

## SIMULTANEOUS PRODUCTION OF DISK AND LOBES: A SINGLE-WIND MHD MODEL FOR THE $\eta$ CARINAE NEBULA

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Received 2004 March 9; accepted 2004 July 16

### ABSTRACT

The luminous blue variable  $\eta$  Carinae is surrounded by a complex and highly structured nebula of ejected material. The best-studied and axisymmetric components of this outflow consist of bipolar lobes (the “homunculus”) and an equatorial “skirt.” Recent proper-motion measurements suggest that the skirt was ejected at the same time as the lobes, contrary to the assumptions of all current theoretical models for the formation of the nebula (which use the skirt to collimate stellar winds into lobes). We present a magnetohydrodynamic (MHD) stellar wind model that produces an outflowing disk and bipolar lobes in a single, steady state wind. The basic model consists of a wind from a rotating star with a rotation axis–aligned dipole magnetic field. The azimuthal component of the magnetic field, generated by stellar rotation, compresses the wind toward the equator and also toward the rotation axis, simultaneously producing an outflowing disk and jet. We use numerical MHD simulations to study the wind for various amounts of stellar rotation and to show a range of wind morphologies. In order to produce wide-angle lobes similar to the homunculus (which have roughly a  $30^\circ$  opening angle), a high-speed polar wind (with enhanced energy density) from the star is also required. In that case, the structure of the wind bears a remarkable resemblance to the skirt plus homunculus morphology of the  $\eta$  Car nebulae, and a significant fraction of the stellar angular momentum is carried away by the wind. Although the model assumes a steady state wind (rather than an eruption) and thermal wind driving (rather than radiation pressure), the structure of the wind is encouraging.

*Subject headings:* circumstellar matter — MHD — stars: individual ( $\eta$  Carinae) — stars: magnetic fields — stars: winds, outflows

### 1. INTRODUCTION

During the “great eruption,” beginning in 1837 and lasting until about 1858, the luminous blue variable  $\eta$  Carinae ejected more than  $1 M_\odot$  of material (for a review, see Davidson & Humphreys 1997). That ejected material has been well studied (e.g., Davidson et al. 2001) and is highly structured, consisting of an outflowing equatorial “skirt” and bipolar lobes (the hourglass-shaped “homunculus”). The presence of the bipolar lobes has been explained by some authors (e.g., Frank et al. 1995; Dwarkadas & Balick 1998; Langer et al. 1999) with interacting stellar wind (ISW) models. In such models, a previously ejected and slowly moving equatorial flow (presumably related to the skirt) acts as an inertial barrier that channels a subsequent flow toward the poles, forming the homunculus (but see also González et al. 2004).

However, recent proper-motion measurements by Morse et al. (2001) reveal that much of the material in the skirt has the same dynamical age as the lobes. These observations suggest that the skirt was ejected at the same time as the lobes (within the uncertainty of a few years) in 1846, during the great eruption, and thus rule out the ISW models. Clearly, there is a need for a theoretical model that produces bipolar lobes and an equatorial flow, simultaneously, in a single wind. As far as we know, no such model yet exists in the literature.

An alternative to the ISW scenario is shaping the outflow by some sort of magnetohydrodynamic (MHD) process. This

explanation has observational support from Aitken et al. (1995), who measured the polarization of mid-infrared emission from the spatially resolved homunculus. Their explanation for the structure and degree of polarization requires a magnetic field within the homunculus with a strength in the range of milligauss. The homunculus is surrounded by material that was ejected prior to the great eruption, so the magnetic field in the homunculus cannot be the result of swept-up and compressed interstellar medium field, and it must have originated from the central source of the wind (Aitken et al. 1995).

The influence of magnetic fields on stellar winds has been studied by many authors. Sakurai (1985) extended the MHD wind theory of Weber & Davis (1967) and studied isotropic, thermally driven stellar winds, embedded in a radial magnetic field. Sakurai showed that when the star rotates, the magnetic field is twisted azimuthally, which results in a collimation of the wind toward the rotation axis, as well as an increased wind speed (see also Michel 1969; Blandford & Payne 1982). Washimi & Shibata (1993, hereafter WS93) considered the effect of a dipole magnetic field on an otherwise isotropic, pressure-driven stellar wind. They showed that when the star rotates rapidly, the azimuthally twisted dipole produces the same collimation shown by Sakurai but also results in an enhancement of the wind along the magnetic equator. In other words, the wind from a rapidly rotating star with an aligned dipolar magnetic field contains both an outflowing disk and a jet.

It seems plausible that the skirt in the  $\eta$  Car nebula was produced by a similar mechanism, as the skirt may correspond to the outflowing disk in such a wind. On the other hand, the homunculus consists of wide-angle lobes and does not match the narrow jet morphology predicted by the standard, rotating

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MHD wind theory. So the idea of an isotropic wind-driving mechanism, plus a rotating dipole magnetic field, must be modified to produce a wider angle polar flow. In this work, we follow the work of WS93 and explore the possibility that the skirt and homunculus were formed during the great eruption in a single wind from a rotating star with a dipole magnetic field. In order to produce the wide-angle, bipolar lobes of  $\eta$  Car, we find that the wind driving must have been more energetic near the poles than at lower latitudes. Section 2 contains a description of the general model and the assumptions therein.

Using numerical, 2.5-dimensional (axisymmetric) MHD simulations, we follow the acceleration of the stellar wind from the surface of the star to beyond 100 stellar radii, self-consistently capturing the interaction between the stellar magnetic field and the wind plasma. In order to understand each physical process active in the wind and to demonstrate a range of wind morphologies, we carry out a limited parameter study. Section 3 contains a description of the simulations and the parameters we explored. Section 4 contains the results of the parameter study, which are quite general and may be applicable to a variety of structured stellar outflows, particularly those with quadrupolar symmetry, such as pulsar winds and proto-planetary nebulae. One of the simulations produces in a steady state wind structures that bear a remarkable resemblance to the skirt and homunculus morphology of  $\eta$  Car.

## 2. GENERAL MODEL AND PHYSICAL PARAMETERS

Axisymmetric outflow nebulae are the result of a coherent but poorly understood process that probably arises in very close proximity to the stellar surface. Thus, one of the ultimate goals of any wind study is to link observed wind parameters (and morphology) to the conditions at or very near the source of the wind. In this paper, we explore the possibility that the outflow from  $\eta$  Car during the great eruption was produced by a wind from an isolated star. We hope to constrain the possible conditions near the surface of  $\eta$  Car during the great eruption. Knowing these conditions, and how they change in time, is part of the bigger picture of understanding the internal structure and evolution of extremely massive stars.

We shall assume that  $\eta$  Car had a dynamically important magnetic field on its surface during outburst. Magnetic fields are capable of producing structure in otherwise isotropic flows, particularly in systems with rotational energy, where the twisting of magnetic field lines naturally leads to a collimation of the flow along the rotation axis. We will employ an MHD formulation of the problem, adopting all of the assumptions therein. This assumption is supported, in principle, by the observations of Aitken et al. (1995).

In general, any MHD stellar wind is characterized by several key components: magnetic field geometry, magnetic field strength, stellar rotation rate and gravity, and the wind-driving mechanism. These components may have time dependence as well as spatial variations along the surface of the star. In addition, they are all likely to be interrelated. For example, the rotation rate may influence a stellar dynamo, which determines the field strength and geometry, which in turn may influence the structure and temperature in a corona. However, since there is currently no comprehensive theory relating all key components with one another, we treat them as independent parameters.

Following the work of WS93, we consider pressure (thermal) driving for the wind, using a polytropic ( $\gamma = 1.05$ ) equation of state, as appropriate for solar wind studies. The assumption of thermal driving is not very appropriate for  $\eta$  Car,

as radiation pressure is likely to have been more important during the great eruption (e.g., Dwarkadas & Owocki 2002). However, since this is an initial study, our assumption is for simplicity, and the general results will be valid for any wind-driving mechanism. Also as in WS93, the magnetic field anchored in the surface of the star is a rotation axis-aligned dipole with a fixed field strength. A dipolar magnetic field is the most reasonable choice, as it is the lowest order multipole occurring in nature and reflects the symmetries inherent to a rotating star. Even the Sun's magnetic field is dominated by the dipole component outside a few solar radii over most of the magnetic cycle (Bravo et al. 1998). Significant progress has been made in recent years toward understanding the effect of dipole fields on stellar winds (e.g., WS93; Babel & Montmerle 1997; Porter 1997; Keppens & Goedbloed 1999, 2000; Matt et al. 2000; ud-Doula & Owocki 2002), and this literature provides a background for the work contained here.

There are four fundamental parameters, one for each of the energies in the system, which can be written in terms of characteristic speeds. The thermal energy is a measure of the wind driving (via pressure gradients) and is represented by the sound speed at the surface of the star,  $c_s$ . The gravitational potential energy must be overcome by the wind and is parameterized by the escape speed from the surface,  $v_{\text{esc}}$ . The rotational kinetic energy is represented by the rotation speed at the equator,  $v_{\text{rot}}$ , and the magnetic energy by the Alfvén speed at the equator,  $v_A$ . The conditions in the wind, including the location of various critical points (where the wind flow speed equals other characteristic speeds), outflow speed, and mass flux will be completely determined, self-consistently, from the conditions present on the stellar surface (i.e., at the base of the wind).

For simplicity, and so that we can first understand the shaping of individual winds, we only consider steady state wind solutions. Below we present winds with features that resemble both the skirt and homunculus, but since the  $\eta$  Car nebula was ejected in an outburst, we are only tackling part of the problem. Specifically, we assume that our steady state solution represents the wind from  $\eta$  Car during the outburst. Since the true outburst lasted for a few years (or decades, at most), the nebula's evolution from outburst until the present day was likely to have been influenced by previous and subsequent winds. Thus, our steady state models do not address the conditions responsible for or resulting from the time-dependent outburst phenomenon as a whole. However, since there are currently no other stellar wind models that produce a disk and lobes simultaneously, we hope that the present work can serve as a starting point for more complex models, and that it may be more widely applicable to other classes of structured stellar outflows.

