Quarks for Dummies ™ *

Modeling (e/ μ /ν)-N Cross Sections from Low to High Energies: from DIS to Resonance, to Quasielastic Scattering

Modified LO PDFs, ξ_w scaling, Quarks and Duality**

NuInt02 Conference http://nuint.ps.uci.edu/

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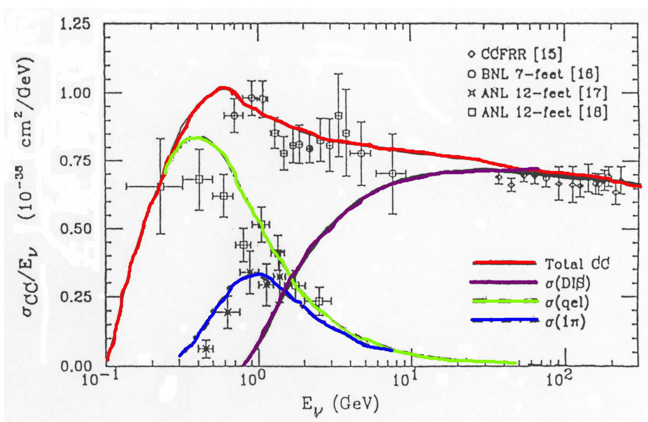
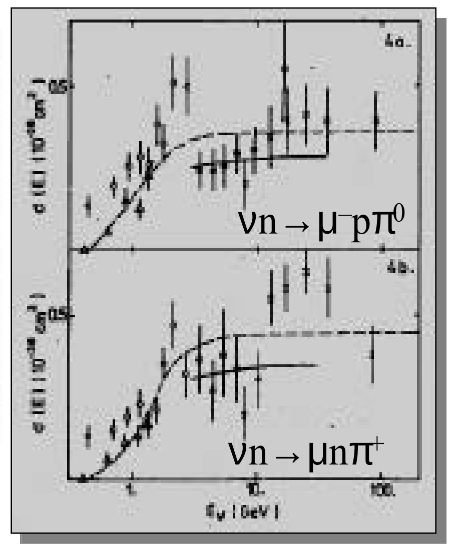
(work done with Un-Ki Yang, Univ. of Chicago)

http://www.pas.rochester.edu/~bodek/NeutrinoIrvine.ppt

Status of Cross-Sections

- Not well-known, especially in region of NUMI 0.7^0 off-axis proposal (~2 GeV)

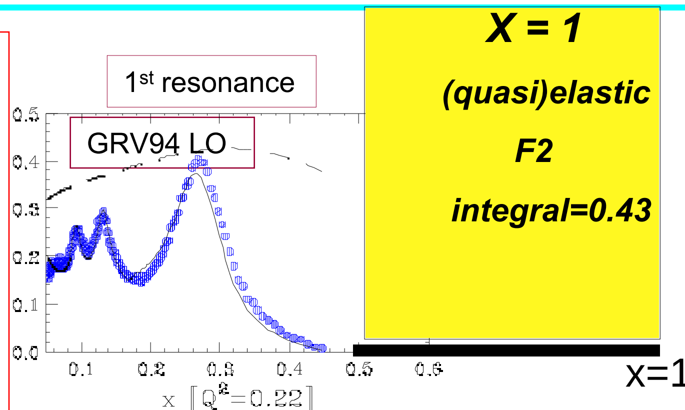




(e/ μ /ν)-N cross sections at low energy

Neutrino interactions --

- **Quasi-Elastic / Elastic (W=Mp)**
  ν_μ + n --> μ^- + p (x =1, W=Mp)
  well measured and described by form factors (but need to account for Fermi Motion/binding effects in nucleus) e.g. Bodek and Ritchie (Phys. Rev. D23, 1070 (1981)
- **Resonance (low Q^2, W< 2)**
  ν_μ + p --> μ^- + p + n π Poorly measured , Adding DIS and resonances together without double counting is a problem. 1st resonance and others modeled by Rein and Seghal. Ann Phys 133, 79, (1981)
- **Deep Inelastic**
  ν_μ + p --> μ^- + X (high Q^2, W> 2)
  well measured by high energy experiments and well described by quark-parton model (pQCD with NLO PDFs), but doesn't work well at low Q^2 region.



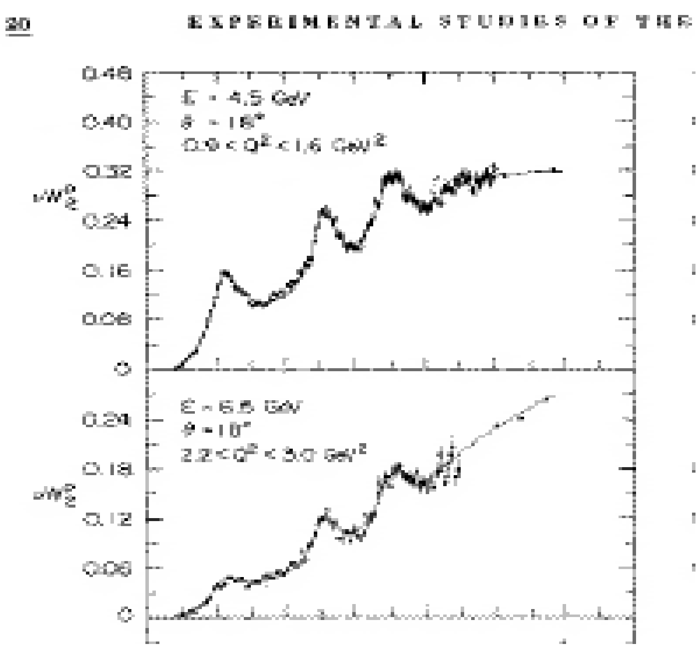
(e.g. SLAC data at Q^2=0.22)

