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Modeling Atmospheric Neutrino Interactions: Duality Constrained Parameterization of Vector and Axial Nucleon Form Factors

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Abstract: We present new parameterizations of vector and axial nucleon form factors. We maintain an excellent descriptions of the form factors at low momentum transfers, where the spatial structure of the nucleon is important, and use the Nachtman scaling variable ξ to relate elastic and inelastic form factors and impose quark-hadron duality constraints at high momentum transfers where the quark structure dominates. We use the new vector form factors to re-extract updated values of the axial form factor from neutrino experiments on deuterium. We obtain an updated world average value from neutrino and pion electroproduction experiments of $M_A = 1.0155 \pm 0.0136\ GeV/c^2$. Our parameterizations are useful in modeling atmospheric neutrino interactions (e.g. for neutrino oscillations experiments).

At low Q^2, a reasonable description of the proton and neutron elastic form factors is given by the dipole approximation.

$$G_D^{V,A}(Q^2) = \frac{C^{V,A}}{\left(1 + \frac{Q^2}{M_{V,A}^2}\right)^2},$$

where $C^{V,A}$= (1,g_A), g_A = -1.267, M_V^2 = 0.71 $(GeV/c^2)^2$, and M_A = 1.015 GeV/c^2.

Here we present parameterizations that simultaneously satisfy constraints at low Q^2 where the spatial structure of the nucleon is important, as well as at high Q^2 where the quark structure is important. Our new quark-hadron duality based parameterization ("BBBA07") includes: (1) Improved functional form that uses Nachtman scaling variable ξ to relate elastic and inelastic vector and axial nucleon form factors; (2) Yield the same values as $fits$ by Arrington and Sick [3] for $Q^2 < 0.64(GeV/c)^2$, while satisfying quark-hadron duality constraints at high-Q^2.

We use our new "BBBA07"vector form factors to re-extract updated values of the axial form factor from a re-analysis of previous neutrino scattering data on deuterium and present a new parameterization for the axial form factor within the framework of quark-hadron duality. For vector form factors our fit functions are $A_N(\xi)$ (i.e. $A_{Ep}(\xi)$, $A_{Mp}(\xi)$, $A_{En}(\xi)$, $A_{Mn}(\xi)$) multiplied by an updated Kelly [3] type parameterization of one of the proton form factors.

$$A_N(\xi) = \sum_{j=1}^{n} P_j(\xi)$$

$$P_j(\xi) = p_j \prod_{k=1,k\neq j}^{n} \frac{\xi - \xi_k}{\xi_j - \xi_k}.$$

$$G^{Kelly}(Q^2) = \frac{\sum_{k=0}^{m} a_k \tau^k}{1 + \sum_{k=1}^{m+2} b_k \tau^k},$$

where $a_0 = 1$, $m = 1$, and $\tau = Q^2/4M_N^2$. (M_N is proton, neutron, or average nucleon mass for proton, neutron, and axial form factors, respectively). The datasets used by Kelly to fit G_{Ep} and G_{Mp}/μ_p ($\mu_p = 2.7928$, $\mu_n = -1.913$) are described in [3]. Our parameterization employs the as-published Kelly parameterization to G_{Ep}^{Kelly} and an updated set of parameters for $G_{MP}^{Kelly}(Q^2)$ that includes the recent BLAST[2] results. The parameters used for $G^{Kelly}(Q^2)$ are given in ref. [1].

