2.34 kyr	...
Period	T_o	5.28 kyr	...
Radius	r_o	5960 AU	$10\Delta x$ at 512^3
Inner buffer	z_i	$2\Delta x$	1190 AU at 512^3
Launch thickness	Δz	3580 AU	$6\Delta x$ at 512^3
Opening angle	θ_o	$0°$	...
Mass loss rate	$\dot{M}$	1.30×10^{-4} $M_\odot$ yr^{-1}	...

3 pc box, 0.2km/s sound

Feedback, SN and Ioniz. Rad

- In terms of energy, SN+SW, ionization radiation need only be of order 0.01 or 0.001 efficient in coupling to gas to stop GMC free fall at even largest scale. Subtlety is in the coupling, so how much momentum is deposited

- HI shells likely accounted for by feedback from SN or ionizing radiation

- Heiles (79) 4 x $10^8 M_{sun}$ in HI shells, 10^7 year life, $40M_{sun}$/yr; estimate can easily be accounted for by SN or ionizing radiation

- absence of H_2 in inter-arm regions gives minimum requirement of $10M_{sun}$/yr to convert to HI, also accommodated by SN or ionizing rad.

Conclusions

- Time dependent virial evolution of spherical clouds subject to inclusion of external pressure shows that clouds “hug” critical radius and mass values as they evolve, if kinetic energy dissipation time scale is ≥ 2 crossing time; near viral equipartition even though time evolving
- can explain why clouds statistically appear to be in critical state, satisfying Larson’s laws, SLWR, density-size scale, subsequent mass conserving fragmentation cascade can explain number distribution, constant ε_{ff}..
- density fluctuations are needed for fragmentation cascade
- Feedback enters through the dissipation time scale
- feedback required to abate star forming infall “momentum” depends on coupling scale (less demanding for smaller scales)
- Reducing SFR from unfettered free-fall value is a distinct issue from supplying overall velocity dispersions to MC; impulsive feedback and feedback on clump scales may leave unfettered “inertial range”; overall: 1) source of background fluctuations 2) velocity dispersions from fragmentation 3) most abatement of SFR from high mass stellar intermettent effect (SN, SW) 4) some from outflows from low mass stars (factor of few)
- Generalize to cylinders
- much opportunity for further study, comparison and iteration with simulations; simple models have value in this regard, and for pondering what ultimately goes in textbooks

END

- Federrath 2013 (very high res) scaling $\sigma \propto R^{1/2}$ ($P(\sigma) \propto k^{-2}$ Burgers) but $P(\rho^{1/3}\sigma) \propto k^{-1.74}$ (solenoidal) $P(\rho^{1/3}\sigma) \propto k^{-19/9}$ (compressive) whereas obs: $P(\rho^{1/3}\sigma) \propto k^{-4/3}$ (need self gravity, cooling); momentum spectra don't quite scale as observed without self-gravity, or even with..

Sources of feedback and their role

Bottom up vs. top down

- Two types of bottom up feedback
    - (1) steady feedback from outflows injecting on on sub-parsec scales that disperse and abate star formation
    - (2) Less frequent but more catastrophic feedback from massive stars that destroy clouds completely
- Turbulence also supplied top down from ambient supernovae driven ISM, spiral arms, and large scale colliding flows
- Distinguish feedback from large scale source of turbulence

Resulting Scaling Relations, (F_M constant)

$$\sigma \propto x^{p_1} \sim x^{0.5}$$

$$\rho \propto x^{2p_1-2} \sim x^{-1}$$

$$M(x) \propto x^{2p_1+1} \sim x^2$$

$$\frac{dN}{dx} \propto x^{-3p_1-1} \sim x^{-2.5}$$

$$\frac{dN}{dM} = \frac{dN/dx}{dM/dx} = x^{-5p_1-1} \propto M^{\frac{-5p_1-1}{2p_1+1}} \sim M^{-1.75}$$

- Not horrible →

Time-Dependent Virial Equation

$$(1) \text{ and } (2) \quad \Rightarrow$$

$$\sigma^2 \simeq 0.8(4\pi G P_e)^{1/2} R_c$$

Size Line Width Relation (SLRW)
….but why should $R=R_c$?