- **Issues at few GeV :**
- **Resonance production and low Q^2 DIS contribution meet.**
- **The challenge is to describe ALL THREE processes at ALL neutrino (or electron) energies**
- **HOW CAN THIS BE DONE? - Subject of this TALK**

MIT SLAC DATA 1972 e.g. E0 = 4.5 and 6.5 GeV

e-P scattering A. Bodek PhD thesis 1972

[PRD 20, 1471(1979)] Proton Data

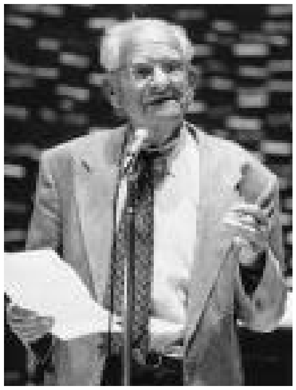


' The electron scattering data in the Resonance Region is the "Frank Hertz Experiment" of the Proton. The Deep Inelastic Region is the "Rutherford Experiment" of the proton' SAID

V. Weisskopf * (former faculty member at Rochester and at MIT when he showed these data at an MIT Colloquium in 1971 (* died April 2002 at age 93)

What do The Frank Hertz" and "Rutherford Experiment" of the proton' have in common?

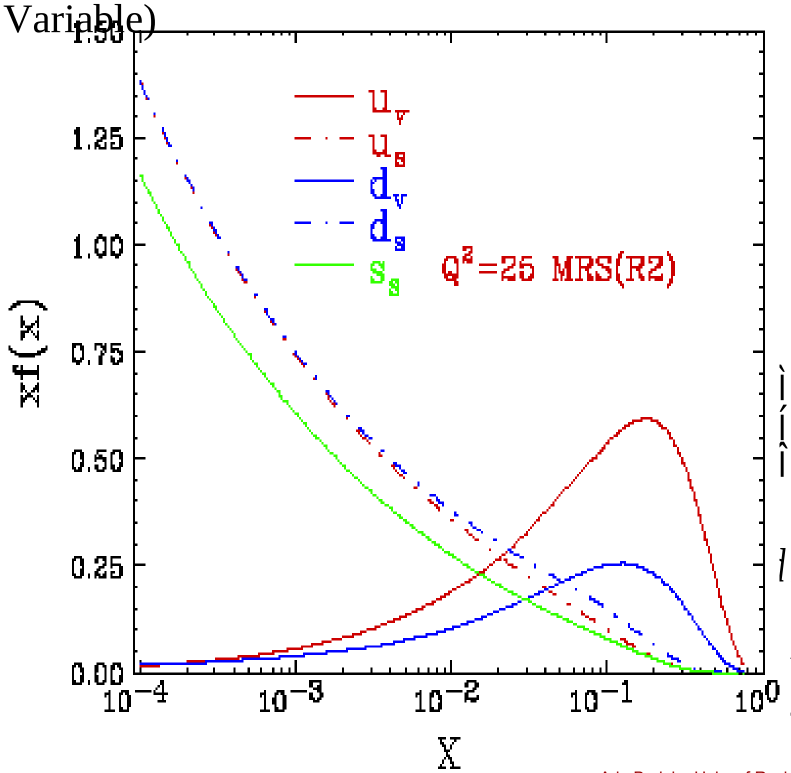
A: Quarks! And QCD



How are PDFs Extracted from global fits to High Q2 Deep Inelastic e/μ/ν Data

Note: additional information on Antiquarks from Drell-Yan and on Gluons from p-pbar jets also used.

MRSR2 PDFs xq is the probability that a Parton q carries fractional momentum x = $Q^2/2M\nu$ in the nucleon (x is the Bjorken Variable)



$$u_V + d_V \xrightarrow{from} F_2^{\nu} \approx x(u+\bar{u}) + x(d+\bar{d})$$

$$xF_3^{\nu} \approx x(u-\bar{u}) + x(d-\bar{d})$$

$$u+\bar{u} \xrightarrow{from} {}^{\mu}F_2^{p} \approx \frac{4}{9}x(u+\bar{u}) + \frac{1}{9}x(d+\bar{d})$$

$$d+\bar{d} \xrightarrow{from} {}^{\mu}F_2^{n} \approx \frac{1}{9}x(u+\bar{u}) + \frac{4}{9}x(d+\bar{d})$$

$$\left\{ \begin{array}{l} \text{nuclear effects} \\ \text{typically ignored} \end{array} \right\} \quad {}^{\mu}F_2^{n} = 2\frac{{}^{\mu}F_2^{d}}{{}^{\mu}F_2^{p}} - 1$$

$$d/u \xrightarrow{from} p\bar{p}W \;\; {}^{Asymmetry} \approx \frac{d/u(x_1) - d/u(x_2)}{d/u(x_1) + d/u(x_2)}$$

At high x, deuteron binding effects introduce an uncertainty in the d distribution extracted from F2d data (but not from the W asymmetry data). X=$Q^2/2M\nu$ Fraction momentum of quark

Building up a model for all Q^2.