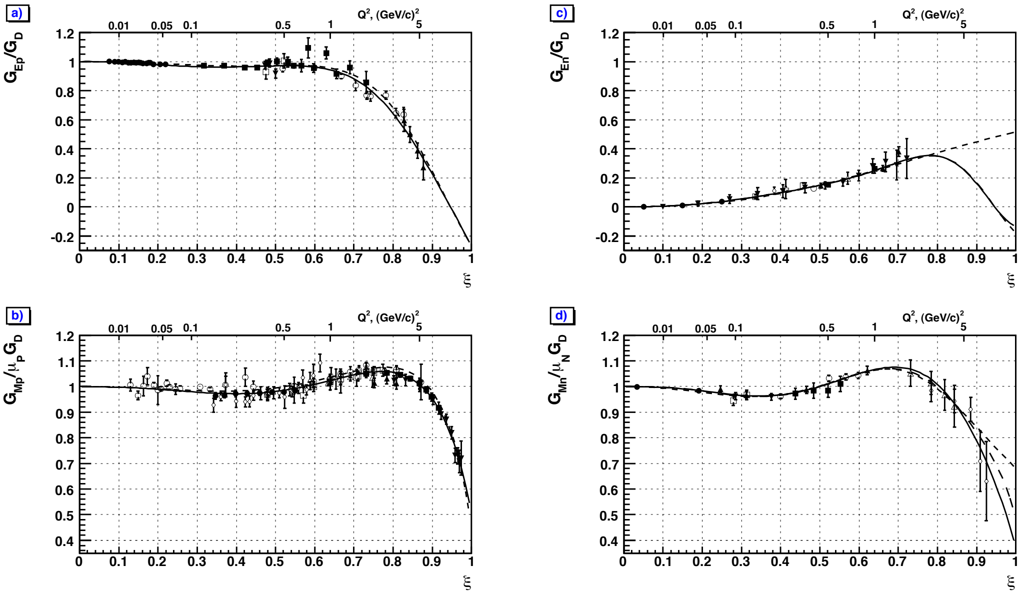

Figure 1: Ratios of G_{Ep} (a), G_{Mp}/μ_p (b), G_{En} (c) and G_{Mn}/μ_n (d) to G_D^V. The short-dashed line in each plot is the old Kelly parameterizations (old Galster for G_{En}). The solid line is our new BBBA07 parameterization for $\frac{d}{u} = 0.0$, and the long-dashed line is BBBA07 for $\frac{d}{u} = 0.2$.

Each P_j is a LaGrange polynomial in the Nachtman variable, $\xi = \frac{2}{(1+\sqrt{1+1/\tau})}$ The ξ_j are equidistant "nodes" on an interval $[0, 1]$ and p_j are the fit parameters that have the property $A_N(\xi_j) = p_j$. The seven p_j parameters are at ξ_j=0, 1/6, 1/3, 1/2, 2/3, 5/6, and 1.0.

In the $fitting$ procedure described below, the parameters of $A_N(\xi)$ are constrained to give the same vector form factors as the recent low Q^2 fit of Arrington and Sick [3] for $Q^2 < 0.64(GeV/c)^2$ (as that analysis includes coulombs corrections which modify G_{Ep},two photon exchange corrections which modify G_{Mp} and G_{Mn}). Since the published form factor data do not have these corrections, this constraint is implemented by including additional $''fake''$ data points for $Q^2 < 0.64(GeV/c)^2$. Our Tts to the form factors are:

$$G_{Mp}(Q^2) = \mu_p A_{Mp}(\xi) \times G_{Mp}^{Kelly}(Q^2)$$
$$G_{Ep}(Q^2) = A_{Ep}(\xi) \times G_{Ep}^{Kelly}(Q^2)$$
$$G_{En}(Q^2) = A_{En}^{a,b}(\xi) \times G_{Ep}(Q^2) \times \left(\frac{a\tau}{1+b\tau}\right)$$
$$G_{Mn}(Q^2) = A_{Mn}^{a,b}(\xi) \times G_{Mp}(Q^2)\mu_n/\mu_p,$$

where we use our updated parameters in the Kelly parameterization of G_{Mp}. For G_{En} the parameters a=1.7 and b=3.3 are the same as in the Galster[3] parametrization and ensure that dG_{En}/dQ^2 at for $Q^2 = 0$ is in agreement with measurements. The values $A(\xi)$=p_1 at ξ_1=0 ($Q^2 = 0$) for G_{Mp}, G_{Ep}, G_{En}, G_{Mn} are set to to 1.0. The value $A(\xi)$=p_7 at ξ_j=1 ($Q^2 \to \infty$) for G_{Mp} and G_{Ep} is set to 1.0. The value $A(\xi)$=p_j at ξ_j=1 for G_{Mn} and G_{En} are $fixed$ by constraints from quark-hadron duality. Quark-hadron duality implies that the ratio of neutron and proton magnetic form factors should be the same as the ratio of the corresponding inelastic structure functions $\frac{F_{2n}}{F_{2p}}$ in the ξ=1 limit. (Here $F_2 = \xi \sum_i e_i^2 q_i(\xi)$)

$$\frac{G_{Mn}^2}{G_{Mp}^2} = \frac{F_{2n}}{F_{2p}} = \frac{1+4\frac{d}{u}}{4+\frac{d}{u}} = \left(\frac{\mu_n^2}{\mu_p^2}\right) A_{Mn}^2(\xi = 1)$$