- external pressure implies a crit- ical mass above which a cloud cannot exist in stable equilibrium. We follow an earlier suggestion by Chie'ze (1987) that the masses of MCs tend to be equal to a critical mass Mc, defined as the largest stable mass for their internal kinetic energy and external pressure. If a cloud's mass is equal to its critical mass, then hydrostatic equilib- rium (generally defined with support from internal kinetic as well as thermal energy) implies a critical column density that is now defined solely by the external pressure.

- If the clouds in the interstellar medium (ISM) tend towards

- this critical column density, a further consequence is that there

- is still a size–linewidth relation, but it is not a simple proportion-

- ality as originally suggested by Larson (1981). Rather the critical

- mass defines a critical radius with the result that the proportional-

- ity between linewidth and size depends on the external pressure,

- $\sigma 2/R \propto \sigma 2/R \propto P1/2$. ce

Virial Equilibrium and Virial Parameter

- $M_v = 5\,\sigma^2 R/G$
- $M = 4\pi\rho R^3/3$
- For p_1=0.5, p_2= -1, M_v / M = constant = α G
- α is virial parameter (e.g. Bertoldi & Mckee)
- α=1 for virial equilibrium
- perhaps larger scales show α=1 more commonly than smaller scales

Basic Picture

- α =1: **Large scale structures unstable to gravitational collapse** *derive observable MC cascade scalings assuming gravity is primary source of velocity dispersion*

- α >1: **Small scale 'pressure' confined structures** (Elmegreen 85; Keto Meyers 86; Bertoldi & Mckee 92): stability of HLC, and "pressure" bound clumps, consistent with p_1=0.5. (Below critical mass, both grav. unstable and pressure stable solutions are available)

- SFR mediation can be "incorporated" via scale dependent mass flux. To suppress given mass deposition rate, needed feedback power decreases with decreasing scale

CASE 1: $\alpha=1$

- "Inertial range" gravity driven
- $\rho R^2 / \sigma^2$ = constant: $p_2 = 2p_1 - 2$
- Nested structure: $x=R/R_1$

$$M_1 = -\int_0^1 M(x) \frac{dN}{dx} dx$$

$$M(x) = M_1 x^{p_4} = M_1 x^{2p_1+1}$$

$$\frac{dN}{dx} \propto x^{p_3}$$

$$F_M = -M(x) \frac{dN}{dx} \frac{dx}{dt}$$

- mass flux

- Constant F_M ⟶ $p_3 = -3p_1 - 1$
- Constant dF_M/dR ⟶ $p_3 = -3p_1$

Resulting Scaling Relations, (dF_M/dR =constant)

$$\sigma \propto x^{p_1} \sim x^{0.5}$$

$$\rho \propto x^{2p_1-2} \sim x^{-1}$$

$$M(x) \propto x^{2p_1+1} \sim x^2$$

$$\frac{dN}{dx} \propto x^{-3p_1} \propto x^{-1.5}$$

$$\frac{dN}{dM} = \frac{dN/dx}{dM/dx} = x^{-5p_1} \propto M^{\frac{-5p_1}{2p_1+1}} \sim M^{-1.25}$$

- Not horrible either →

$$\frac{dN}{dM} \propto M^{-p_5}$$

Author Year	p_5
Blitz et al. 2007	-1.7
Casoli et al. 1984	-1.4 to -1.6
Loren 1989a	-1.1
Myers et al. 1983	-1 to -1.5
Snell et al. 2002	-1.9
Solomon et al. 1987	-1.5
Williams & Blitz 1995	-1.3

Comment on p_1 and conservation laws

- Would like to derive p_1 from conservation relations
- conservation of momentum/mass $\longleftrightarrow$ $p_1 = 0.5$
- Maybe physically motivated for supersonic grav driven flow (note alternate context of outflow e.g. Matzner 07)