The rotation rate of  $\eta$  Car is unknown, so we consider a full range of rotation rates in our models, from no rotation to a significant fraction of breakup speed. Similarly, there are currently no direct measurements of a magnetic field on the surface of  $\eta$  Car. Furthermore, the possible field strength on the star during the great eruption, which is relevant here, is completely unconstrained by observations. However, we assume that the field was strong enough to dynamically influence the wind, and we pick a single value for the field strength. We choose not to vary the magnetic field for simplicity and since the effect of various dipole field strengths has been studied by Keppens & Goedbloed (2000) and ud-Doula & Owocki (2002). In general, for the magnetic field to be dynamically important, the magnetic energy density must be comparable to

the kinetic energy density in the wind. This energy balance gives a rough lower limit on the magnetic field strength on the stellar surface of

$$B_s \gtrsim 2.8 \times 10^3 \text{ G} \left( \frac{100 R_\odot}{R_*} \right) \times \left( \frac{\dot{M}_w}{10^{-1} M_\odot \text{ yr}^{-1}} \right)^{1/2} \left( \frac{v_\infty}{600 \text{ km s}^{-1}} \right)^{1/2}, \quad (1)$$

where we have assumed that, during the outburst,  $\eta$  Car had a radius of  $R_* = 100 R_\odot$  and drove a wind with a mass outflow rate of  $\dot{M}_w = 10^{-1} M_\odot \text{ yr}^{-1}$  and a speed of  $v_\infty = 600 \text{ km s}^{-1}$ . These assumptions are consistent with observationally determined values (Davidson & Humphreys 1997). For the solar wind, equation (1) gives a minimum field of 0.1 G, although the Sun maintains a 1–2 G dipole magnetic field throughout most of the solar cycle (Bravo et al. 1998). By drawing on the example of the Sun, we choose a dipole field strength of  $2.5 \times 10^4 \text{ G}$ , measured at the equator of  $\eta$  Car. This is a strong field for a giant star, but it is required if magnetic shaping is to be important. Note that the wind from  $\eta$  Car during outburst was extremely powerful, compared to other classes of stellar winds. If the process that produces an increase in wind energy during outburst also produces an increase in dynamo activity, it may be natural to assume that the magnetic field will also exist in a heightened state during outburst.

We know from the work of WS93 that the effect of a rotating dipolar field on an otherwise isotropic wind is to simultaneously compress the flow toward the magnetic equator and collimate it toward the rotation axis. The resulting outflowing disk superficially resembles the morphology of the skirt around  $\eta$  Car, but the collimated jet is too narrow to explain the wide-angle, bipolar lobes that comprise the homunculus. In order to produce these lobes, the wind driving could be more energetic near the poles than at lower latitudes; a similar modification to the rotating dipole wind model was used by Tanaka & Washimi (2002) to reproduce the three-ring structure of SN 1987A. This enhancement of the polar wind effectively reduces the dynamical influence of the magnetic field at high latitudes, reducing the collimation in that region and resulting in a polar flow with a wide opening angle. To this end, we present some models with an increased temperature on the stellar poles, and since we are considering a thermally driven wind, the higher temperature results in a more energetic flow from the poles. There is justification for an enhanced polar wind from  $\eta$  Car. First, radiatively driven winds from rapidly rotating, luminous blue variable stars are expected to be more energetic in the polar direction (Dwarkadas & Owocki 2002). Second, observations of  $\eta$  Car (Smith et al. 2003; van Boekel et al. 2003) reveal that the present-day stellar wind is significantly enhanced in the polar direction. Furthermore, for more general support, observations of the solar wind show that the kinetic energy and outflow speed is larger (by a factor of  $\sim 2$ ) above  $30^\circ$  latitude than at lower latitudes (e.g., Goldstein et al. 1996).

In summary, we consider steady state, MHD winds from a star with a rotation axis–aligned dipole magnetic field and varying amounts of rotation. The wind is thermally driven, and we consider two possibilities: cases with isotropic wind driving, and cases with an enhanced polar wind. The resulting winds (described in § 4) display an encouraging range of morphologies. The case with the fastest rotation and with an enhanced polar wind most resembles the  $\eta$  Car nebula.

### 3. MHD SIMULATION DETAILS

Here we present numerical MHD simulations of several models, each with a different set of stellar surface values of  $c_s$ ,  $v_{\text{esc}}$ ,  $v_{\text{rot}}$ , and  $v_A$ . It is not our intention to carry out a complete parameter study, which should include variations of all of these parameters, plus variations of the magnetic field geometry, and wind-driving anisotropy. Instead, our aim is to determine what surface conditions may be required to reproduce the sort of structures seen in the  $\eta$  Car outburst (i.e., the skirt and homunculus).

The cases we ran were chosen to illustrate, in a step-by-step manner, the individual effect of each physical process, leading up to a “final” model that resembles the  $\eta$  Car nebula. In each case, the parameters are chosen and held fixed on the stellar surface, and the simulations run until the entire computational domain is filled with a steady wind emanating from the stellar boundary. This method gives the steady state solution of the flow within the computational grid and determined solely by the conditions held fixed on the stellar surface boundary.

#### 3.1. Numerical Method

We use the 2.5-dimensional MHD code of Matt et al. (2002), which we describe briefly here. The reader will find further details of the code in Matt et al. (2002, and also in Goodson et al. 1997; Matt 2002), which solves the ideal (nonresistive) MHD equations using a two-step Lax-Wendroff finite-difference scheme (Ritchmyer & Morton 1967) in cylindrical  $(r, \phi, z)$  geometry. The formulation of the equations allows for a polytropic equation of state (we use  $\gamma = 1.05$ ), includes a source term in the momentum and energy equations for point-source gravity, and makes the explicit assumption of axisymmetry ( $\partial/\partial\phi = 0$  for all quantities). The fundamental plasma quantities in the equations are the vector magnetic field  $\mathbf{B}$ , mass density  $\rho$ , vector momentum density  $\rho\mathbf{v}$  (where  $\mathbf{v}$  is the velocity), and energy density  $e = 0.5\rho v^2 + P/(\gamma - 1)$ , where  $P$  is the thermal pressure. Because of the geometry and symmetry in the system, it is often useful to decompose the vectors into the azimuthal and poloidal components. The azimuthal component is that in the cylindrical  $\phi$ -direction and denoted by a subscript “ $\phi$ ,” and the poloidal component is that in the  $r$ - $z$  plane denoted by a subscript “ $p$ ” (e.g.,  $\mathbf{v} = v_p\hat{p} + v_\phi\hat{\phi}$  and  $v_p\hat{p} = v_r\hat{r} + v_z\hat{z}$ ).

The computational domain consists of four nested grids (or “boxes”) in the cylindrical  $r$ - $z$  plane. Each box contains  $401 \times 400$  grid points (in  $r$  and  $z$ , respectively) with constant grid spacing. The boxes are nested concentrically, so that the inner box represents the smallest domain at the highest resolution. The next outer box represents twice the domain size with half the spatial resolution (and so on for other boxes). With four boxes, the total computational domain is 8 times larger than that of the innermost grid, but requiring only 4 times the computational expense. This computational efficiency allowed us to run the several cases necessary for the current study. A circular boundary, centered on the origin and with a radius of 30 grid points, represents the surface of the star. Assuming  $R_* = 100 R_\odot$  for  $\eta$  Car during outburst, the innermost grid then has a spatial resolution of roughly  $3.33 R_\odot$  and a domain size of  $13.3R_*$ . Similarly, the outermost (fourth) box has a resolution and domain size of  $26.7 R_\odot$  and  $107R_*$ , respectively.

#### 3.2. Boundary and Initial Conditions

We use standard outflow conditions on the box boundaries, appropriate for wind studies. Namely, along the outermost

boundary in  $r$  and  $z$  in the fourth box, a continuous boundary condition (in which the spatial derivative across the boundary is zero for all quantities) allows the stellar wind to flow out of the computational domain undisturbed. In all grids, we enforce reflection symmetry across the equatorial ( $z = 0$ ) plane. Along the rotation axis ( $r = 0$ ), we require the azimuthal and radial components of magnetic field and velocity to be zero, and all other quantities are equal to their value at  $r = dr$  (where  $dr$  is the grid spacing).

The surface of the star is represented by a spherical inner boundary, centered at the origin ( $r = z = 0$ ). This boundary is the source of the wind that is to fill the computational domain, and so the conditions here entirely determine the solution of the system. Following the wind from the very surface of a star and self-consistently capturing the interaction between the wind plasma and a magnetic field that is anchored into the rotating stellar surface is a complex problem. In a real star, the properties of the wind (e.g., mass flux and velocity) are determined by the balance of forces resulting from, e.g., thermal pressure gradients, gravity, centrifugal forces, and through interaction with the poloidal magnetic field (which distorts when pushed and also pushes back) and the azimuthal magnetic field, which is generated by a twisting of the poloidal field as outflowing plasma tries to conserve its angular momentum.

In order to capture these interactions within the framework of our second-order finite-difference scheme, we employ a four-layer boundary for the star, on which the various parameters are set as follows. For all grid points such that  $R \leq 34.5$  [where  $R = (r^2 + z^2)^{1/2}$  in units of the grid spacing in the innermost box], the poloidal velocity is forced to be parallel with the poloidal magnetic field ( $v_p \parallel B_p$ ). Where  $R \leq 33.5$ ,  $\rho$  and  $P$  are held constant (in time) at their initial values. For  $R \leq 32.5$ ,  $v_p$  is held at zero, while  $v_\phi$  is held at corotation with the star. For  $R \leq 31.5$ ,  $B_p$  is held at its initial, dipolar value, while  $B_\phi$  is set so that there is no poloidal electric current at that layer (which gives it a dependence on the conditions in the next outer layer,  $31.5 < R \leq 32.5$ ). We consider the spherical location  $R = 30$  to be the surface of the star, as this is where all quantities are held fixed and is thus the absolute base of the wind.

These boundary conditions capture the behavior of a wind accelerated thermally from the surface of a rotating magnetized star as follows. There is a layer on the stellar boundary ( $R > 32.5$ ) outside of which the velocity not fixed, but is allowed to vary in time. In this way, the wind speed and direction is determined by the code in response to all of the forces. By holding  $P$  fixed at its initial value for all  $R \leq 33.5$ , we constrain the pressure gradient force (thermal driving) behind the wind to be constant. Also, holding the density fixed at  $R \leq 33.5$  allows the region from where the wind flows to be instantly replenished with plasma. Thus, the base of the wind maintains a constant temperature and density, regardless of how fast or slow the wind flows away from that region. The existence of a layer in which  $v_p = 0$  and  $B_p$  can evolve (namely, at  $31.5 < R \leq 32.5$ ) allows  $B_p$  (and  $v_p$ ) to reach a value that is self-consistently determined by the balance of magnetic and inertial forces (and is also necessary to maintain a negligible  $\nabla \cdot \mathbf{B}$ ). We set the poloidal velocity parallel to the poloidal magnetic field for the next two outer layers (which ensures a smoother transition from the region of pure dipole field and zero velocity to that with a perturbed field and outflow) for reasons of numerical stability. Setting  $B_\phi$  so that the poloidal electric current is zero inside some radius ensures that the field is completely anchored in a rotating conductor (the surface of the star). This way,  $B_\phi$  evolves appropriately outside

the anchored layer according to the interaction with the wind plasma.

