MUSCL Scheme and Sweep Scheme in AstroBEAR

Baowei Liu

University of Rochester

January 21, 2013

Table of contents

MUSCL: Monotone Upwind(Upstream)-centered Schemes for Conservation Laws

MUSCL-Hancock Method

- Spatial reconstruction:
  $U^{LR}i = U_i^n \mp \frac{1}{2}\Delta_i$
- Temporal evolution (CFL condition):
  $\bar{U}_i^{LR} = U_i^{LR} + \frac{1}{2}\frac{\Delta t}{\Delta x}\left[f(U_i^L) - f(U_i^R)\right]$
- Solving intercel flux $f^*_{i+\frac{1}{2}}$ with Piece-Wise Riemann Problem:
  $U_t + f(U)_x = 0$
  $U(x,0) = \begin{cases} \bar{U}_i^R, x<0 \\ \bar{U}_i^L, x>0 \end{cases}$
- Conservative update:
  $U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x}\left[f^*_{i-\frac{1}{2}} - f^*_{i+\frac{1}{2}}\right]$

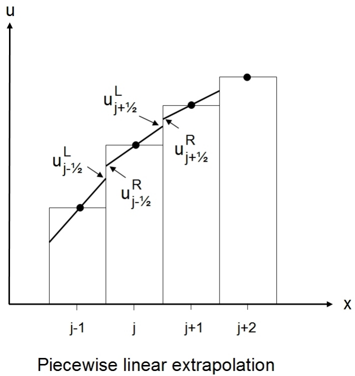

Figure 1: MUSCL Scheme:Spatial Reconstruction

Splitting Methods

Splitting Methods:

1. $U^{*n+1}_{i,j} = U^n_{i,j} + \frac{\Delta t}{\Delta x}\left[f_{i-\frac{1}{2},j} - f_{i+\frac{1}{2},j}\right]$ for all i
2. $U^{n+1}_{i,j} = U^{*n+1}_{i,j} + \frac{\Delta t}{\Delta y}\left[f_{i,j-\frac{1}{2}} - f_{i,j+\frac{1}{2}}\right]$ for all j
3. Exchange directions every step in 3D $(x, y, z), (y, z, x), (z, x, y), \cdots$

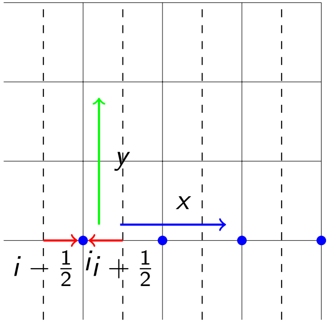

Figure 2: Splitting methods: Update $U_{x,y}$ with f_x first, then with f_y

Unsplitting Methods

1. $U_{i,j}^{n+1} = U_{i,j}^{n} + \frac{\Delta t}{\Delta x}\Delta f_i + \frac{\Delta t}{\Delta y}\Delta f_j$ for all i, j
2. Exchange directions every step in 3D $(x, y, z), (y, z, x), (z, x, y), \cdots$

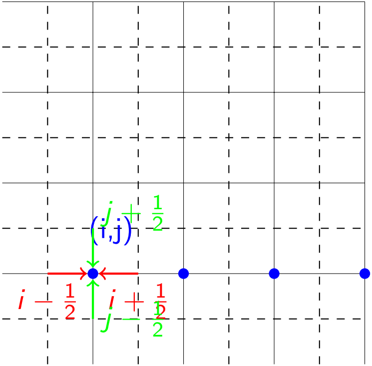

Figure 3: Unsplitting Methods: Update $U_{x,y}$ with f_x and f_y

Splitting Vs. Unsplitting

- 3D update: $x \to y \to z$
- 1D stencil
- Straight forward

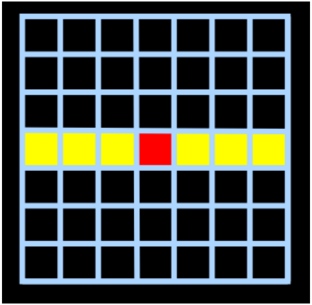

Figure 4: splitting method

- 3D update: $x + y + z$
- 3D stencil
- Corner Transport Upwind (CTU)

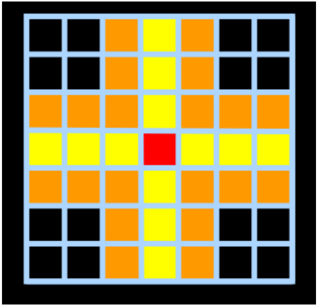

Figure 5: unsplitting method

Sweep scheme in AstroBEAR

MUSCL Scheme

1. Spatial reconstruction: $q^{n}_{LR,x}$
2. Temporal evolution: $q^{n+\frac{1}{2}}_{LR,x}$
3. Riemann solver: $f^{n+\frac{1}{2}}_{LR,x}$
4. Conservative update: $q^{n+1} = q^{n} + \Delta f_x$
5. repeat in y and z

Sweep Scheme in AstroBEAR

1. Spatial reconstruction in x, y, z: $q^{n}_{LR,x,y,z}$
2. Temporal evolution in x, y, z: $q^{n+\frac{1}{2}}_{LR,x,y,z}$
3. Calculate predicted fluxes: $f^{n+\frac{1}{2}*}_{x,y,z}$
4. Transvers flux update (CTU): $q^{n+\frac{1}{2}}_{LR,x,y,z}$
5. Calculate final fluxes: $f^{n+\frac{1}{2}}_{x,y,z}$
6. Update: $q^{n+1} = q^{n} + \Delta f_x + \Delta f_y + \Delta f_z$

Results from 1D MUSCL Euler Equation Solver and AstroBEAR

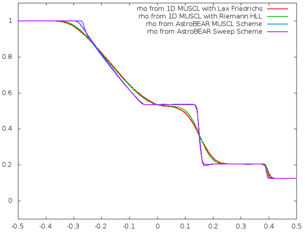

Figure 6: Sod Shock Tube: Density

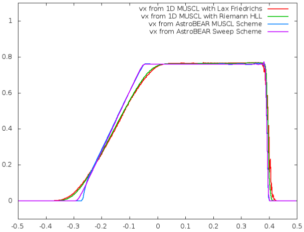

Figure 7: Sod Shock Tube: Velocity

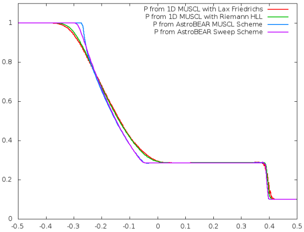

Figure 8: Sod Shock Tube: Pressure

- A simple alternative scheme for AstroBEAR
- Things to do: add tracer, get multi-dimention work
- Things to Add: Fluxes/slope Limiters, How CTU implemented in AstroBEAR