Challenges

- **Can we build up a model to describe all Q^2 region** from high down to very low energies **? [Resonance, DIS, even photo production]**
- Advantage if we describe it in terms of the **quark-parton model.**
- With PDFs is straightforward to convert charged-lepton scattering cross sections into neutrino cross section. **(just matter of different couplings)**

o **Final state hadrons implemented in terms of fragmentation functions.**

o **Nuclear dependence of PDFs and fragmentation functions can be included.**

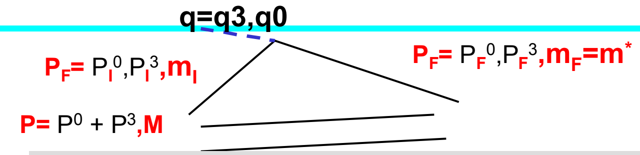
- Understanding of high x PDFs at very low Q^2
- **There is a of wealth SLAC, JLAB data**, but it requires understanding of non-perturbative QCD effects.
- **Need better understanding of resonance** scattering in terms of the quark-parton model? (duality works, many studies by JLAB)
- **Need to satisfy photoproduction limits at Q^2=0 and describe photoproduction.**
- **Should have theoretical basis. E.g. At high Q^2 should agree with QCD PDFs and sum rules - e.g. Momentum Sum Rule**
- **At ALL Q2 should agree with Current Algebra sum rules - Adler Sum rule is EXACT down to Q^2=0**
- ***If one knows where the road begins (high Q^2 PDFs) and ends (Q2=^{2} photo-production), it is easier to build it.***
- ***like the old Mayan Road from Coba to Chichen Itza - Very Straight and Very Level, Still there above the planes, but overgrown***

Initial quark mass m_I and final mass $m_F = m^*$ bound in a proton of mass M -- Summary: INCLUDE quark initial Pt) Get ξ scaling (not $x = Q^2/2M\nu$)

for a general parton Model

ξ Is the correct variable which is Invariant in any frame : q3 and P in opposite directions.

ξ



$PI, P0 \quad q3\ q0$

$$\xi = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \qquad \underset{quark}{\longrightarrow} \quad \underset{photon}{\longleftarrow}$$

$$(q + P_I)^2 = P_F^2 \quad \to q^2 + 2P_I\, q + P_I^2 = m_F^2$$

$$\xi_W = \frac{Q^2 + m_F^2 + A}{\{M\nu[1 + \sqrt{(1 + Q^2/\nu^2)}] + B\}} \qquad for\ m_I^2, Pt = 0$$

Special cases:

(1) Bjorken x, $x_{BJ} = Q^2/2M\nu$, ξ, -> x
For $m_F^2 = m_I^2 = 0$ and High ν^2,

(2) Numerator m_F^2 : Slow Rescaling ξ as in charm production

(3) Denominator: Target mass term
ξ =Nachtman Variable
ξ =Light Cone Variable
ξ =Georgi Politzer Target Mass var. (all the same ξ)

Most General Case: (Derivation in Appendix)

ξ'_w = $[Q'^2 + B]$ / $[M\nu (1+(1+Q^2/\nu^2))^{1/2} + A]$ (with A=0, B=0)

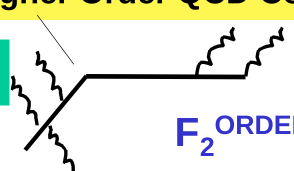
where $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + \{ (Q^2 + m_F^2 - m_I^2)^2 + 4Q^2 (m_I^2 + P^2t) \}^{1/2}$

Bodek-Yang: Add B and A to account for effects of additional Δm^2 from NLO and NNLO (up to infinite order) QCD effects. For case ξ_w with **P^2t** =0 see R. Barbieri et al Phys. Lett. 64B, 1717 (1976) and Nucl. Phys. B117, 50 (1976)

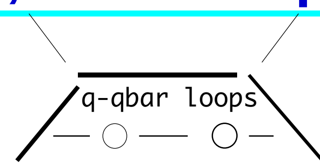
ORIGIN of A, B: QCD is an asymptotic series, not a converging series- at any order, there are power corrections

Higher Order QCD Corr.

Pti,Mi



Mi



Renormalon Power Corr.