We have extensively tested these conditions for a wide range of all system parameters and have been able to reproduce the work of other authors (e.g., the results of Keppens & Goedbloed 1999 and WS93). The advantage of our boundary conditions is in their versatility: they capture the expected behavior for a wide range of conditions. Of particular note, they produce the correct hydrodynamic solution for the case with  $\mathbf{B} = 0$  and give the same solution for a similar case with a weak (dynamically insignificant) dipole magnetic field in which dipole field lines are stripped open to a radial, split monopole configuration by the spherical wind (see, e.g., Matt et al. 2000). In cases with a strong dipole, closed magnetic loops near the equator overcome thermal and centrifugal forces to “shut off” the flow, resulting in a magnetically closed region with  $v_p = 0$  and that corotates with the star (i.e., self-consistently forming a “dead zone”; see Keppens & Goedbloed 2000).

For the simulations with enhanced polar winds (see § 3.3), the pressure at high latitudes (above  $74^\circ$ ) on the star is held fixed at twice the pressure at the same spherical radius but at lower latitudes. Formally, for all regions on the star such that  $R \leq 33.5$  and  $z/r \geq 3.5$ , the pressure is doubled. The resulting factor of 2 pressure discontinuity across  $74^\circ$  latitude on the star is only slightly larger than the typical radial pressure gradients near the star, in which, at the resolution of our simulations, the variation in pressure between grid points is typically a factor of 1.3. Furthermore, the code has been tested and behaves well under shock conditions, in which discontinuities can be much larger than a factor of 2. Thus, the enhanced polar wind does not present a difficulty for the numerical method.

Ultimately, the conditions in the steady state wind that emanates from the stellar surface depends only on the parameters held fixed on the star. However, an initial state for the system must be specified on the entire simulation grid, so we initialized the grid with the spherically symmetric wind solution of Parker (1958). As discussed above, the pressure gradient and density given by this initial Parker (1958) solution is held fixed for all time on the star, although the velocity is allowed to vary to respond to the rotation and magnetic field of the star. This approach is similar to the work of Keppens & Goedbloed (1999). For the cases with an enhanced polar wind, the pressure is doubled on the polar region of the star, so the fixed pressure gradient is also doubled.

### 3.3. Parameter Space Explored

Our approach is to illustrate, in a logical progression of cases, the individual effects of each additional parameter. To this end, we ran the 12 simulation models listed in Table 1. The cases named “ISO” have isotropic thermal wind driving, in which the pressure from the initial Parker (1958) solution is held fixed at all latitudes on the star. The number in the last part of the name corresponds to the rotation period in days, assuming the fiducial radius for  $\eta$  Car during outburst used in equation (1). The rotation period is also listed in column (2) of Table 1. The cases named “HTC” are those in which the thermal wind driving is enhanced on a “high-temperature cap” on the star. They are identical to the corresponding ISO cases, except that for all regions on the star above a latitude of  $74^\circ$ , the pressure ( $P$ ) is set and held at double the value of the ISO case. Thus, the star has a polar cap with twice the temperature (and twice the pressure gradient and thermal energy) as for lower latitudes, which results in a more energetic flow from the polar region.

TABLE 1  
SIMULATION PARAMETERS

Case Name <sup>a</sup> (1)	$T$ (days) (2)	$v_{\text{rot}}/v_{\text{esc}}$ <sup>b</sup> (3)	$v_A/v_{\text{esc}}$ <sup>b</sup> (4)
ISO <sup>0</sup> $\infty$ .....	$\infty$	0	0
ISO $\infty$ .....	$\infty$	0	0.35
ISO2000.....	2000	0.004	0.35
ISO200.....	200	0.041	0.35
ISO100.....	100	0.082	0.35
ISO50.....	50	0.165	0.35
HTC <sup>0</sup> $\infty$ .....	$\infty$	0	0
HTC $\infty$ .....	$\infty$	0	0.35
HTC2000.....	2000	0.004	0.35
HTC200.....	200	0.041	0.35
HTC100.....	100	0.082	0.35
HTC50.....	50	0.165	0.35

NOTE.—The value of  $c_s/v_{\text{esc}}$  is the same in all models and equal to 0.223; see text.

<sup>a</sup> The ISO models have isotropic wind driving, while the HTC models include a high-temperature polar region on the star.

<sup>b</sup> Given is the value at the equator and surface of the star.

In all models, the mass and radius of the star are assumed to be  $100 M_{\odot}$  and  $100 R_{\odot}$ , respectively, which corresponds to an escape speed of  $v_{\text{esc}} = 617 \text{ km s}^{-1}$ . The sound speed  $[=(\gamma P/\rho)^{1/2}]$  on the surface of the entire star in the ISO models and below  $74^\circ$  latitude for the HTC models is  $c_s = 137 \text{ km s}^{-1}$  (and so  $c_s = 194 \text{ km s}^{-1}$  in the polar region of the HTC models). The values of  $c_s$  and  $v_{\text{esc}}$  determine the solution to the velocity at all radii in a (nonrotating, nonmagnetic) hydrodynamic wind. We chose this value of  $c_s$  so that the outflow speed reaches several hundred  $\text{km s}^{-1}$ , far from the star, and we set our density normalization (which is the same for all models) such that the mass outflow rate is around  $10^{-1} M_{\odot} \text{ yr}^{-1}$ , appropriate for  $\eta$  Car during outburst. The magnetic field, when nonzero, is always a rotation axis-aligned dipole with a strength of  $B_* = 2.5 \times 10^4 \text{ G}$  on the equator of the star (and twice that on the pole). With our density normalization, this gives an Alfvén speed  $[=B/(4\pi\rho)^{1/2}]$  in cgs units] on the surface of the star at the equator of  $v_A = 218 \text{ km s}^{-1}$ . We explore a full range of stellar rotation up to a rotation of 23% of breakup speed. Thus, the equatorial rotation speed on the star for different models ranges from  $v_{\text{rot}} = 0$  to  $102 \text{ km s}^{-1}$ .

The ratios of  $v_{\text{rot}}/v_{\text{esc}}$  and  $v_A/v_{\text{esc}}$  at the equator and surface of the star are listed in columns (3) and (4) of Table 1 for each model. The ratio of  $c_s/v_{\text{esc}}$  is 0.223 for all models (except on the polar region of the HTC models, where  $c_s/v_{\text{esc}} = 0.315$ ). In order to establish a baseline for the ISO cases, we first ran a purely hydrodynamic case ( $B = 0$ ) with no rotation, called ISO<sup>0</sup> $\infty$ . Then, to show the effect of a  $2.5 \times 10^4 \text{ G}$  dipolar field, we ran the ISO $\infty$  case, also with no rotation. Next, to illustrate the twisting of magnetic fields, but before the twisting is dynamically important, we ran the ISO2000 case, in which the rotation is so slow that the solution is virtually identical to the ISO $\infty$  case (see § 4). The effects of dynamically significant rotation are demonstrated with the ISO200, ISO100, and ISO50 cases. Finally, we ran an HTC model corresponding to each of the ISO models, so that we could see the effect of the anisotropic thermal wind driving for each set of parameters.

The results of these simulations (presented in § 4) are scalable to any star with the same values of  $v_{\text{rot}}/v_{\text{esc}}$ ,  $v_A/v_{\text{esc}}$ , and  $c_s/v_{\text{esc}}$ . Note that the rotation period (and any physical times)

should then scale with  $R_*/v$ , where  $v$  represents any of the characteristic speeds in the system (e.g.,  $v_{\text{esc}}$ ). In § 4.2 we calculate the outflow rates of mass ( $\dot{M}_w$ ), angular momentum ( $\dot{J}_w$ ), and energy ( $\dot{E}_w$ ), which can also be scaled. The mass outflow rate scales with  $vR_*^2$  times the density normalization,  $\dot{J}_w \propto \dot{M}_w v R_*$ , and  $\dot{E}_w \propto \dot{M}_w v^2$ . Finally, the magnetic field strength  $B$  scales with  $v$  times the square root of the density normalization [so that  $B \propto (\dot{M}_w v)^{1/2}/R_*$ ]. As an example, the Sun has the same  $v_{\text{esc}}$  and roughly the same coronal sound speed and  $v_A$  as our models. Using  $\dot{M}_w \sim 10^{-14} M_{\odot} \text{ yr}^{-1}$  for the Sun, scaling down  $\dot{M}_w$  and  $R_*$  in our models corresponds to a dipole field strength of  $\sim 1 \text{ G}$ , which is appropriate (Bravo et al. 1998). Thus, the ISO2000 model is similar to the Sun, although with a rotation period of 20 days. Since the solar rotation does not have a significant effect on the solar wind (see, e.g., WS93), we expect the ISO2000 case to display global properties similar to the solar wind (although the solar wind is further modified by, e.g., anisotropic coronal heating, high-order magnetic fields).

### 3.4. Accuracy of the Numerical Solution

After the start of the simulations, a flow emanates from the surface of the star. In all cases, that flow accelerates to a speed faster than all information-carrying waves (i.e., slow, Alfvén, and fast magnetosonic waves) within the computational domain, usually within the first or second grid. To obtain a numerical steady state solution, we ran each case for  $\sim 2 \text{ yr}$  of physical time. Since it takes only  $\sim 0.6 \text{ yr}$  for the wind, traveling with a typical velocity of  $400 \text{ km s}^{-1}$ , to cross our largest simulation box, material that leaves the surface of the star at the beginning of the simulation has traveled well beyond the entire domain by the end. A measure of the fractional rate of change of each of the quantities  $\rho$ ,  $P$ ,  $v_r$ ,  $v_\phi$ ,  $v_z$ ,  $B_r$ ,  $B_\phi$ , and  $B_z$  at every grid point quantifies the steadiness of the numerical solution. In the largest box, for each case, the average grid point was changing by less than 1% per wind crossing time by the end of the simulations. Since the computations require roughly 16,000 time steps per crossing time, this corresponds to a fractional change of less than  $10^{-6}$  per time step. As another simple test of the steadiness and conservation of our numerical solutions, we calculated the outflow rates of  $\dot{M}_w$ ,  $\dot{J}_w$ , and  $\dot{E}_w$  (see § 4.2) at all radii within the computational domain and found that these global quantities are constant (i.e., no variation with radius), within 3%, for all cases. Thus, our solutions are sufficiently steady for the present study.