CASE 2: $\alpha > 1$

- Viral theorem with ext pressure

$$\frac{3GM^2}{20\pi R^4} - \frac{3M\sigma^2}{4\pi R^3} + P_e = 0$$

$$y^2 - 2y + y_0 = 0$$

$$y = \frac{M}{\pi R^2} \frac{2\pi GR}{5\sigma^2} \qquad y_0 = \frac{P_e}{P_c}; \; P_c = \frac{15\sigma^4}{16\pi GR^2}$$

$$y = 1 \pm (1 - y_0)^{1/2}$$

- No equilib for $P_e > P_{e,c}$ and two solns for $P_e < P_{e,c}$

For given P_e, M_c is the M for which $P_e = P_{e,c}$

- $GM_c^2/RV \approx M_c \sigma^2/V^{4/3}$ at critical point
- $V \approx M_c\sigma^2/P_e$ for a given P_e
- $M_c \approx \sigma^4/(GP_e^{1/2})$
- For $M > M_c$ unstable collapse
- For $M < M_c$ pressure confined solutions, collapse not inevitable for pressure branch (grav branch are unstable Keto Field 05)
- Requires source of P_e

Application to Observations

Pressure bound HLC (Magnani et al 85, Keto & Meyers 86; Heithhause 1990)

- p_1=0.5 is measured, σ^2/R measured, then compute $P_{e,c}$=1.2 x 10^{-10} dyn/cm^2
- Infer $P_e/P_{e,c}$ = 10^{-3} observationally
- Solve for surface density in above formalism: =1.3 x 10^{-4} g/cm^2 and A_v=0.2
- Agrees with Keto & Meyers 86

Part 1: A few “problems” with MRI simulations and viscous disk models

Part 2: Minimalist model of Cloud Fragmentation and Feedback

Eric Blackman (U. Rochester)

A few “Concerns” with MRI simulations and alpha disks

$$\alpha_{\rm mag} \equiv -\frac{2f(\Gamma)\langle b_y b_x\rangle}{3c_{\rm s}^2} = \frac{C_{\rm mag}(\Gamma,\beta)}{\beta},$$

$$\alpha_{\rm kin} \equiv \frac{2f(\Gamma)\langle v_y v_x\rangle}{3c_{\rm s}^2} \equiv \frac{C_{\rm kin}(\Gamma,\beta)}{\beta},$$

$\alpha_{tot}=\alpha_{mag}+\alpha_{kin}$

(Blackman, Penna, Varniere 07)