$F_2^{ORDER\text{-}N}$

$$F_2^{ORDER\text{-}N} = F_2^{QCD\text{-}0} \{ 1 + C_1(x,Q)\, \alpha_S + C_2(x,Q)\, \alpha_S^2 + \ldots\ldots\ C_N(x,Q)\, \alpha_S^N \}$$

$\Delta F_2 = F_2^{ALL\ ORDERS} - F_2^{ORDER\ N}$ --> (The series is Truncated)

$$\Delta F_2 = \text{Power Corrections} = (1/Q^2)\ a_{2,N}\, D_2(x,Q^2) + (1/Q^4)\ a_{4,N}\, D_4(x,Q^2)$$

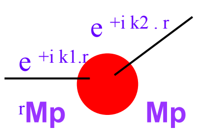
1. In pQCD the $(1/Q^2)$ terms from the interacting quark are the missing higher order terms. Hence, $a_{2,N}$ and $a_{4,N}$ should become smaller with N.
2. The only other HT terms are from the final state interaction with the spectator quarks, which should only affect the low W region.
3. Our studies have shown that to a good approximation, if one includes the known target mass (TM) effects, the spectator quarks do not affect the average level of the low W cross section as predicted by pQCD if the power corrections from the interacting quark are included.

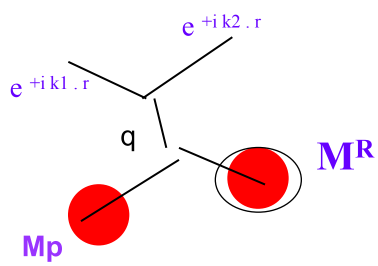
A, B in ξ'_w model multi-gluon emission as Δm^2 added to m_f, m_I Pt

What are Higher Twist Effects - Page 2-details

- **Nature has "evolved" the high Q^2 PDF from the low Q^2 PDF,** therefore, **the high Q^2 PDF** include the information about the higher twists .
- High Q^2 manifestations of higher twist/non perturbative effects include: difference between u and d, the difference between d-bar, u-bar and s-bar etc. High Q^2 PDFs "remember" the higher twists, which originate from the non-perturbative QCD terms.
- **Evolving back the high Q^2 PDFs *to low Q^2* (e.g. NLO-QCD) and comparing to low Q^2 data is one way to check for the effects of higher order terms.**
- What do these higher twists come from?
  - **Kinematic higher twist** – initial state target mass binding (Mp, $\xi_{TM)}$ initial state and final state quark masses (e.g. charm production)- ξ_{TM} important at high x
  - **Dynamic higher twist** – correlations between quarks in initial or final state.==> Examples : Initial or final state multiquark correlations: diquarks, elastic scattering, excitation of quarks to higher bound states e.g. resonance production, exchange of many gluons: important at low W
  - **Non-perturbative effects** to satisfy gauge invariance and connection to photo-production [e.g. $F_2(\nu, Q^2=0) = Q^2 / [Q^2 + C] = 0$]. important at very low Q2.
  - **Higher Order QCD effects/power corrections -** to e.g. NNLO+ multi-gluon emission"looks like" Power higher twist corrections since a LO or NLO calculation do not take these into account, also quark intrinsic P_T (terms like P_T^2/Q^2). Important at all x (look like **Dynamic Higher Twist**)

Old Picture of fixed W scattering - form factors (the Frank Hertz Picture)





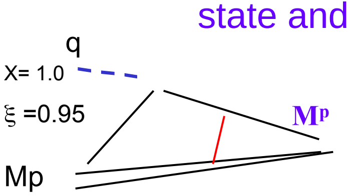
- **OLD Picture fixed W: Elastic Scattering, Resonance Production. Electric and Magnetic Form Factors (G_E and G_M) versus Q^2 measure size of object (the electric charge and magnetization distributions).**
- **Elastic scattering $W = M^p = M$**, single final state nucleon: Form factor measures size of nucleon.Matrix element squared $| \langle p_f | V(r) | p_i \rangle |^2$ between initial and final state lepton plane waves. Which becomes:
- $| \langle e^{-i\,k2.\,r} | V(r) | e^{+i\,k1\,.\,r} \rangle |^2$
- $q = k1 - k2$ = momentum transfer
- $G_E(q) = \int \{ e^{i\,q\,.\,r} \rho(r)\, d^3r \}$ = Electric form factor is the Fourier transform of the charge distribution. Similarly for the magnetization distribution for G_M Form factors are relates to structure function by:
- $2xF_1(x, Q^2)_{elastic} = x^2\, G_M{}^2{}_{elastic}\,(Q^2)\,\delta\,(x-1)$
- **Resonance Production, $W=M^R$,** Measure transition form factor between a quark in the ground state and a quark in the first excited state. For the Delta 1.238 GeV first resonance, we have a Breit-Wigner instead of δ (x-1).
- $2xF_1(x, Q^2)_{resonance} \sim x^2\, G_M{}^2{}_{Res.\ transition}\,(Q^2)\ BW\,(W-1.238)$

Duality: Parton Model Pictures of Elastic and Resonance Production at Low W (High Q2)

Elastic Scattering, Resonance Production: Scatter from one quark with the correct parton momentum ξ, and the two spectator are just right such that a final state interaction A_w (w, Q^2) makes up a proton, or a resonance.

Elastic scattering W = M^p = M, single nucleon in final state.

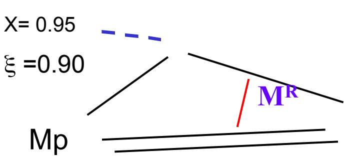
The scattering is from a quark with a very high value of ξ, is such that one cannot produce a single pion in the final state and the final state interaction makes a proton.



