MUSCL Scheme and Sweep Scheme in AstroBEAR

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MUSCL Scheme
- One-Dimensional MUSCL Scheme
- Multi-Dimensional: Splitting VS UnSplitting

Sweep Scheme in AstroBEAR

Testing Results

Summary MUSCL Scheme in AstroBEAR
MUSCL: Monotone Upwind(Upstream)-centered Schemes for Conservation Laws

MUSCL-Hancock Method

- Spatial reconstruction:
  \[ U_{LR}^i = U_i^n \pm \frac{1}{2} \Delta_i \]

- Temporal evolution (CFL condition):
  \[ \overline{U}_{i}^{LR} = U_{LR}^i + \frac{1}{2} \frac{\Delta t}{\Delta x} \left[ f(U_{L}^i) - f(U_{R}^i) \right] \]

- Solving intercel flux \( f_{i+\frac{1}{2}} \) with Piece-Wise Riemann Problem:
  \[ U_t + f(U)_x = 0 \]
  \[ U(x, 0) = \begin{cases} \overline{U}^R_i, & x < 0 \\ \overline{U}^L_i, & x > 0 \end{cases} \]

- Conservative update:
  \[ U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x} \left[ f_{i-\frac{1}{2}} - f_{i+\frac{1}{2}} \right] \]

Figure 1: MUSCL Scheme: Spatial Reconstruction
Splitting Methods

Splitting Methods:

1. \[ U_{i,j}^{n+1} = U_{i,j}^n + \frac{\Delta t}{\Delta x} \left[ f_{i-\frac{1}{2},j} - f_{i+\frac{1}{2},j} \right] \] for all \( i \)

2. \[ U_{i,j}^{n+1} = U_{i,j}^{n+1} + \frac{\Delta t}{\Delta y} \left[ f_{i,j-\frac{1}{2}} - f_{i,j+\frac{1}{2}} \right] \] for all \( j \)

3. Exchange directions every step in 3D \((x, y, z), (y, z, x), (z, x, y), \cdots\)

Figure 2: Splitting methods: Update \( U_{x,y} \) with \( f_x \) first, then with \( f_y \)
Unsplitting Methods

1. \[ U_{i,j}^{n+1} = U_{i,j}^n + \frac{\Delta t}{\Delta x} \Delta f_i + \frac{\Delta t}{\Delta y} \Delta f_j \] for all \( i, j \)
2. Exchange directions every step in 3D \((x, y, z), (y, z, x), (z, x, y), \cdots\)

Figure 3: Unsplitting Methods: Update \( U_{x,y} \) with \( f_x \) and \( f_y \)
Splitting Vs. Unsplitting

- 3D update: $x \rightarrow y \rightarrow z$
- 1D stencil
- Straight forward

- 3D update: $x + y + z$
- 3D stencil
- Corner Transport Upwind (CTU)

Figure 4: splitting method

Figure 5: unsplitting method
MUSCL Scheme

1. Spatial reconstruction: \( q^n_{LR,x} \)
2. Temporal evolution: \( q^{n+\frac{1}{2}}_{LR,x} \)
3. Riemann solver: \( f^{n+\frac{1}{2}}_{LR,x} \)
4. Conservative update: \( q^{n+1} = q^n + \Delta f_x \)
5. repeat in \( y \) and \( z \)

Sweep Scheme in AstroBEAR

1. Spatial reconstruction in \( x, y, z \): \( q^n_{LR,x,y,z} \)
2. Temporal evolution in \( x, y, z \): \( q^{n+\frac{1}{2}}_{LR,x,y,z} \)
3. Calculate predicted fluxes: \( f^{n+\frac{1}{2}*}_{x,y,z} \)
4. Transvers flux update (CTU): \( q^{n+\frac{1}{2}}_{LR,x,y,z} \)
5. Calculate final fluxes: \( f^{n+\frac{1}{2}}_{x,y,z} \)
6. Update: \( q^{n+1} = q^n + \Delta f_x + \Delta f_y + \Delta f_z \)
Results from 1D MUSCL Euler Equation Solver and AstroBEAR

**Figure 6:** Sod Shock Tube: Density

**Figure 7:** Sod Shock Tube: Velocity

**Figure 8:** Sod Shock Tube: Pressure
- A simple alternative scheme for AstroBEAR
- Things to do: add tracer, get multi-dimension work
- Things to Add: Fluxes/slope Limiters, How CTU implemented in AstroBEAR