One general physical test of numerical MHD solutions is to quantify the divergence of  $\mathbf{B}$ , which should always and everywhere be zero. The initial state of the system has  $\nabla \cdot \mathbf{B} = 0$ , and the code maintains this physical requirement to second order in space and time. However, numerical errors do lead to nonzero values of  $\nabla \cdot \mathbf{B}$ , and we deal with these errors in two ways. First, the code solves the momentum equation using a nonconservative formulation of the magnetic force that prevents numerical  $\nabla \cdot \mathbf{B}$  errors from producing unphysical forces (Brackbill & Barnes 1980). Second, we check that these numerical errors do not contribute significantly to the structure of the magnetic field, by computing the ratio of  $\nabla \cdot \mathbf{B}$  to  $|\nabla B|$ —that is, the magnitude of the gradient of the magnitude of  $\mathbf{B}$ —at every grid point. This ratio determines the maximum possible contribution of nonzero  $\nabla \cdot \mathbf{B}$  to any spatial change in the magnetic field. For the vast majority of grid points, in all cases, this ratio is always much less than 1%. Near the stellar boundary, this can be larger, although never exceeding 5% in all magnetic cases. These  $\nabla \cdot \mathbf{B}$  errors are acceptable.

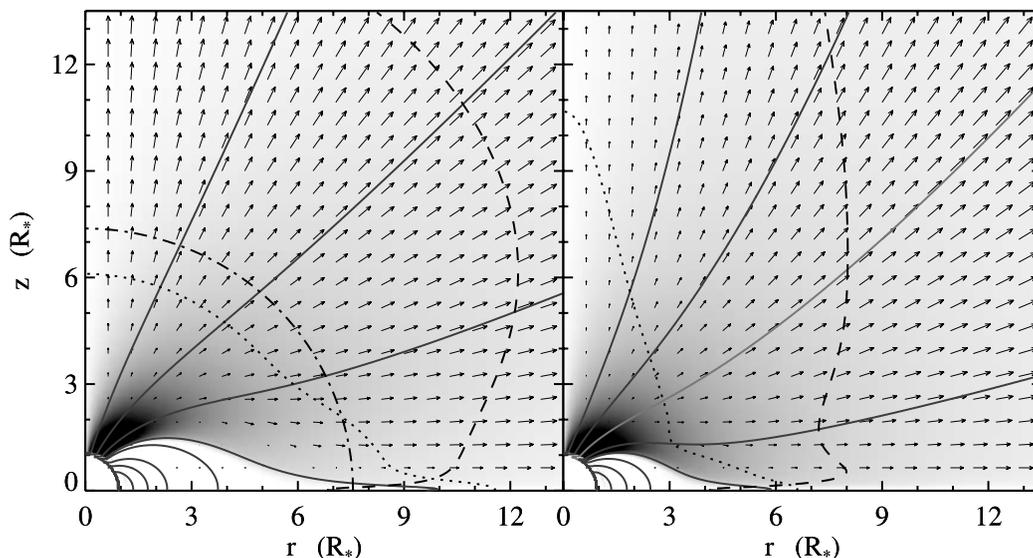


FIG. 1.—Gray scale of  $B_\phi$  (white is zero; black is strong) with selected magnetic field lines overdrawn in gray for two cases in which the star has a constant temperature across its surface. The dashed line indicates the radial Alfvén surface, and the dotted line is the sonic surface. The left panel contains data for the very weakly rotating case ( $T = 2000$  days), while the right panel corresponds to faster rotation ( $T = 100$  days). The dash-dotted line in the left panel is the location of the sonic surface for a simulation without rotation and without a magnetic field. The light gray field line in the right panel intersects the stellar surface at  $65^\circ$  and indicates the path  $s$  followed in Fig. 2.

For a more comprehensive check on the precision of our solutions, we follow the error analysis of Ustyugova et al. (1999), who list five integrals of motion  $K$ ,  $\Lambda$ ,  $\Omega$ ,  $S$ , and  $E$  (their eqs. [1]–[5]), corresponding to the conservation of mass, angular momentum, helicity, entropy, and energy, respectively. Under the conditions of ideal, axisymmetric steady state MHD, these integrals should be constant along a given magnetic field line and thus serve as a stringent test of our numerical solution. In every magnetic model, we calculated  $K$ ,  $\Lambda$ ,  $\Omega$ ,  $S$ , and  $E$  along two reference field lines: one that connects to the stellar surface at  $65^\circ$  latitude, and one that connects to  $80^\circ$ . In this way, we sampled two latitudes in the flow, one of which is within the hot polar cap of the HTC models. In all cases, all five integral functions are conserved to within 5%, along both reference field lines.

For an even more stringent test, we compared the absolute value of the conserved quantity  $\Omega$  to the known value at the stellar surface, which equals the constant angular rotation rate of the star. Following the error analysis of Keppens & Goedbloed (2000), we calculated  $\Omega$  for all grid points in our domain for all of the magnetic models. Over 90% of the domain, the error in  $\Omega$  ranges from 2% to 7%, for all cases, except one. This exception is the ISO50 case, in which this error is 12%. A larger error in  $\Omega$  exists in all cases over an area of less than 10% of the domain that covers a narrow region near the equator with an opening angle of less than  $10^\circ$ , as seen by the star. This error occurs in the portion of the flow that travels along the lowest latitude open field lines near the dead zone (see § 4). In that portion of the flow, the maximum error in  $\Omega$  ranges from 20% to 40% in all cases, except one. This exception is the HTC2000 case, in which the maximum error in this portion of the flow reaches 80%. These errors are comparable to those of Keppens & Goedbloed (2000), who found errors in  $\Omega$  reaching 40% in the same portion of the flow as where our largest errors occur.

#### 4. SIMULATION RESULTS

For the cases with nonzero magnetic field, the wind is significantly altered from the purely hydrodynamic solution. The

behavior of the dipole magnetic field in a wind and its effect on the thermally driven flow is discussed by many authors (see especially Keppens & Goedbloed 2000). In particular, the dipole field lines at high latitudes are nearly radial, and thus wind can flow along them with little impedance. Furthermore, they reach to distances far from the star, where the field becomes very weak. As the dipole field energy density falls off with  $R^{-6}$ , and the kinetic energy density in the wind falls off more slowly than  $R^{-2}$  (for an accelerating wind), these high-latitude field lines will open up as they are dominated by and carried out in the wind. On the other hand, at lower magnetic latitudes, the dipole field lines are more perpendicular to the flow, and they do not reach as far away from the star. If the field is strong enough to counteract the outward thermal pressure forces, the flow will be shut off. This produces a region of closed-field lines around the magnetic equator, a dead zone that corotates with the star and from which no wind can flow (in the absence of magnetic diffusion).

If the star rotates, wind material tries to conserve its angular momentum as it travels outward so that its angular velocity decreases with distance from the rotation axis. When wind plasma is connected to the stellar surface by magnetic fields, the difference in angular rotation rate between the star and wind twists the field azimuthally, generating a  $B_\phi$ -component to the field (see, e.g., Parker 1958). If the stellar rotation is fast enough, magnetic forces associated with  $B_\phi$  significantly influence the flow (e.g., WS93). These effects will be further discussed below.

##### 4.1. Wind Acceleration and Shaping

The left panel of Figure 1 illustrates the conditions in the steady state wind for the ISO2000 case. Shown is a gray-scale image of  $B_\phi$  (black represents 57 G, and white is zero), the poloidal magnetic field lines (*solid lines*), and poloidal velocity vectors (*arrows*). The data are from the innermost (highest resolution) computational box. The star is at the lower left and has a radius of  $R_*$ . Poloidal magnetic field lines are drawn so that they originate at roughly evenly spaced intervals in latitude on the star (their spacing is not proportional to field

strength). We have also drawn a field line that marks the transition between closed- and open-field regions (i.e., the line that encloses the dead zone). Note that under steady state, ideal MHD conditions (as we have here) magnetic field lines are also in the same direction as plasma flow streamlines. The rotation is so slow in the ISO2000 case that the resulting wind is identical (within a few percent) to the ISO $\infty$  case, except that  $B_\phi$  is exactly zero for ISO $\infty$ . Thus, the left panel of Figure 1 simply demonstrates the effects of a  $2.5 \times 10^4$  G dipole magnetic field on an otherwise completely isotropic flow. The results here are comparable to other, similar studies (e.g., Keppens & Goedbloed 2000), and they form the base to our understanding of the effects of additional processes in the wind (i.e., rotation and enhanced polar winds).

In the left panel of Figure 1, the dead zone covers a significant fraction of the stellar surface near the equator and extends beyond  $5R_*$  above the star. Thus, all of the wind from the star originates from the open-field region at high latitudes. There is flow at low latitudes, outside several stellar radii, but (as indicated by the field lines and velocity vectors) that flow originates from the polar region of the star. In other words, the wind from the poles diverges very rapidly near the star and tends more toward radial divergence far from the star. The flow actually converges toward the equator within roughly  $8R_*$ , as the open field lines curve around the dead zone. The dotted line in the left panel of Figure 1 marks the location of the sonic surface, where the wind velocity equals the local sound speed. For reference, the dash-dotted line marks the sonic surface for the ISO $^0\infty$  case, in which there is no magnetic field, and the wind is completely isotropic. The effect of a dipole field is to shift the sonic surface farther from the star near the equator. This is because the convergent flow on the equator leads to a shallower radial pressure gradient, and the wind is accelerated more slowly there. Conversely, the sonic surface is moved closer to the star near the pole because the flow diverges faster than radial divergence, making the pressure gradient steeper. The dashed line marks the location of the poloidal Alfvén surface, where the poloidal wind velocity equals the local poloidal Alfvén speed (calculated from the  $B_p$ -component of the field only). The Alfvén surface is farthest from the star along the pole, since the magnetic field is strongest there. Because of the opening of field lines in the wind, the poloidal magnetic field strength goes to zero on the equator (across which there is a direction reversal of the field, supported by a current sheet), leading to a “cusp” in the Alfvén surface there.