A_w **(w, Q^2) =** δ **(x-1) and the level is the {integral over** ξ, from pion threshold to ξ =1 } : **local duality**

(This is a check of local duality in the extreme, better to use measured Ge,Gm, Ga, Gv)

Note: in Neutrinos (axial form factor within 20% of vector form factor)



Resonance Production, W=M^R, e.g. delta 1.238 resonance. The scattering is from a quark with a high value of ξ, is such that that the final state interaction makes a low mass resonance. A_w **(w, Q^2) includes** Breit-Wigners. *Local duality*

Also a check of local duality for electrons and neutrinos

With the correct scaling variable, and if we account for low W and low Q2 higher twist effects, the prediction using QCD PDFs q (ξ, Q^2) should give an average of F2 in the elastic scattering and in the resonance region. (including both resonance and continuum contributions). If we **modulate** the PDFs with a final state interaction **resonance** A **(w, Q^2)** we could also reproduce the various Breit-Wigners + continuum.

Photo-production Limit $Q^2=0$
Non-Perturbative - QCD evolution freezes

- Photo-production Limit: Transverse Virtual and Real Photo-production cross sections must be equal at $Q^2=0$. Non-perturbative effect.
- There are no longitudinally polarized photons at $Q^2=0$
  - $\sigma_L (\nu, Q^2) = 0$ limit as $Q^2 \to 0$
  - Implies $R (\nu, Q^2) = \sigma_L / \sigma_T \sim Q^2 / [Q^2 + const] \to 0$ limit as $Q^2 \to 0$
  -
  - Real $\sigma(\gamma\text{-proton}, \nu) =$ virtual $\sigma_T (\nu, Q^2)$ limit as $Q^2 \to 0$
  - virtual $\sigma_T (\nu, Q^2) = 0.112$ mb $2xF_1 (\nu, Q^2) / (JQ^2)$ limit as $Q^2 \to 0$
  - virtual $\sigma_T (\nu, Q^2 = 0.112$ mb $F_2 (\nu, Q^2) D / (JQ^2)$ limit as $Q^2 \to 0$
  - or $F_2 (\nu, Q^2) \sim Q^2 / [Q^2 + C] \to 0$ limit as $Q^2 \to 0$
- Since $J= [1 - Q^2/ 2M\nu] =1$ and $D = (1+ Q^2/ \nu^2)/(1+R) =1$ at $Q^2=0$
- Therefore Real $\sigma(\gamma\text{-proton}, \nu) = 0.112$ mb $F_2 (\nu, Q^2) / Q^2$ limit as $Q^2 \to 0$
- If we want PDFs down to $Q^2=0$ and pQCD evolution freezes at $Q^2 = Q^2_{min}$
- Then $F_2 (\nu, Q^2) = F_{2\,QCD} (\nu, Q^2)\, Q^2 / [Q^2 + C]$
  and Real $\sigma(\gamma\text{-proton}, \nu) = 0.112$ mb $F_{2QCD} (\nu, Q^2 = Q^2_{min}) / C$
- The scaling variable x does not work since $\sigma(\gamma\text{-proton}, \nu) = \sigma_T (\nu, Q^2)$
  - At $Q^2 = 0$ $F_2 (\nu, Q^2) = F_2 (x, Q^2)$ with $x = Q^2 /(2M\nu)$ reduces to one point x=0
  - However, a scaling variable
  - $\xi_w = [Q^2 + B] / [M\nu (1+(1+Q^2/\nu^2))^{1/2} + A]$ works at $Q^2 = 0$
  - $F_2 (\nu, Q^2) = F_2 (\xi_c, Q^2) = F_2 [B/ (2M\nu), 0]$ limit as $Q^2 \to 0$

How do we "measure" higher twist (HT)

- Take a set of QCD PDF which were fit to high Q^2 (e/μ/ν) data (in Leading Order-LO, or NLO, or NNLO)
- Evolve to low Q2 (NNLO, NLO to Q^2=1 GeV^2) (LO to Q^2=0.24)
- Include the "known" kinematic higher twist from initial target mass (proton mass) and final heavy quark masses (e.g. charm production).
- Compare to low Q^2data in the DIS region (e.g. SLAC)
- The difference between data and QCD+target mass predictions is the extracted "effective" dynamic higher twists+Power Corrections.
- Describe the extracted "effective" dynamic higher twist within a specific HT Power Correction model (e.g. QCD renormalons, or a purely empirical model).
- Obviously - results will depend on the QCD order LO, NLO, NNLO (since in the 1 GeV region $1/Q^2$and $1/LnQ^2$ are similar). In lower orders, the "effective higher twist" will also account for missing QCD higher order terms. The question is the relative size of the terms.