We ran $fits$ with two different values of $\frac{d}{u}$ at the ξ=1 limit: $\frac{d}{u}$ =0 and 0.2 (corresponding to $\frac{F_{2n}}{F_{2p}}$ = 0.25 and 0.4286). The fit utilizing $\frac{d}{u} = 0$ is

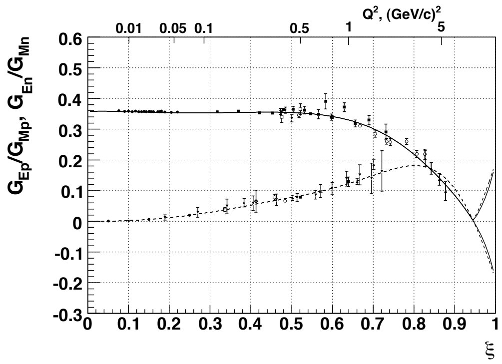

Figure 2: The constraint used in fitting G_{En} stipulates that $G^2_{En}/G^2_{Mn} = G^2_{Ep}/G^2_{Mp}$ at high ξ. The solid line is $\frac{G_{Ep}}{|G_{Mp}|}$ and $\frac{|G_{Ep}|}{|G_{Mp}|}$, and the short-dashed line is $\frac{G_{En}}{|G_{Mn}|}$ and $\frac{|G_{En}|}{|G_{Mn}|}$.

$G^2 5_{Mn}$, and the fit utilizing $\frac{d}{u} = 0.2$ is $G^4 3_{Mn}$. The value $A(\xi)$=p_j at ξ_j=1 for G_{En} is set by another duality-motivated constraint. R is $defined$ as the ratio of deep-inelastic longitudinal and transverse structure functions. In the elastic limit:

$$R_n(x = 1; Q^2) = (4M^2_N/Q^2) \times (G^2_{En}/G^2_{Mn}).$$

For inelastic scattering, as $Q^2 \to \infty$, $R_n = R_p$. If we assume quark-hadron duality, the same is for the elastic form factors at ξ=1 ($Q^2 \to \infty$)

$$G^2_{En}/G^2_{Mn} = G^2_{Ep}/G^2_{Mp}$$

The new form factors G_{Ep}, G_{Mp}/μ_p, G_{Mn}/μ_n, and G_{En} are plotted in Figure 1 as ratios to the dipole form G^V_D. as ratios to the dipole form factor, G^V_D. $A_N(\xi)$ is not needed for G_{Mp} as it is very close to 1.0. For G_{Ep} it yields a correction of 1% at low Q^2 (because it is required to agree with the $fits$ of Arrington and Sick[3] which include two photon exchange and Coulomb corrections. For G_{En} and G_{Mn} it is used to impose quark-hadron duality asymptotic constraints. Figure 2 shows plots of $\frac{G_{En}}{|G_{Mn}|}$ and $\frac{G_{Ep}}{|G_{Mp}|}$ (for the $\frac{d}{u}$ = 0 at ξ = 1 case). The long-dashed line is a simple quark-hadron duality model [4].

Using the updated vector form factors, we perform a complete reanalysis of published neutrino quasielastic [6] data on deuterium. (Because of uncertain nuclear corrections, neutrino data on heavier nuclear targets are not used.) We extract new values of M_A, and updated values of $F_A(Q^2)$. The average of the corrected neutrino measurements is $M_A = 1.0142 \pm 0.0266$. This is to be compared to the average value of 1.016 ± 0.016 extracted from pion electroproduction experiments[5] after corrections for hadronic effects. The average of the two average values is 1.0155 ± 0.0136. The new form factors G_{Ep}, G_{Mp}/μ_p, G_{Mn}/μ_n, and G_{En} are plotted in Figure 1.

$Experiment$	M_A (published)	ΔM_A new–old
$Miller - D - ANL_{82,77,73}$	1.00 ± 0.05	-0.035
$Baker - D - BNL_{81}$	1.07 ± 0.06	-0.032
$Kitagaki - D - FNAL_{83}$	$1.05^{+0.12}_{-0.16}$	-0.024
$Kitagaki - D - BNL_{90}$	$1.070^{+0.040}_{-0.045}$	-0.039

Table 1: M_A GeV/c^2 values published by neutrino-deuterium experiments and updated corrections ΔM_A when re-extracted with updated vector form factors.

