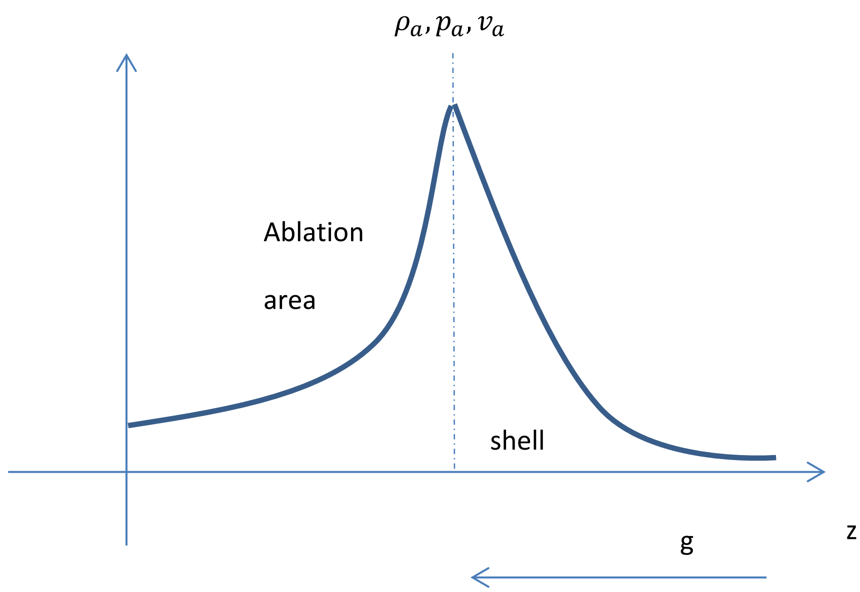
Ablative Equilibrium Condition



We define an Ablative RT equilibration problem by defining ρ, p, v at the ablation front (dot-dash line in Fig. 1) and the gravity g. The hydro profiles are then calculated with different approximations in two regions below and above the ablation front.

(1) In the shell above the ablation front, density is high, temperature is low, so the thermal conduction is neglected here. The shell is performing like a solid piston. So the hydrodynamic equations are approximated as:

$$\begin{cases} v = v_a \\ p = s(z)\ \rho^{\gamma} \\ \dfrac{d}{dz}(p + \rho v^2) = -\rho g \end{cases}$$

where $\gamma = 5/3, v_a$ is the fluid velocity at the ablation front and should have a negative value. $s(z)$ is a prescribed entropy function which is used to adjust the shape of the shell.

Combining these 3 equations above we obtain:

$$\frac{d\rho}{dz} = -\frac{s'(z)\rho^{\gamma} + \rho\, g}{s(z)\,\gamma\,\rho^{\gamma-1} + v_a^2}$$

$$\rho(z = z_{ablation}) = \rho_a$$

This ODE can be readily solved to get the density profile and each quantity is then determined for the shell.

(2) In the ablation area below the ablation front, density is low, temperature is very high. The profile in this region is in equilibrium, that is d/dt=0. Simply drop the d/dt terms in hydrodynamic equations:

$$\begin{cases} \rho\, v = \rho_a\, v_a \\ \dfrac{d}{dz}(p + \rho v^2) = -\rho g \\ \dfrac{d}{dz}\left[v\left(\dfrac{\gamma\, p}{\gamma - 1} + \rho \dfrac{v^2}{2}\right) - \kappa \dfrac{dT}{dz}\right] = -\rho g v \\ p = \rho T / A \end{cases}$$

A is the average particle mass $A = M_i/(Z+1)$. For notational convenience, define a constant $\sigma \equiv \rho_a v_a$. Combine the 4 equations above to 2 equations:

$$\begin{cases} \dfrac{d}{dz}\left(\dfrac{\sigma^2}{\rho} + \dfrac{\rho T}{A}\right) = -\rho g \\ \dfrac{d}{dz}\left[\dfrac{\gamma \sigma T}{(\gamma - 1)A} + \dfrac{\sigma^3}{2\rho^2} - \kappa \dfrac{dT}{dz}\right] = -g\sigma \end{cases}$$

The second equation can be integrated analytically and yield:

$$\left[\frac{\gamma \sigma T}{(\gamma - 1)A} + \frac{\sigma^3}{2\rho^2} - \kappa \frac{dT}{dz}\right] = -g\sigma z + const$$

The integration constant is chosen as $const = \ g\sigma z_{albation}$ so the RHS terms cancel out at the ablation front.

Now we need to solve the ODE system simultaneously:

$$\begin{cases} \dfrac{dT}{dz} = \left[\dfrac{\gamma \sigma T}{(\gamma - 1)A} + \dfrac{\sigma^3}{2\rho^2} + g\sigma z - const\right] / \kappa \\ \dfrac{dQ}{dz} = -\rho g \\ Q = \dfrac{\sigma^2}{\rho} + \dfrac{\rho T}{A} \end{cases}$$

This system need to be solved numerically. Also note that κ is a function of T in Spitzer's model. After T and Q are advanced in each step, ρ can be found via

$$\rho = \frac{Q + \sqrt{Q^2 - 4\sigma^2 T/A}}{2T/A}$$