  - **Studies in NLO - Yang and Bodek: Phys. Rev. Lett 82, 2467 (1999) ;ibid 84, 3456 (2000)**
  - **Studies in NNLO - Yang and Bodek: Eur. Phys. J. C13, 241 (2000)**
  - **Studies in LO - Bodek and Yang: hep-ex/0203009 and hep-ex 0210024**
  - **Studies in QPM 0th order - Bodek, el al PRD 20, 1471 (1979)**

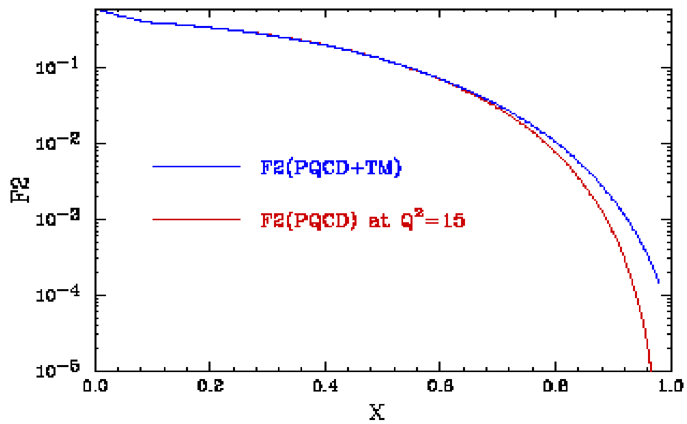
Lessons from Two 99,00 QCD studies

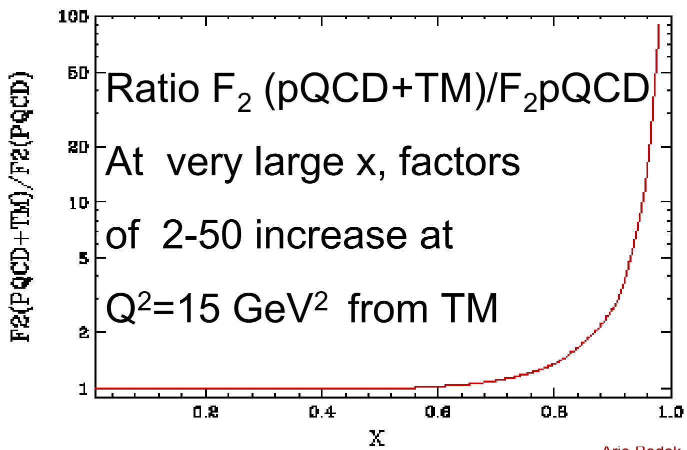
- Our NLO study comparing **NLO PDFs** to DIS SLAC, NMC, and BCDMS e/μ scattering data on H and D targets shows (for $Q^2 > 1$ GeV^2)
  [ref:Yang and Bodek: Phys. Rev. Lett 82, 2467 (1999)]
  - o *Kinematic Higher Twist* (**target mass**) effects are **large** and important at large x, and must be included in the form of Georgi & Politzer ξ_{TM} scaling.
  - o *Dynamic Higher Twist* -e.g. power correction effects are smaller, but need to be included. (A second NNLO study established their origin)
  - o The ratio of d/u at high x must be increased if nuclear binding effects in the deuteron are taken into account (not subject of this talk)
  - o *The Very high x (=0.9) region* - is described by **NLO QCD** (if target mass and renormalon higher twist effects are included) to better than 10%. SPECTATOR QUARKS modulate A(W,Q^2) ONLY.
  - o *Resonance region*: NLO pQCD + Target mass + Higher Twist describes average F_2 in the resonance region (duality works). ***Include A_w* (w, Q^2)** resonance modulating function from spectator quarks later.
- A similar NNLO study using **NNLO QCD** we find that the "empirically measured "effective" **Dynamic Higher Twist Effects/Power Corrections** in the NLO study come from the **missing NNLO higher order QCD terms**. [ref: Yang and Bodek Eur. Phys. J**. C13**, 241 (2000)]

Denominator: Kinematic Higher-Twist (target mass)

Georgi and Politzer Phys. Rev. D14, 1829 (1976):

$\xi_{TM} = [Q^2 / [M\nu (1+(1+Q^2/\nu^2))^{1/2}]$





$\xi_{TM+c} = \{ 2x / [1 + k] \} [1+ Mc^2 / Q^2]$

(last term only for heavy charm product)

$k = (1 + 4x^2 M^2 / Q^2)^{1/2}$ (target mass part)

(Derivation of ξ_{TM} in Appendix)