The gray scale of  $B_\phi$  in the left panel of Figure 1 shows that the dead zone is corotating with the star, since  $B_\phi \approx 0$  there. However, in the open-field region, the field lines are “trailing” the star as it rotates beneath the expanding wind, and so  $B_\phi \neq 0$ . Note that  $B_\phi$  is zero along the rotation axis because the rotation speed goes to zero at  $90^\circ$  latitude;  $B_\phi$  is also zero everywhere along the equator because the radial component of the magnetic field goes to zero there, for a dipole, and it is the radial component of the magnetic field that is twisted to generate  $B_\phi$ . Thus, for a twisted dipole field,  $B_\phi$  has a maximum value at midlatitudes (Washimi 1990). There is a magnetic force associated with gradients in  $B_\phi$  (proportional to  $-\nabla B_\phi^2$ ), which point toward the rotation axis (as studied by, e.g., Sakurai 1985) and also toward the equator (as pointed out by WS93). For the ISO2000 case, these magnetic forces are too small to have a significant effect, and the wind is accelerated hydrodynamically, although modified by the greater (less) than radial divergence near the polar (equatorial) region, imposed by the poloidal magnetic field.

For the cases with more rapid rotation, however, the forces associated with  $B_\phi$  become important and act both perpendicular to and parallel to the poloidal field to modify the flow. This is evident in the right panel of Figure 1, which illustrates the steady state flow for the ISO100 case. The quantities shown in the right panel are the same as for the left panel, except that the gray-scale image of  $B_\phi$  is scaled so that black corresponds to 860 G and white is zero. The only difference between the two cases is that the star in the ISO100 case rotates 20 times faster than for ISO2000, so a stronger  $B_\phi$  is generated that influences the wind. A comparison between the two panels of Figure 1 reveals this influence. First, the dead zone is significantly smaller for ISO100 because the magnetic pressure gradient in  $B_\phi$ , acting perpendicular to the poloidal field, is strong enough to “squash” the closed-field region. In addition, the field lines in the ISO100 case exhibit greater than radial divergence from the polar region of the star (as in the ISO2000 case) but then undergo a change in curvature outside a few  $R_*$  and begin to diverge more slowly than radially near the rotation axis. This gives rise to an asymptotic collimation of the flow and is caused by the magnetic pressure gradients in  $B_\phi$  that are perpendicular to the poloidal field and directed toward the axis. This collimation results in shallower pressure gradients along the pole, leading to slower acceleration of the wind there, and the sonic point is shifted outward relative to the ISO2000 case. This effect (and due to the more slowly diverging poloidal magnetic field) also moves the Alfvén surface farther out on the poles. The most important difference between the left and right panels of Figure 1—that a rotating dipole field simultaneously compresses the flow on the magnetic equator and collimates it along the rotation axis—was first shown by WS93 (e.g., compare with their Fig. 1).

Fast rotation also affects the flow in the direction parallel to the poloidal magnetic field. Figure 2 shows these forces along a reference magnetic field line that connects to the star at  $65^\circ$  latitude, for the ISO100 case. This field line is indicated by the light gray line in the right panel of Figure 1, and the data in Figure 2 are from the second computational box. The forces per mass along a field line are given by (e.g., Ustyugova et al. 1999)

$$f_P = -\frac{1}{\rho} \frac{\partial P}{\partial s}, \quad (2)$$

$$f_G = -\frac{GM_*}{R^2} \hat{R} \cdot \hat{s}, \quad (3)$$

$$f_C = v_\phi r \hat{r} \cdot \hat{s}, \quad (4)$$

$$f_M = -\frac{1}{8\pi\rho r^2} \frac{\partial (rB_\phi)^2}{\partial s}, \quad (5)$$

where the subscripts P, G, C, and M correspond to the pressure gradient, gravitational, centrifugal, and magnetic forces, respectively. Here,  $s$  is the distance along the poloidal field,  $G$  is the gravitational constant, and  $M_*$  is the mass of the star. Also note that we have used both the cylindrical ( $r$ ) and spherical ( $R$ ) radial coordinate, where appropriate. Figure 2 shows these forces in units of  $\text{cm s}^{-2}$  (for  $\eta$  Car parameters, where the surface gravity is  $270 \text{ cm s}^{-1}$ ), as a function of position along  $s$ , where  $1R_*$  corresponds to the stellar surface. It is evident that the pressure gradient force along the reference field line dominates near the star, but the centrifugal and magnetic forces are significant. We find that the ratio of  $f_C/f_P$  reaches a maximum value of  $\sim 30\%$  at  $\sim 10R_*$  along  $s$ . Consequently, material

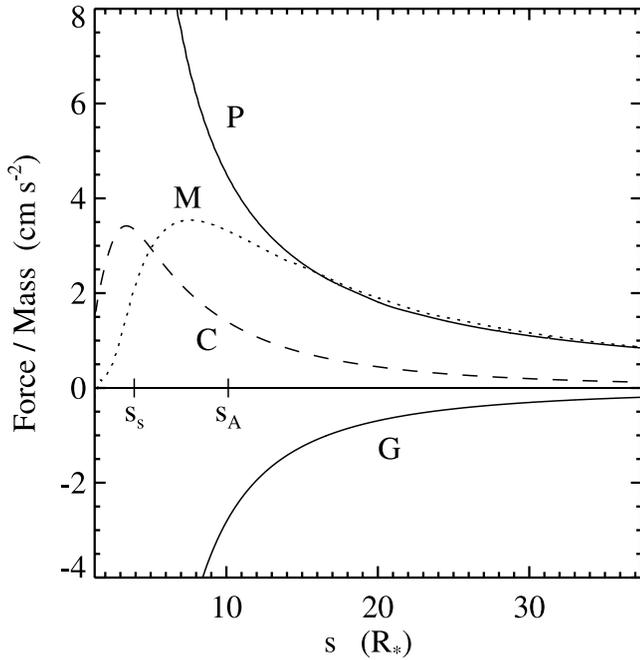


FIG. 2.—Forces parallel to the reference field line indicated by the light gray line (Fig. 1, *right*), as a function of position  $s$  along the field line. Lines labeled P, G, C, and M represent the pressure gradient, gravitational, centrifugal, and magnetic forces, respectively (eqs. [2]–[5]). The locations  $s_s$  and  $s_A$  indicate the sonic and Alfvénic critical points, respectively.

at midlatitudes is partially magnetocentrifugally accelerated, leading to a faster outflow. As a result, the sonic and Alfvénic surfaces are closer to the star than for the ISO2000 case (see Fig. 1).

The left and right panels of Figure 3 show the steady state winds of the HTC2000 and HTC100, respectively, in the same format as Figure 1 (the gray scaling for  $B_\phi$  is also the same for each respective panel). The two cases shown in Figure 3 are identical to those in Figure 1 except that the star now has a

high-temperature polar region. In each panel of Figure 3, there is a light gray field line that is anchored on the stellar surface at  $74^\circ$  latitude. Since that is the latitude of the edge of the high-temperature region, and since the field line is also a streamline, it marks the division between material that originates from the hot polar cap and material that flows from lower latitudes.

A comparison of the left panels of Figures 1 and 3 reveals the effect of the hot polar cap on the wind from a slowly rotating star with a dipole field. The increase in thermal energy on the pole more quickly accelerates the wind, which moves the sonic and Alfvén surface much closer to the star near the pole. However, since the hot polar wind is not in pressure equilibrium with the wind at lower latitudes, it expands rapidly toward the equator as it flows outward (more than for ISO2000), until there is a meridional balance in pressure between the high- and low-latitude flows. Far from the star, the flow in a large range of latitudes originates from the hot polar cap (further discussed in § 4.3). The convergence of flow toward the equator reduces radial pressure gradients there, which moves the sonic and Alfvén surface outward (relative to the ISO2000 case). The azimuthal magnetic field exhibits the same qualitative behavior in both the ISO and HTC cases.

A comparison between the left and right panels of Figure 3 reveals the effect of significant rotation in the HTC models. As with the ISO cases, centrifugal flinging accelerates the flow more rapidly in the equatorial direction, moving the location of the sonic and Alfvén surface inward. Also, the dead zone is somewhat squashed by the magnetic forces (associated with  $B_\phi$ ) directed toward the equator. However, in contrast to the ISO cases, the dynamical effects of the rotation in the HTC case are not as important at high latitudes, because of the stronger pressure gradient there. Along a field line connected to the HTC100 star at  $80^\circ$  latitude, for example,  $f_C/f_P$  reaches a maximum value of less than 1%, compared to a ratio of greater than 9% along a field line connected to the star at  $65^\circ$ . This difference between the high- and low-latitude flow is evident in the right

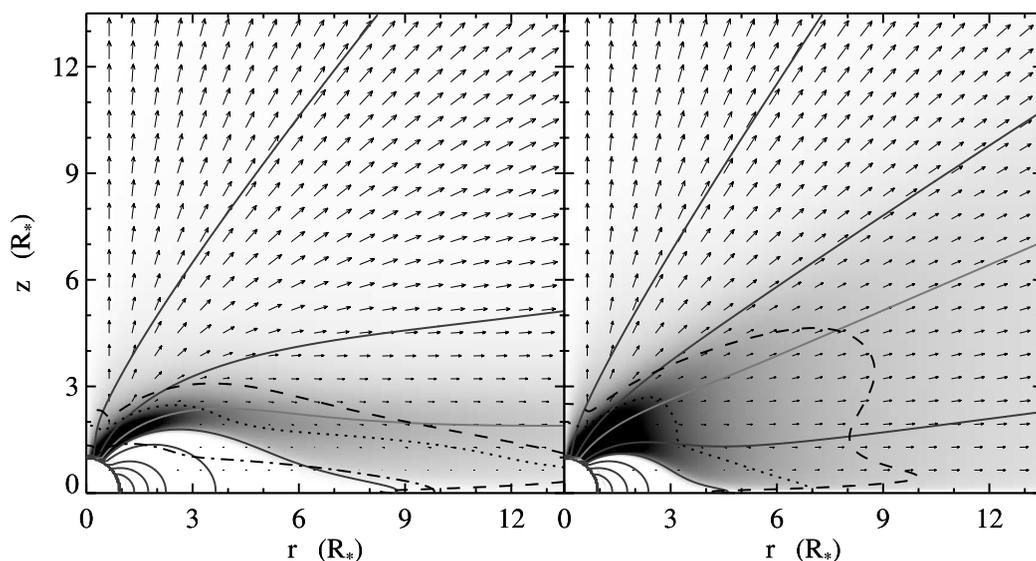


FIG. 3.—Gray scale of  $B_\phi$  (white is zero; black is strong) with selected magnetic field lines overlaid in gray for two cases in which the star has a high-temperature polar cap. The light gray field line in each panel intersects the stellar surface at  $74^\circ$ , the latitude of the temperature discontinuity on the star. The dashed line indicates the radial Alfvén surface, and the dotted line is the sonic surface. The left panel contains data for the very weakly rotating case ( $T = 2000$  days), while the right panel corresponds to faster rotation ( $T = 100$  days). The dash-dotted line in the left panel is the location of the sonic surface for a simulation without rotation and without a magnetic field.

