Scales for Ablative RT Problem

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1 Betti's 2D parameters

All variables are in SI unit except Temperature which is in Joule Equation of State

$$P = 4.7904E26\rho T \tag{1}$$

Conductivity

$$\kappa_0 = 3.734E69 \tag{2}$$

Capacity

$$C_v = 7.186E26 \tag{3}$$

Adiabatic Index/Heat Capacity Ratio

$$\gamma = 5/3 \tag{4}$$

2 Calculated using Betti's Parameters

2.1 Mean Molecular Weight

$$\begin{aligned} X_\mu &= 1.0/(4.7904E26 * h_{mass}) \\ &= 1.0/(4.7904E26 * 1.67E-27) \\ &= 1.25 \end{aligned} \tag{5}$$

Eq.(5) is derived from

$$\begin{aligned} P &= nRT \\ &= \frac{\rho}{N_A X_\mu h_{mass}} \frac{R}{K_B}(K_B T) \\ &= \frac{1.0}{X_\mu h_{mass}} \rho (K_B T) \\ &= 4.7904E26\rho T' \end{aligned} \tag{6}$$

where T' is the temperature in Joule as in Betti's

2.2 Specific Heat Capacity C_v

$$
\begin{aligned}
C_v &= 4.7904E26/(\gamma-1) \\
&= 7.186E26
\end{aligned} \tag{7}
$$

Eq.(7) can be derived from the specific inertial energy (internal energy per unit mass) $e = C_v T = \frac{U}{\rho V}$. Since the internal energy

$$
\begin{aligned}
U &= \frac{1}{\gamma-1} nRT \\
&= \frac{1}{\gamma-1} PV
\end{aligned} \tag{8}
$$

So $P = (\gamma - 1)C_v \rho T$.

3 AstroBEAR Scales

3.1 pSCale

pScale=rScale/hMass/Xmu*TempScale*Boltzmann As

$$
\begin{aligned}
PV &= nRT \\
&= \frac{\rho}{N_A X_\mu h_{mass}} * R * T \\
&= \frac{\rho K_B}{X_\mu h_{mass}} T
\end{aligned} \tag{9}
$$

3.2 ScaleFlux

In AstroBEAR we have

$$
\frac{1}{\gamma-1}\rho\frac{\partial T}{\partial t} = \frac{\partial Q}{\partial x} \tag{10}
$$

So the heat flux scale "ScaleFlux=rScale*TempScale*VelScale" And since "VelScale=SQRT(pScale/rScale)", "ScaleFlux=TempScale*SQRT(pScacle*rScale)"

3.3 ScaleDiff

ScaleDiff is the scale for "Kappa1", it comes from the equation

$$
\frac{1}{\gamma-1}\rho\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\kappa_1 T^n \frac{\partial T}{\partial x}\right] \tag{11}
$$

So

$$
\begin{aligned}
(\kappa_1 T^n)_{scale} &= \frac{1}{1}(rScale * VelScale * lScale) \\
&= \frac{lScale * \sqrt{pScale * rScale}}{1}
\end{aligned} \tag{12}
$$

So "ScaleDiff=lScale*SQRT(pScale*rScale)/TempScale**nDiff"

3.4 EnergyScale

"EnergyScale=pScale"

4 AstroBEAR Variables

hydro/2dpressure: $P = (\gamma - 1)(E - 0.5 * (px^2 + py^2)/\rho)$, where $E - 0.5 * (px^2 + py^2)/\rho$ is the internal energy density (per unit mass). This is the $e = C_v T$ and comes from the equation $P = (\gamma - 1)C_v \rho T$. And $C_v \rho T$ is the internal energy density(per volume)
And hydro/3Dpressure: $P = (\gamma - 1)(E - 0.5 * (p_x^2 + p_y^2 + p_z^2)/\rho)$
Correspondingly, Energy density (per volumn): $E = 0.5 * (p_x^2 + p_y^2 + p_z^2)/\rho + P/(\gamma - 1)$
hydro/Temp: P/ρ This comes from $P = \frac{K_B}{X_\mu h_{mass}} \rho T$ and considering the relation between "pScale" and "TempScale"

$$\frac{pSCale}{TempScale} = \frac{rScale * Boltzmann}{Xmu * hmass} \tag{13}$$

5 Rui's Parameters

5.1 Original Doc

$L_0 = 1E - 3cm$ $\kappa_0 = 2.0637032E12 erg/(cm - s - K)$ $Cv_0 = 8.2548129E + 07 erg/g - deg$ $Q_0 = 6.2500000E + 20 erg/cm^2 - sec$

5.2 Compare with Riccardo's 2D Parameter

Parameters	2D	3D
Dimensions	(0.0348,1.2,0)E-2 cm	(0.1,0.1,1.2)E-2cm
Conductivity(κ_0)	3.734E69 J/(m-s-J)	2.0637032E12 erg/(cm-s-K)
Capacity (C_v)	7.186E26 J/(kg-J)	8.2548129E+07 erg/g-K
Bottom Heat flux (q_0)	5.876E18 W/m^2	5.1819941E+21 or 5.1853192E+21 $(erg/cm^2 - sec)$
Gravity (g)	1E14 m/s^2	2.5E16 cm/s