For Q^2 large (valence) $F_2 = 2\xi F_1 = \xi F_3$

$F_2^{pQCD+TM}(x,Q^2) = F_2^{pQCD}(\xi, Q^2)\, x^2 / [k^3\xi^2]$

$+J_{1*} (6M^2x^3 / [Q^2k^4]) + J_{2*}(12M^4x^4 / [Q^4k^5])$

$2F_1^{pQCD+TM}(x,Q^2) = 2F_1^{pQCD}(\xi, Q^2)\, x / [k\xi]$

$+J_{1*}(2M^2x^2 / [Q^2k^2]) + J_{2*}(4M^4x^4 / [Q^4k^5])$

$F_3^{pQCD+TM}(x,Q^2) = F_3^{pQCD}(\xi, Q^2)\, x / [k^2\xi]$

$+J_{1F3*}(4M^2x^2 / [Q^2k^3])$

For charm production replace x above

With -> x $[1+ Mc^2 / Q^2]$

$$J_1 = \int_{\xi}^{1} du\, F_2^{pQCD}(u,Q^2)\, /u^2$$

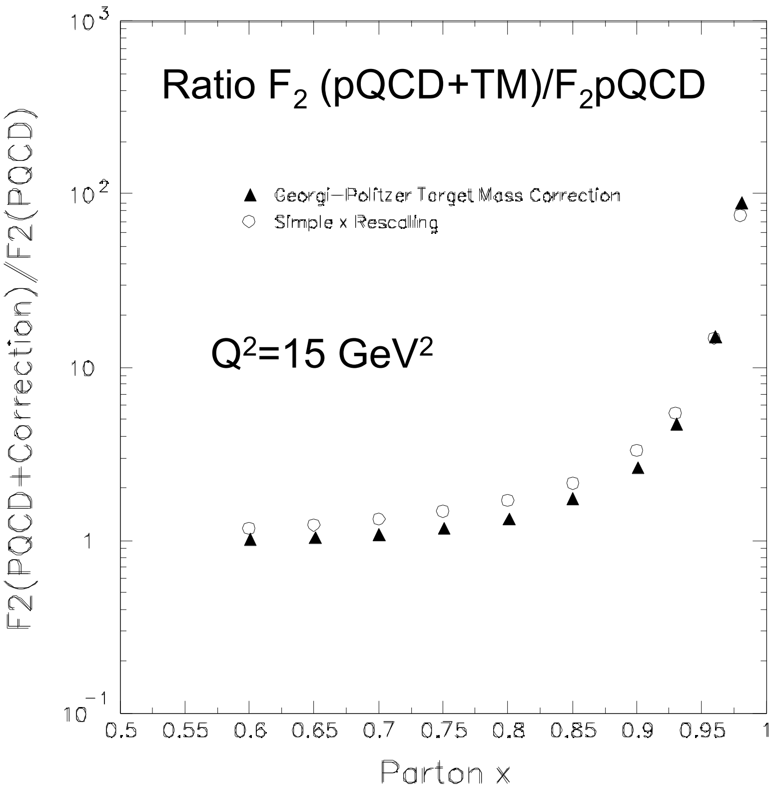
$$J_{1F3} = \int_{\xi}^{1} du\, F_3^{pQCD}(u,Q^2)\, /u$$

$$J_2 = \int_{\xi}^{1} du \int_{u}^{1} dV\, F_2^{pQCD}(V,Q^2)\, /V^2$$

Kinematic Higher-Twist (target mass:TM)

$\xi_{TM} = Q^2 / [M\nu (1 + (1 + Q^2/\nu^2)^{1/2})]$

Compare complete Target-Mass calculation to simple rescaling in ξ_{TM}



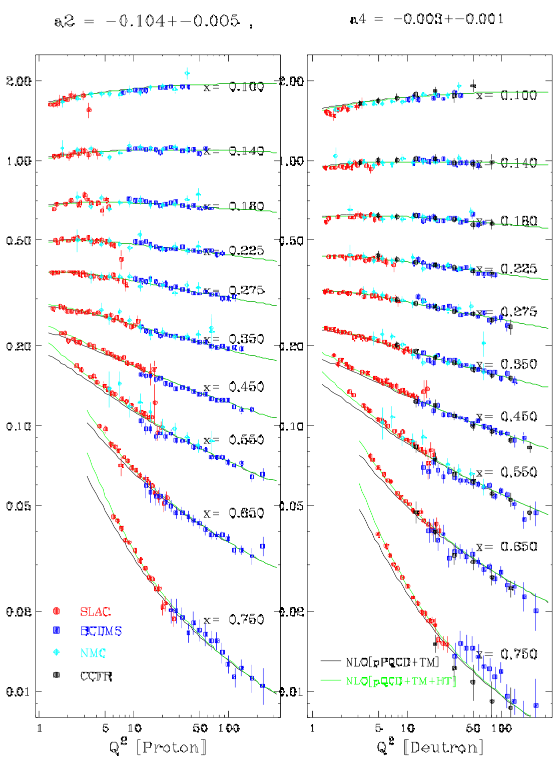
- The Target Mass Kinematic Higher Twist effects comes from the fact that the quarks are bound in the nucleon. They are important at low Q^2 and high x. They involve change in the scaling variable from x to ξ_{TM} and various kinematic factors and convolution integrals in terms of the PDFs for xF1, F_2 and xF_3
- Above x=0.9, this effect is mostly explained by a simple rescaling in ξ_{TM}.

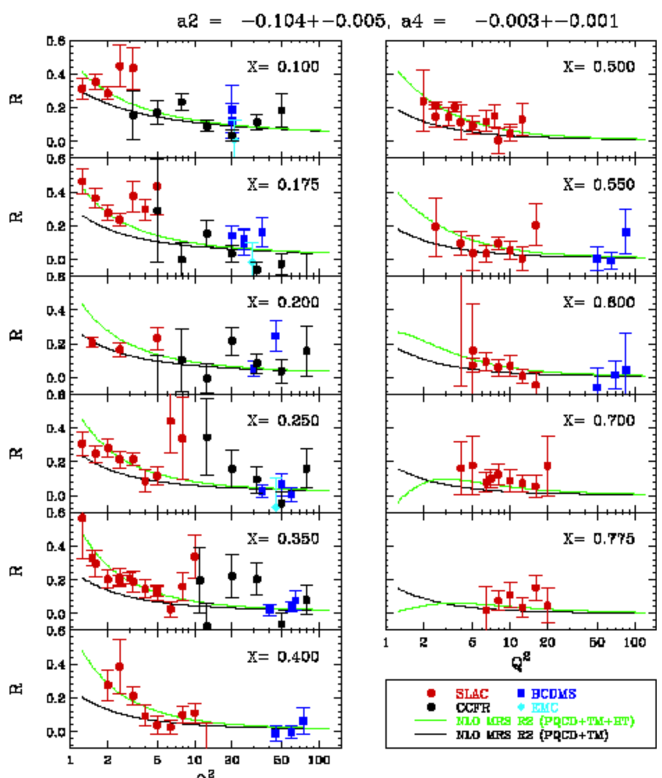
$F_2^{pQCD+TM}(x,Q^2) = F_2^{pQCD}(\xi_{TM},Q^2)$