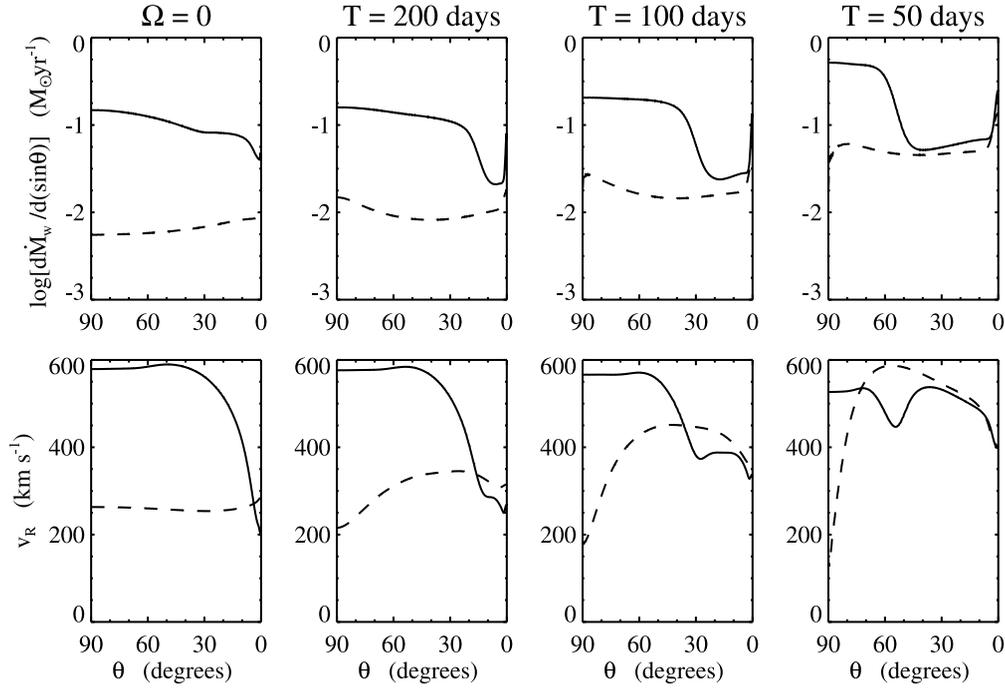


FIG. 4.—Mass outflow rate  $\partial\dot{M}/\partial(\sin\theta)$  (top) and radial velocity of the wind (bottom) as a function of latitude  $\theta$  for cases with (from left to right) no rotation,  $T = 200$ ,  $T = 100$ , and  $T = 50$  days. The dashed lines represent cases with spherical thermal wind driving, while the solid lines are for the cases with a high-temperature polar wind. All panels are calculated at a distance of  $40R_*$  from the star.

panel of Figure 3, as the lowest latitude open field line is bending slightly poleward (collimating), while the higher latitude field lines are not. The high-latitude flow is inherently more energetic, so it would require a stronger field and/or faster rotation than the lower latitude flow, to be significantly altered. Consequently, the effect of significant rotation on the HTC models is to “clear out” the wind at midlatitudes as it is compressed toward the equator and also against the side of the polar flow. Note that the direction of the light gray line points to higher latitudes in the HTC100 case than in the HTC2000 case, indicating that when the star spins faster, the energetic polar flow is confined to a narrower opening angle.

#### 4.2. Global Outflow Properties

The presence of a dipole field and the rotation of the magnetized star influences the global outflow rates of mass ( $\dot{M}_w$ ), angular momentum ( $\dot{J}_w$ ), and energy ( $\dot{E}_w$ ). We calculate these quantities from the simulation data by integrating the fluxes over an enclosed spherical surface at a radius  $R$ . Thus,

$$\dot{M}_w = 4\pi R^2 \int_0^1 \rho v_R d(\sin\theta), \quad (6)$$

$$\dot{J}_w = 4\pi R^2 \int_0^1 \rho v_R \Lambda d(\sin\theta), \quad (7)$$

$$\dot{E}_w = 4\pi R^2 \int_0^1 \rho v_R E' d(\sin\theta), \quad (8)$$

where  $\theta$  is the latitude,  $v_R$  is the velocity component in the spherical radial direction ( $v_R = \mathbf{v} \cdot \hat{R}$ ), and

$$\Lambda = v_\phi r - \frac{B_\phi B_p r}{4\pi\rho v_p}, \quad (9)$$

$$E' = \frac{v_p^2 + v_\phi^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P}{\rho} - \frac{GM_*}{R} + \frac{B_\phi^2}{4\pi\rho} - \frac{v_\phi B_\phi B_p}{4\pi\rho v_p}. \quad (10)$$

Here,  $\Lambda$  is the integral of motion discussed in § 3.4 that gives the angular momentum per mass carried in the wind, and  $E' = E + \Lambda\Omega$  gives the specific energy carried by the wind. The first three terms in equation (10) represent the kinetic ( $E_K$ ), thermal, and gravitational energy, while the last two terms together represent the magnetic energy ( $E_M = \text{Poynting flux divided by } \rho v_R$ ). Note that the equations (6)–(8) have been multiplied by a factor of 2 to take both hemispheres into account.

The top row of panels of Figure 4 shows the differential mass loss rate,  $\partial\dot{M}_w/\partial(\sin\theta) = 2\pi R^2 \rho v_R$  (“mass flux”). The bottom row of panels shows the radial velocity  $v_R$ . Both the mass flux and  $v_R$  are plotted versus  $\theta$ , at a constant spherical radius of  $40R_*$  (which equals 300 grid points in the third computational box), for several models. From left to right, the panels for both mass flux and  $v_R$  show cases with increasing rotation rate. In each panel, the dashed line is for the ISO case, while the solid line represents the HTC case. Note that for the ISO2000 and HTC2000 cases, which are featured in the left panels of Figures 1 and 3, the rotation is so slow that the mass flux and  $v_R$  for those models are the same (within a few percent) as the data plotted in the left panels of Figure 4 for the ISO $\infty$  and HTC $\infty$  cases. Effectively then, Figure 4 displays data from all 10 of the models with nonzero magnetic fields.

The dashed lines in the left panels of Figure 4 reveal that steady state wind in the nonrotating ISO $\infty$  case is not completely isotropic. For reasons discussed in § 4.1, the flow is densest near the equator (see also Matt et al. 2000; our § 4.3). This is the effect of the dipole field on an otherwise isotropic wind. For the HTC models, the flow is generally faster at

TABLE 2  
SIMULATION RESULTS

Case Name (1)	$\dot{M}_w$ ( $10^{-1} M_\odot \text{ yr}^{-1}$ ) (2)	$\dot{J}_w$ ( $10^{45}$ ergs) (3)	$\dot{E}_w$ ( $10^{39}$ ergs $\text{s}^{-1}$ ) (4)	$\dot{E}_K^a$ ( $10^{39}$ ergs $\text{s}^{-1}$ ) (5)	$\dot{E}_M^a$ ( $10^{39}$ ergs $\text{s}^{-1}$ ) (6)	$v_{\text{max}}/v_{\text{min}}^b$ (7)	$\rho_{\text{max}}/\rho_{\text{min}}^b$ (8)
ISO $^0_\infty$ .....	0.15	...	2.2	0.44	...	1.0	1.0
ISO $_\infty$ .....	0.14	...	2.1	0.42	...	1.1	1.5
ISO2000.....	0.14	0.16	2.1	0.42	0.0029	1.1	1.5
ISO200.....	0.19	1.6	3.4	0.86	0.22	1.6	2.9
ISO100.....	0.34	3.7	7.3	2.4	0.84	2.6	5.0
ISO50.....	1.1	11	28	11	3.9	4.2	4.3
HTC $^0_\infty$ .....	2.6	...	86	27	...	2.0	1.9
HTC $_\infty$ .....	1.8	...	61	20	...	2.9	1.7
HTC2000.....	1.8	0.13	61	20	0.0043	2.9	1.7
HTC200.....	1.9	2.0	62	20	0.48	2.3	4.0
HTC100.....	2.0	4.9	66	20	2.0	1.7	6.5
HTC50.....	2.7	13	88	26	7.9	1.3	10.2

<sup>a</sup> Given is the value at a radius of  $80R_*$ .

<sup>b</sup> Given is the value at a radius of  $40R_*$ .

higher latitudes, and the mass flux is higher than for the ISO cases because of the more energetic wind driving in the HTC cases.

The integrated outflow rates  $\dot{M}_w$ ,  $\dot{J}_w$ , and  $\dot{E}_w$  are listed in columns (2), (3), and (4) of Table 2 for each case. Since  $\dot{E}_w$  is dominated by the thermal energy within the computational domain because of our chosen value of  $\gamma$  near unity, we list the individual components of  $\dot{E}_K$  and  $\dot{E}_M$  in columns (5) and (6) of Table 2. These are calculated with equation (8) but using only the kinetic or magnetic terms of equation (10). The table lists these values at a spherical radius of  $80R_*$  (which equals 300 grid points in the fourth computational box), where the gravitational energy is negligible. If one scales  $\dot{M}_w$  and  $R_*$  down to solar values for the ISO2000 case (as described in § 3.3), this case predicts  $\dot{J}_w \approx 1.1 \times 10^{30}$  ergs for the solar wind. This value is comparable to the predictions of  $\dot{J}_w \sim 10^{30}$  ergs from the solar models of Keppens & Goedbloed (2000), and this further verifies our numerical solution.

It is evident from Table 2 that  $\dot{M}_w$  and  $\dot{E}_w$  are slightly less in the ISO $_\infty$  case than for ISO $^0_\infty$  (and similarly for HTC $_\infty$  and HTC $^0_\infty$ ). This is because the existence of the dead zone shuts off part of the flow. However,  $\dot{M}_w$  and  $\dot{E}_w$  are not simply reduced by the amount proportional to the fractional area covered by the dead zone on the star, since a reduced wind flux near the equator is counteracted, somewhat, by an increased flow from the poles, driven by the faster than radial divergence of the wind there. In order to quantify the anisotropy of the wind, we use a crude measure given by the ratio of the maximum value of  $v_R$  to the minimum  $v_R$  found in the wind at a spherical radius of  $40R_*$ . This ratio is listed in column (7) of Table 2 for all cases, while column (8) contains a similar ratio for the anisotropy in the mass density.