In computational units

Parameters	2D	3D
Dimensions	(0.0348,1.2,0)E-4	(0.1,0.1,1.2)E-4
Conductivity(κ_0)	3.734E69 J/(m-s-J)	2.0637032E12 erg/(cm-
Capacity (C_v)	7.186E26 J/(kg-J)	8.2548129E+07 erg/g
Bottom Heat flux (q_0)	5.876E18 W/m^2	5.1819941E+21 or 5.1853192E+21
Gravity (g)	1E14 m/s^2	2.5E16 cm/s

5.3 AstroBEAR Variables

6 Equations

6.1 SI and Joule for Temperature

Equation solved in Betti's code (SI units)

$$\rho C_v \frac{\partial(K_B T)}{\partial t} = \frac{\partial}{\partial x}\left[\kappa_0 (K_B T)^n \frac{\partial(K_B T}{\partial x}\right] \tag{14}$$

where K_B is the Boltzmann constant and the flux is

$$q_0 = \kappa_0 (K_B T)^n \frac{\partial(K_B T}{\partial x_{SI}} \tag{15}$$

To convert this to cgs units we have

$$10^3 \rho_{cgs} \frac{\partial T}{\partial t} = \frac{\kappa_0 K_B^n}{C_v} \frac{\partial}{10^{-2}\partial x_{cgs}}\left[T^n \frac{\partial T}{10^{-2}\partial x_{cgs}}\right] \tag{16}$$

That is

$$\rho_{cgs} \frac{\partial T}{\partial t} = \frac{10\kappa_0 K_B^n}{C_v} \frac{\partial}{\partial x_{cgs}}\left[T^n \frac{\partial T}{\partial x_{cgs}}\right] \tag{17}$$

In AstroBEAR we define

$$\kappa_1 = \frac{10\kappa_0 K_B^n}{C_v} \tag{18}$$

and

$$q^*_{cgs} = \kappa_1 \frac{\partial T}{\partial x_{cgs}} \tag{19}$$

Comparing the definition of q_0 and q^*_{cgs} we have

$$q_0 = 10 q^*_{cgs} C_v K_B \tag{20}$$

In Betti's data, $C_v = 7.816 \times 10^{26}$ so

$$q_0 = 7.816 * 1.38 * 10^4 * q^*_{cgs} \tag{21}$$

6.2 AstroBEAR

In Betti's definition

$$C_v = \frac{K_B}{X_\mu h_{mass}} \tag{22}$$

While in AstroBEAR

$$pScale = \frac{rScale}{h_{mass} X_\mu} TempScale * K_B \tag{23}$$

and

$$C_v = \frac{pScale}{TempScale * rScale} \tag{24}$$

So

$$Cv = \frac{K_B}{(\gamma - 1)x_\mu h_{mass}} \tag{25}$$

To include the $\frac{1}{\gamma-1}$ term, we modify the equation to

$$\frac{1}{\gamma - 1}\rho\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\frac{1}{\gamma - 1}\kappa_1 T^n \frac{\partial T}{\partial x}\right] \tag{26}$$

Since in the AstroBEAR code there was already a $\gamma 7$ or $\frac{1}{\gamma-1}$ in the left side of the equation, we just include $\gamma 7$ in the κ_1. So now the new κ_1 becomes

$$\begin{aligned}\kappa_1 &= \frac{10\kappa_0 K_B^n}{(\gamma - 1)C_v} \\ &= \frac{10 * 3.734E69 * (1.38E - 23)^{2.5}}{(\frac{5}{3} - 1) * 7.186E26} \\ &= 5.52 \times 10^{-14}\end{aligned} \tag{27}$$

6.3 Rui's cgs Units and AstroBEAR

$$\rho C_v \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\kappa_0 T^n \frac{\partial T}{\partial x}\right] \tag{28}$$

and the flux is

$$q_0 = \kappa_0 T^n \frac{\partial T}{\partial x} \tag{29}$$

In AstroBEAR again it's

$$\frac{1}{\gamma - 1}\rho\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\frac{1}{\gamma - 1}\kappa_1 T^n \frac{\partial T}{\partial x}\right] \tag{30}$$

So $\kappa_1 = \frac{1}{\gamma-1}\frac{\kappa_0}{C_v}$ and $q^*_{cgs} = \kappa_1 T^n \frac{\partial T}{\partial x}$. Since $q_0 = \kappa_0 T^n \frac{\partial T}{\partial x}$. So $q^*_{cgs} = \frac{1}{(\gamma-1)C_v} q_0$

So in physics.data we should put the diffusion kappa1 (in cgs unit) as

$$\begin{aligned}\kappa_1 &= \frac{1}{\frac{5}{3} - 1}\frac{2.0637032E + 12}{8.2548129E + 07} \\ &= 3.75E4\end{aligned} \tag{31}$$