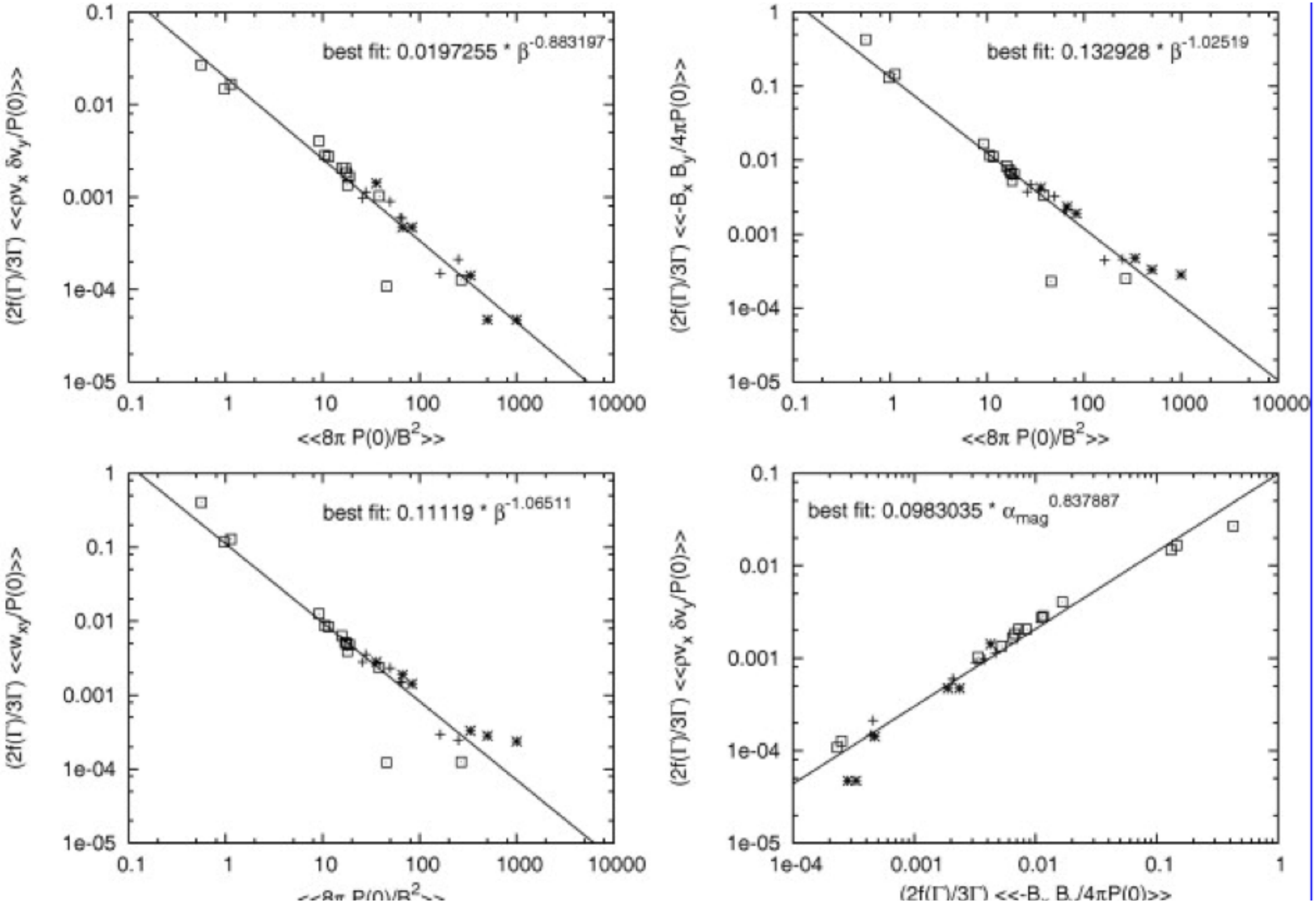


Fig. 1. Isothermal data of Table 8, Table 9 and Table 10 and best fit lines. Top row: Log $\alpha_{kin}(P(0))$ and Log $\alpha_{mag}(P(0))$ vs. Log $\beta(P(0))$, respectively. Bottom row: Log $\alpha_{tot}(P(0))$ vs. Log $\beta(P(0))$ and Log $\alpha_{kin}(P(0))$ vs. Log $\alpha_{mag}(P(0))$. The double brackets indicate that the data represent a combination of spatial average and late time average (e.g. after 15 orbits). The symbols indicate the following specific data sets, respectively: ✳ – Fleming and Stone (2003); □ – Miller and Stone (2000); + – Stone et al. (1996). Values of the magnetic field energy, Maxwell stress, and Reynolds stress are all normalized with respect to the midplane gas pressure, $P(0)$, and for all of these runs, $P(0) = 5 \times 10^{-7}$. For the most part, despite the different initial and boundary conditions and wide ranges of α_{mag}, α_{kin}, and α_{tot}, products of form $\alpha\beta$ lie close to the best fit lines shown. The line equations are at the top of each panel.

$$\frac{\bar{T}_{r\phi}}{\bar{P}} = q\alpha \simeq 0.75\left(\frac{L}{H}\right)^{5/3} \times \begin{cases} \Delta/L & \text{if} \quad \lambda_{\rm MRI} \le \Delta \\ \lambda_{\rm MRI}/L & \text{if} \quad \Delta < \lambda_{\rm MRI} \le L \\ 0 & \text{if} \quad \lambda_{\rm MRI} > L \end{cases}$$