For deep-inelastic scattering, the vector and axial parts of the inelastic structure functions W_2 are equal. Local quark-hadron duality at large Q^2 implies that the axial and vector components of $W_2^{elastic}$ are also equal, yielding:

$$[F_A(Q^2)_{A2=V2}]^2 = \frac{(G^V_E)^2(Q^2) + \tau(G^V_M(Q^2))^2}{(1+\tau)}, \quad (1)$$

where $G^V_E(Q^2) = G_{Ep}(Q^2) - G_{En}(Q^2)$ and $G^V_M(Q^2) = G_{Mp}(Q^2) - G_{Mn}(Q^2)$. We do a duality based fit to the updated values of the axial form factor $F_A(Q^2)$, including pion electroproduction data. Here the fit function is a sum of LaGrange polynomials, A^a_{FA}, multiplied by $G^A_D(Q^2)$ (with $M_A = 1.015$).

$$F_A(Q^2) = A^a_{FA}(\xi) \times G^A_D(Q^2).$$

We impose the constraint $A^a_{FA}(\xi_1 = 0) = p_1 = 1.0$. We also constrain the fit by requiring that $A^a_{FA}(\xi)$ yield $F_A(Q^2) = F_A(Q^2)_{A2=V2}$ for $\xi > 0.9$ ($Q^2 > 7.2(GeV/c)^2$). Figure 3(a) shows values of $F_A(Q^2)$ extracted from neutrino-deuterium experiments divided by $G^A_D(Q^2)$, with M_A =

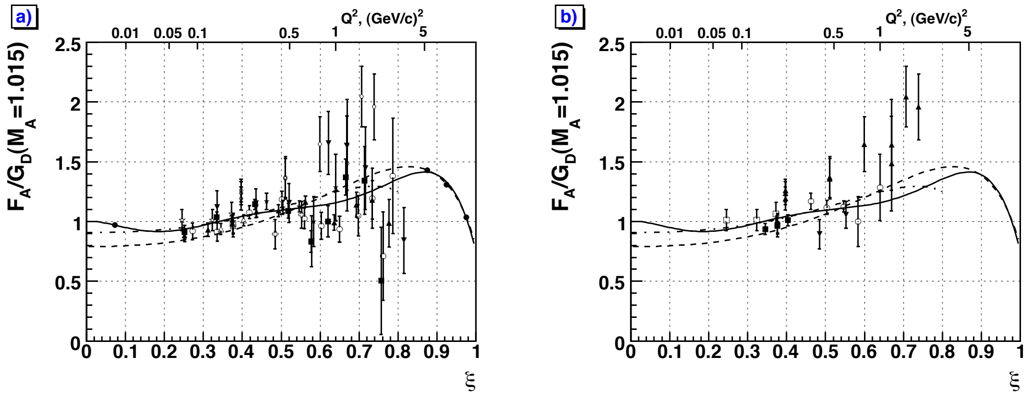

Figure 3: (a) $F_A(Q^2)$ re-extracted from neutrino-deuterium data divided by $G_D^A(Q^2)$ (with $M_A = 1.015$). (b) $F_A(Q^2)$ from pion electroproduction divided by $G_D^A(Q^2)$ (corrected for for hadronic effects[5]). The solid line is our duality based fit. The short-dashed line is $F_A(Q^2)_{A2=V2}$. The dashed-dot line is a prediction from a constituent quark model.

1.015. Figure 3(b) shows values of $F_A(Q^2)$ extracted from pion electroproduction experiments divided by $G_D^A(Q^2)$. These pion electroproduction values can be directly compared to the neutrino results because they are multiplied by a factor $F_A(Q^2, M_A = 1.014)/F_A(Q^2, M_A = 1.069)$ to correct for $\Delta M_A = 0.055$ originating from hadronic effects[5]. The solid line is our duality based fit. The short-dashed line is $F_A(Q^2)_{A2=V2}$. The long-dashed line is $F_A(Q^2)_{A1=V1}$. The dashed-dot line is a prediction from a constituent quark model[7]. Our new parameterizations of vector and axial nucleon form factors use quark-hadron duality constraints at high momentum transfers and maintain a very good descriptions of the form factors at low momentum transfers. These parameterizations are useful in modeling neutrino interactions (e.g. for neutrino oscillations experiments). Our predictions for $G_{En}(Q^2)$ and $F_A(Q^2)$ at high (Q^2) can be tested in upcomng electron scattering and neutrino experiments at Jefferson Laboratory and at Fermilab (MINERvA). The $final$ parameters are given in Ref. [1].

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