From the data in Table 2 and plotted in Figure 4, it is evident that as the stellar rotation rate increases,  $\dot{M}_w$ ,  $\dot{J}_w$ ,  $\dot{E}_w$ , and the anisotropy of the flow increase. The effect of the strong azimuthal magnetic field is to clear out material from midlatitudes, where it has a maximum strength, as it both compresses material toward the equator, forming an outflowing disk, and collimates material toward the pole. For the ISO models, a jet appears as an increase in mass flux along the rotation axis, as is evident in the top row of Figure 4. For the HTC models, the opening angle of the energetic flow from the pole of the star is decreased—an outflowing “lobe” appears—as rotation

increases. These features are discussed further in § 4.3. Note that for the ISO50 and HTC50 cases, the mass fluxes and  $v_R$  are similar at latitudes less than roughly  $40^\circ$ .

Also evident in Figure 4 is that in each case there is a general anticorrelation between the mass flux and the wind velocity. That is, where the mass flux is high relative to the nonrotating case, the velocity is relatively low. This is because the convergent flow that increases the local mass flux also makes the thermal pressure gradients shallower, leading to a slower acceleration of the flow. However, in spite of the general decrease in  $v_R$  near the rotation axis and equator, we find that the linear momentum and kinetic energy flux is generally largest where the mass flux is largest.

An exception to the mass flux and wind velocity anticorrelation occurs for the ISO50 case (Fig. 4, *top rightmost panel, dashed line*), where the mass flux is enhanced near the rotation axis (as in other cases) but decrease to a minimum value on the axis, resulting in a “hollow” jet structure. This behavior is similar to cold, magnetocentrifugally driven winds, in which the inner part of the jet is generally hollow (e.g., Ouyed & Pudritz 1997). The flux in the center of this jet is small because the magnetic field cannot centrifugally fling material along the exact rotation axis, where the rotation speed goes to zero. In our models, we find that along a field line that connects at  $\theta = 65^\circ$  on the star,  $f_C/f_P$  reaches a maximum value of 10%, 30%, and 70% for the ISO200, ISO100, and ISO50 cases, respectively. Thus, the sequence of ISO models is showing a transition from a purely thermal wind to a thermocentrifugal wind (see WS93). This is also evident in Table 2, where the ratio of  $\dot{E}_M/\dot{E}_w$  increases with the rotation rate.

#### 4.3. Morphology: Disks, Jets, and Bipolar Lobes

The outermost (fourth) simulation box reaches beyond  $100R_*$ , and the conditions in the steady state wind there are indicative of the shape of the flow very much farther from the star. The three panels of Figure 5 show logarithmic density contours from the fourth box for three ISO cases with different stellar rotation rates (from left to right: ISO $_\infty$ , ISO200, and ISO100). For reference, the thickest contour line in each panel corresponds to a density of  $1.7 \times 10^{-13}$  g  $\text{cm}^{-3}$ . The left panel shows the effect of the dipole field alone; the wind is denser near the equator than along the pole. For stars that rotate quickly (*middle and right panels*), note the formation of the jet along the rotation ( $z$ ) axis

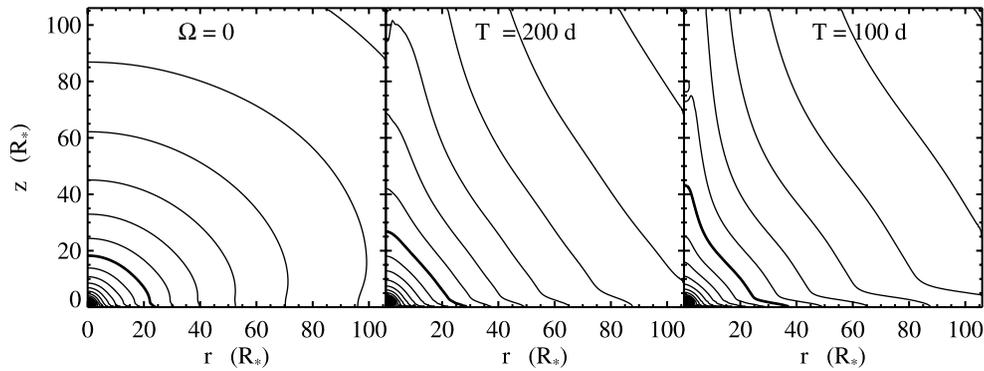


FIG. 5.—Logarithmic density contours in the  $r$ - $z$  plane for cases with (from left to right) no rotation,  $T = 200$ , and  $T = 100$  days and where the star has a constant temperature across its surface. Shown are data from the outermost computational grid, and the star is at the origin. The thickest contour line corresponds to a density of  $1.7 \times 10^{-13}$  g cm $^{-3}$ , and the spacing between each contour is a factor of 2.

and the disk along the equator. Also, compare the middle panel with Figure 2 of WS93.

Figure 6 is the same as Figure 5, but for the HTC $\infty$ , HTC100, and HTC50 models (from left to right). The left panel shows the conditions in the wind from a star with both a dipole magnetic field and an energetic polar wind. The density is enhanced along the equator, because of the dipole field, but also along the poles from the increased thermal driving there. The dashed line in each panel follows the magnetic field line that originates at a latitude of  $74^\circ$  on the stellar surface (as the light gray lines in Fig. 3). So the material that exists at a higher latitude than the dashed line originates from the high-temperature region on the star. In the left panel (where the star does not rotate), only material at very low latitudes originates from the cooler region on the star. When stellar rotation becomes important (*middle and right panels*), the midlatitudes are cleared out as magnetic pressure gradients compress material toward the equator and against “walls” of the polar flow. Instead of a narrow jet, wide-angle, bipolar lobes are formed. Ultimately, the opening angle of the lobes is determined by the balance between the thermal pressure in polar flow and magnetic pressure (associated with  $B_\phi$ ) in the low-latitude flow.

For the HTC50 case (Fig. 6, *right panel*), the density contours superficially resemble  $\eta$  Car’s homunculus. The morphology of the flow is more clearly seen in the projected, three-dimensional isodensity surfaces plotted in the top row of Figure 7. On the top left is an isodensity surface in the steady state wind of the ISO50 case, and the top right is from the HTC50 case. The surface on the top right corresponds to the same density as the thick line in the right panel of Figure 6, which serves as a size reference for the three-dimensional projections of Figure 7. To generate

these images, we have exploited the symmetries in the system by rotating the two-dimensional density contour from the simulation about the rotation axis and reflecting it across the equator. Note that the disk is similar for the two cases, but the polar flows are drastically different. For comparison, we have also included *Hubble Space Telescope* (*HST*) images of two nebulae on the bottom row of the figure. The bottom left is the proto-planetary nebulae IRAS 17106–3046 (from Kwok et al. 2000), and the bottom right is  $\eta$  Car (from Morse et al. 1998). Both nebulae have their symmetry axes along a line from bottom left to top right, both exhibit equatorial structures, and both are oriented such that the axis pointing toward bottom left of the image is tilted out of the plane by (very roughly)  $30^\circ$ .

The top right panel of Figure 6 bears a remarkable resemblance to the skirt and homunculus morphology of  $\eta$  Car (*bottom right*; see also Davidson et al. 2001). However, the comparison is only superficial because we have considered only the conditions in a steady state wind, while the lobes and skirt of  $\eta$  Car were ejected in an outburst. Also, the lobes of  $\eta$  Car are hollow (Davidson et al. 2001). It is not clear from this preliminary study what sorts of morphologies would result, if the wind were, for example, “turned on” for a few years (simulating an outburst) and then “turned off” again afterward and left to drift for 150 yr. Such a study is left for future work.

## 5. SUMMARY AND DISCUSSION

We have studied a thermally driven wind from a star with a rotation axis–aligned dipole magnetic field and varying amounts of rotation. We have considered both isotropic thermal driving and that which is enhanced on the pole of the star. Using MHD

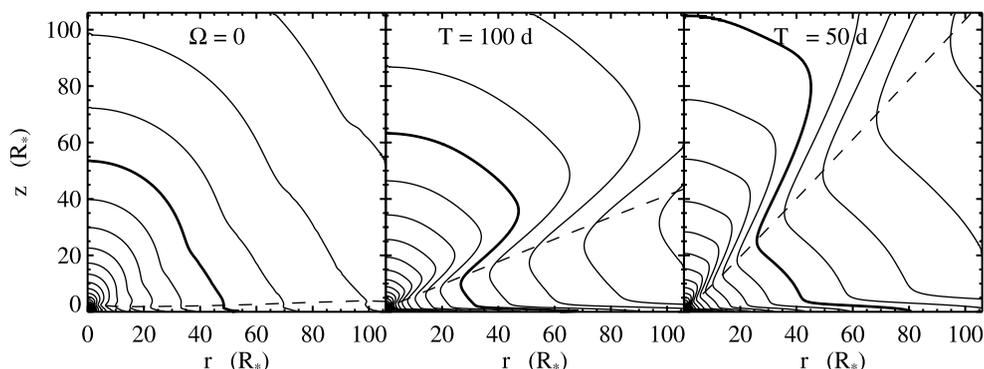


FIG. 6.—Same as Fig. 5, but for cases with (from left to right) no rotation,  $T = 100$ , and  $T = 50$  days and where the star has a high-temperature polar cap. The dashed line in each panel shows the magnetic field line that intersects the stellar surface at  $74^\circ$ , the latitude of the temperature discontinuity on the star.