Pessah et al 2007

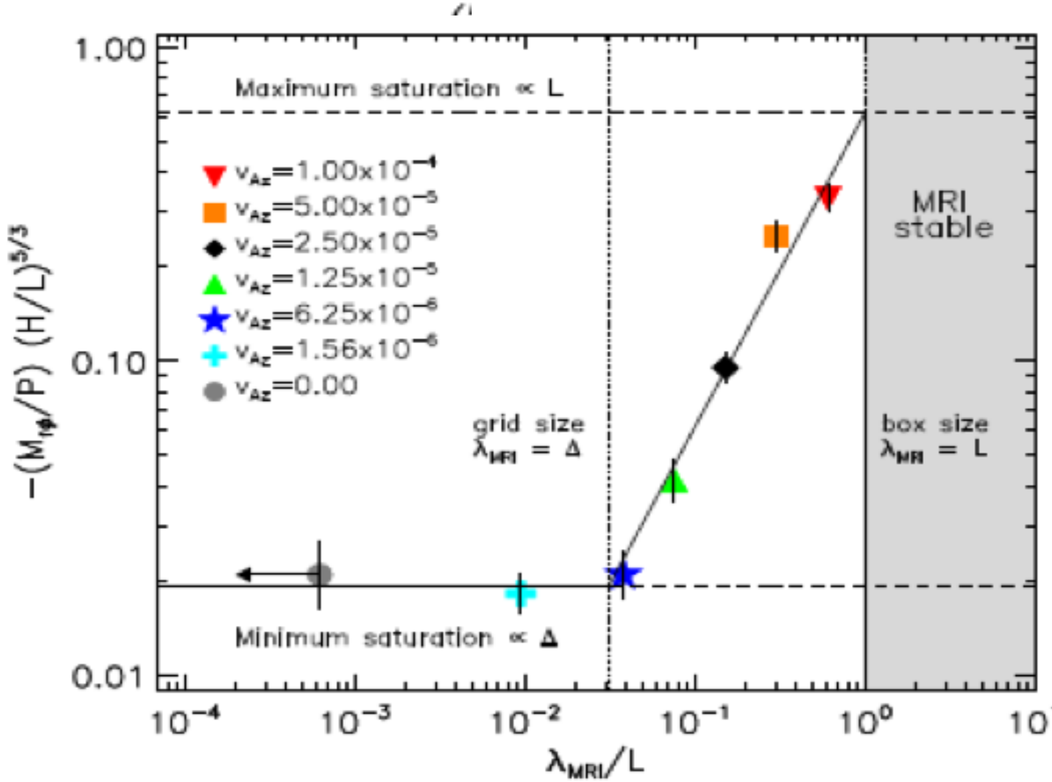


FIG. 2.— Dimensionless magnetic stress, $-\bar{M}_{r\phi}/\bar{P}$, multiplied by $(H/L)^{5/3}$, as a function of the wavelength corresponding to the most unstable MRI mode, $\lambda_{\rm MRI}$. The various types of symbols correspond to the best fit values characterizing each class of simulations according to the value of the ratio $\bar{v}_{{\rm A}z}/L\Omega_0$, as inferred from Figure 1. The simulations with zero mean magnetic flux, i.e., $\bar{v}_{{\rm A}z} = 0$, are displayed at some arbitrary value for visualization purposes only. The error bars quantify the scatter within each class of simulations around the corresponding mean values. Vertical *dotted lines* represent the values at which the most unstable MRI wavelength equals the grid size and the size of the box, respectively, i.e., $\lambda_{\rm MRI} = \Delta = 1/32$ and $\lambda_{\rm MRI} = L = 1$. The overall dependence of the saturation level on the ratio $\lambda_{\rm MRI}/L$ (*solid line*) is given by the saturation predictor (3). Numerical simulations for which $\lambda_{\rm MRI} > L$ are stable to the MRI (*shaded region*).

Pessah et al. 2006

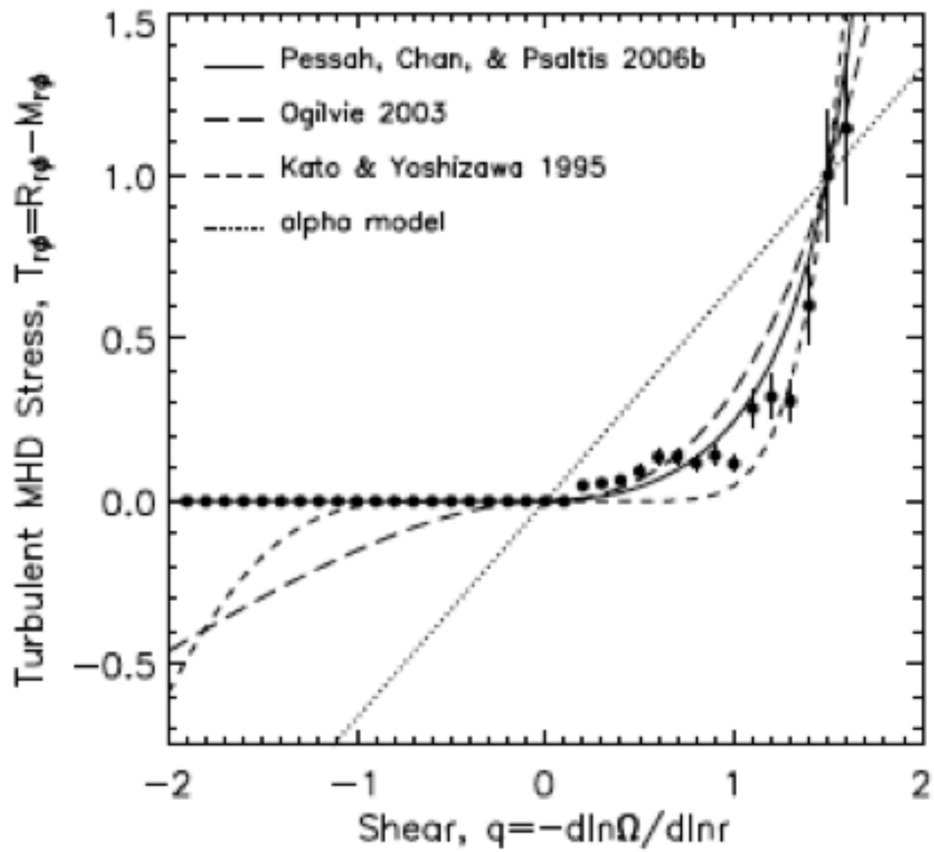


Figure 1. Total turbulent stress at saturation as a function of the shear parameter $q \equiv -d\ln\Omega/d\ln r$. The various lines show the predictions corresponding to the three models for turbulent MHD angular momentum transport discussed in §3. The filled circles correspond to the volume and time averaged MHD stresses calculated from the series of shearing box simulations described in §4. Vertical bars show the typical rms spread (roughly 20%) in the stresses as calculated from ten numerical simulations for a Keplerian disc. The dotted line corresponds to a linear relationship between the stresses and the local shear, like the one assumed in the standard model for alpha viscosity. All the quantities in the figure are normalized to unity for a Keplerian profile, i.e., for $q = 3/2$.

Handful of “Concerns”

- Pr_M indeterminate in many simulations (other than Axel’s)
- α ranges 4+ orders of magnitude but $\alpha\beta$=constant
- Box size and initial mean field strength determine saturation
- Without mean field effective alpha is too low when scaled
- Saturation in “ideal MHD” periodic boxes has linear dependence on initial vertical field seed strength. Weird.
- periodic boxes: chaos in “restricted homotopy class”
- α interpretation of MRI does not scale correctly with shear

- pure viscosity “model” of disk turbulence misses explicit transport term in surface density equation (Hubbard and Blackman in prep.).
- What are minimum properties that turbulence must have to produce outward angular momentum transfer?
- Improve closure model to allow non-viscous physics but still allow tractable analytic theory