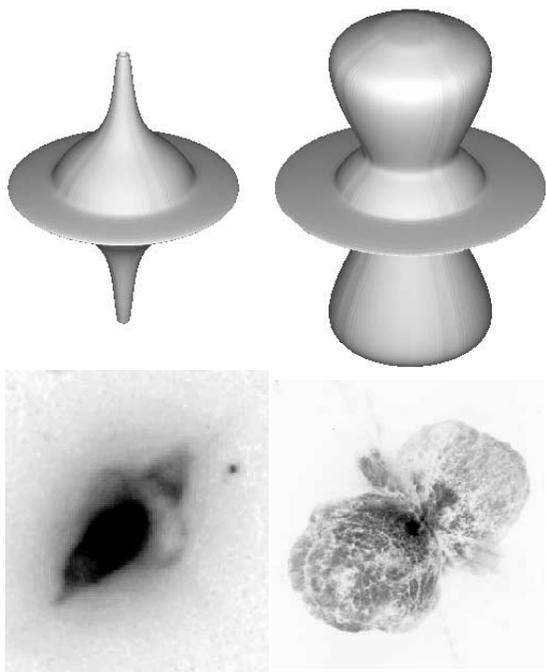


Fig. 7.—Isodensity surface in the steady state wind for two different models (top), each with a stellar rotation period of 50 days, compared to two *HST* WFPC2 images of nebulae (bottom). Top left: Density surface at  $5.2 \times 10^{-14}$  g cm $^{-3}$  for a case with isotropic thermal wind driving. Top right: Density surface at  $1.7 \times 10^{-13}$  g cm $^{-3}$  for a case with an enhanced polar wind. These are projections with the rotational symmetry axis tilted  $30^\circ$  from the plane of the sky. Bottom left: Proto-planetary nebula IRAS 17106–3046 (Kwok et al. 2000). Bottom right:  $\eta$  Carinae (Morse et al. 1998).

simulations, we determined the 2.5-dimensional (axisymmetric), steady state wind solution. Our parameter study resulted in winds with a wide range of shapes. Winds are slightly enhanced along the magnetic equator by the presence of a dipole magnetic field. When the star rotates, the poloidal magnetic field is twisted azimuthally, and magnetic pressure gradients associated with  $B_\phi$  direct material away from midlatitudes: toward the rotation axis, and toward the magnetic equator. For cases with isotropic thermal driving, fast stellar rotation results in an outflowing disk and jet morphology. For cases with an enhanced polar wind, there is an outflowing disk and wide-angle bipolar lobes. As far as we know, no physical model for producing such a wind (with a simultaneous disk and lobe morphology) has yet been demonstrated in the literature.

### 5.1. Implications for $\eta$ Car

The disk and lobe morphology of our steady state wind solutions resembles the skirt and homunculus morphology of the  $\eta$  Car nebula, although the latter was formed in a single eruptive outburst and is much more complex than the flows of our study. The opening angle of the lobes in our model HTC50 most resembles that of the homunculus. In addition, the total energy outflow rate in that model is  $\sim 9 \times 10^{40}$  ergs s $^{-1}$  (Table 2), which is comparable to the observed luminosity of  $\eta$  Car during an outburst of  $\sim 8 \times 10^{40}$  ergs s $^{-1}$  (Davidson & Humphreys 1997). For a comparison between the HTC50 model and the  $\eta$  Car nebula to be appropriate,  $\eta$  Car needs to have had a roughly axis-aligned dipole magnetic field of the order of  $2.5 \times 10^4$  G during the great eruption in the mid-1800s. It also needed to have a wind-driving mechanism that produced a flow that was inherently more energetic on the poles and needed to be rotating at a significant fraction of breakup speed

(both of which are also predicted by Dwarkadas & Owocki 2002). Assuming for now that  $\eta$  Car's wind resembled the HTC50 model during the great eruption, our model implies several things.

First, we find that  $B_\phi \approx 30$  G at  $40R_*$  and at midlatitudes. Assuming  $r^{-1}$  dilution of the field and a size of 37,500 AU for the  $\eta$  Car nebula (Davidson & Humphreys 1997), this predicts a  $\sim 15$  mG (azimuthally directed) field in the present-day homunculus. This is larger than the  $\sim$ mG fields detected by Aitken et al. (1995), but considering the uncertainties in our assumed parameters, and that the magnetic field can be dissipated by other processes, this comparison should not be stringent. Another unique prediction of magnetic wind theory is that the entire flow should be rotating (as verified for protostellar jets; Bacciotti et al. 2002). The HTC50 model has a rotational speed of  $\sim 30$  km s $^{-1}$  at  $40R_*$  in the disk. Assuming angular momentum conservation in the wind from that point outward, this predicts a rotation speed of  $\sim 20$  m s $^{-1}$  in the skirt at 25,000 AU, which would be difficult to detect observationally.

The rotation of the outflow extracts angular momentum from the star, and we find a loss rate of  $\approx 1.3 \times 10^{46}$  ergs (Table 2) during outburst. The angular momentum of  $\eta$  Car, for the assumed parameters, is  $J_* = k^2 M_* \Omega_* R_*^2 \approx 1.4 \times 10^{55} k^2$  erg s (where  $k$  is the normalized radius of gyration). Assuming  $k^2 \sim 0.1 - 0.2$ , the total amount of angular momentum extracted by the wind during the outburst would be comparable to  $J_*$  in only 3.5–7 yr! The reason for such a large  $\dot{J}_w$  is mainly because of the large mass of the homunculus, which is  $\geq 1\%$  of the mass of the star. Particularly, if this much mass is shaped magnetocentrifugally, as we have considered here,  $\dot{J}_w$  will necessarily be large. This result suggests either that  $\eta$  Car can have only one or two such outbursts during its lifetime, or that the star must gain angular momentum from an external source. Note that  $\eta$  Car may have a binary companion (see, e.g., Smith et al. 2004), which could likely further influence the structure of the wind and possibly spin up  $\eta$  Car (Smith et al. 2003), although a detailed calculation remains to be done.

### 5.2. General Application

The range of morphologies exhibited in our parameter study may be applicable (with appropriate modifications) to a wide range of observed outflow nebulae. A general conclusion of our study is that, for any magnetic rotator wind in which collimation occurs, if the source has an aligned dipolar field, the formation of an outflowing disk is inevitable. So any outflow nebulae that exhibits quadrupolar symmetry (i.e., outflowing disk plus jets or disk plus lobes) may be explained with winds similar to those presented here. For example, the general torus plus jet morphology of the crab pulsar X-ray nebula (e.g., Hester et al. 2002) could be explained by a wind accelerated in a rotating dipole field, although relativistic effects and other pulsar physics should be considered. In addition, proto-planetary nebulae all show axisymmetries, and the overwhelming majority of optically visible proto-planetary nebulae have disks seen in silhouette (e.g., see IRAS 17106–3046 in Fig. 7, bottom left panel). A mechanism that generates disks and lobes together may turn out to be important. The general model presented here is also appealing because of the relatively small number of parameters, and its ability to link observed parameters to the conditions very near the source of the wind.

It is important to note, however, that many astrophysical nebulae of various types are evidently shaped by processes

that are inherently time-dependent, either through the interaction of multiple winds or in outburst-type events. Time dependence adds many additional parameters to any outflow model, making parameter studies challenging but greatly enriching the range of possible outcomes. Future work with magnetocentrifugal stellar winds should include time-dependent effects.

We would like to thank the anonymous referee, whose suggestions led to significant improvements in the paper. This research was supported by NASA grant GO 9050, awarded from the Space Telescope Science Institute, and by the National Science and Engineering Research Council of Canada, McMaster University, and the Canadian Institute for Theoretical Astrophysics.

## REFERENCES

- Aitken, D. K., Smith, C. H., Moore, T. J. T., & Roche, P. F. 1995, *MNRAS*, 273, 359
- Babel, J., & Montmerle, T. 1997, *A&A*, 323, 121
- Bacciotti, F., Ray, T. P., Mundt, R., Eisloffel, J., & Solf, J. 2002, *ApJ*, 576, 222
- Blandford, R. D., & Payne, D. G. 1982, *MNRAS*, 199, 883
- Brackbill, J. U., & Barnes, D. C. 1980, *J. Comput. Phys.*, 35, 426
- Bravo, S., Stewart, G. A., & Blanco-Cano, X. 1998, *Sol. Phys.*, 179, 223
- Davidson, K., & Humphreys, R. M. 1997, *ARA&A*, 35, 1
- Davidson, K., Smith, N., Gull, T. R., Ishibashi, K., & Hillier, D. J. 2001, *AJ*, 121, 1569
- Dwarkadas, V. V., & Balick, B. 1998, *AJ*, 116, 829
- Dwarkadas, V. V., & Owocki, S. P. 2002, *ApJ*, 581, 1337
- Frank, A., Balick, B., & Davidson, K. 1995, *ApJ*, 441, L77
- Goldstein, B. E., et al. 1996, *A&A*, 316, 296
- González, R. F., de Gouveia Dal Pino, E. M., Raga, A. C., & Velazquez, P. F. 2004, *ApJ*, 600, L59
- Goodson, A. P., Winglee, R. M., & Böhm, K. H. 1997, *ApJ*, 489, 199
- Hester, J. J., et al. 2002, *ApJ*, 577, L49
- Keppens, R., & Goedbloed, J. P. 1999, *A&A*, 343, 251
- . 2000, *ApJ*, 530, 1036
- Kwok, S., Hrivnak, B. J., & Su, K. Y. L. 2000, *ApJ*, 544, L149
- Langer, N., García-Segura, G., & Mac Low, M. 1999, *ApJ*, 520, L49
- Matt, S. P. 2002, Ph.D. thesis, Univ. Washington
- Matt, S. P., Balick, B., Winglee, R., & Goodson, A. 2000, *ApJ*, 545, 965
- Matt, S. P., Goodson, A. P., Winglee, R. M., & Böhm, K. 2002, *ApJ*, 574, 232
- Michel, F. C. 1969, *ApJ*, 158, 727
- Morse, J. A., Davidson, K., Bally, J., Ebbets, D., Balick, B., & Frank, A. 1998, *AJ*, 116, 2443
- Morse, J. A., Kellogg, J. R., Bally, J., Davidson, K., Balick, B., & Ebbets, D. 2001, *ApJ*, 548, L207
- Ouyed, R., & Pudritz, R. E. 1997, *ApJ*, 482, 712
- Parker, E. N. 1958, *ApJ*, 128, 664
- Porter, J. M. 1997, *A&A*, 324, 597
- Ritchmyer, R. D., & Morton, K. W. 1967, *Difference Methods for Initial-Value Problems* (2nd ed.; New York: Wiley)
- Sakurai, T. 1985, *A&A*, 152, 121
- Smith, N., Davidson, K., Gull, T. R., Ishibashi, K., & Hillier, D. J. 2003, *ApJ*, 586, 432
- Smith, N., Morse, J. A., Collins, N. R., & Gull, T. R. 2004, *ApJ*, 610, L105
- Tanaka, T., & Washimi, H. 2002, *Science*, 296, 321
- ud-Doula, A., & Owocki, S. P. 2002, *ApJ*, 576, 413
- Ustyugova, G. V., Koldoba, A. V., Romanova, M. M., Chechetkin, V. M., & Lovelace, R. V. E. 1999, *ApJ*, 516, 221
- van Boekel, R., et al. 2003, *A&A*, 410, L37
- Washimi, H. 1990, *Geophys. Res. Lett.*, 17, 33
- Washimi, H., & Shibata, S. 1993, *MNRAS*, 262, 936
- Weber, E. J., & Davis, L. J. 1967, *ApJ*, 148, 217