F_2, R comparison of NLO QCD+TM $_{\text{black}}$ (Q^2>1)

vs. NLO QCD+TM+HT$_{\text{green}}$ (use QCD Renormalon Model for HT)

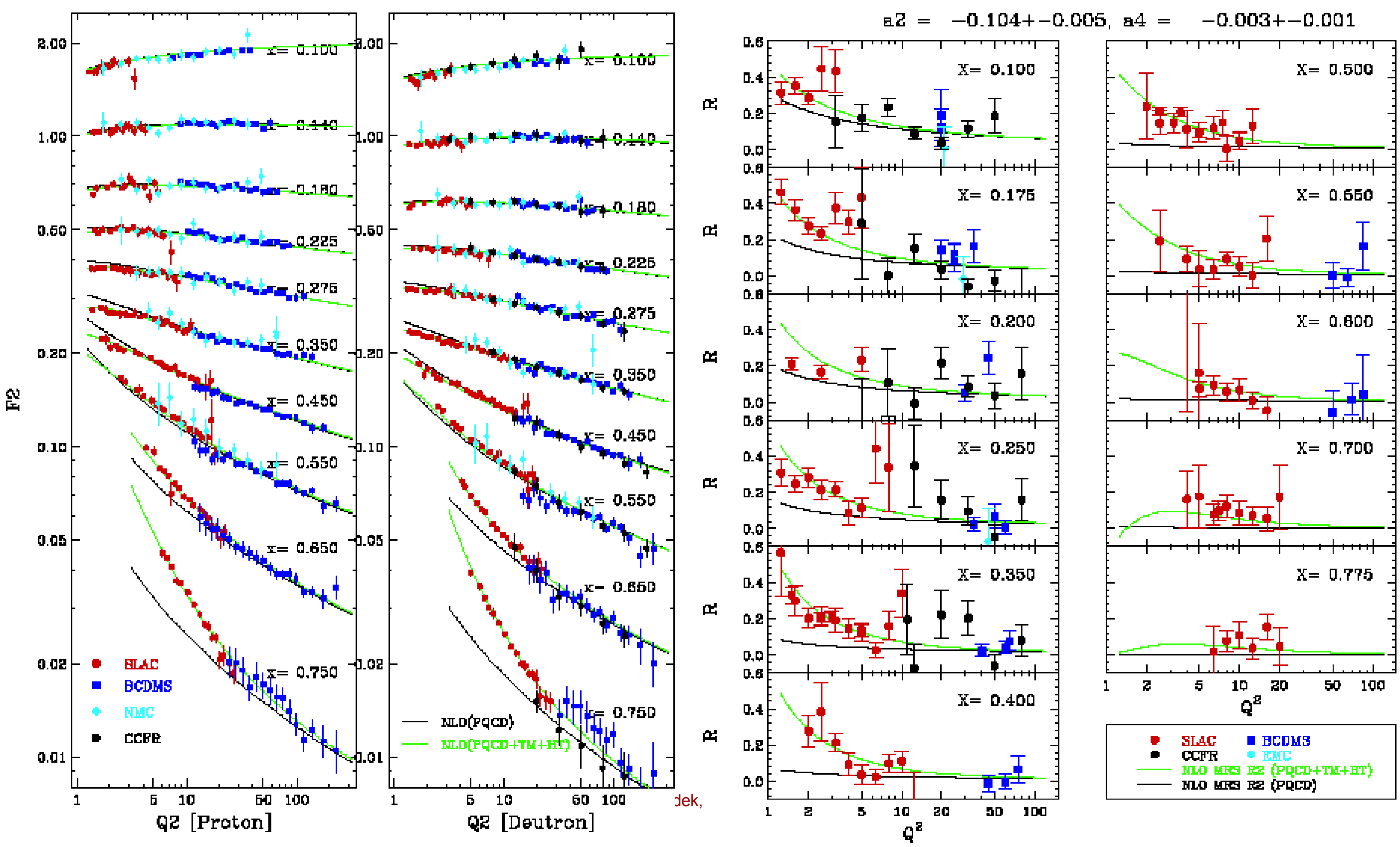
PDFs and QCD in NLO + TM + QCD Renormalon Model for Dynamic HTdescribe the F2 and R data very well, with only 2 parameters. Dynamic HT effects are there but small





Same study showing the NLO QCD-only black (Q^2>1) vs. NLO QCD+TM+HT green (use QCD Renormalon Model for HT)

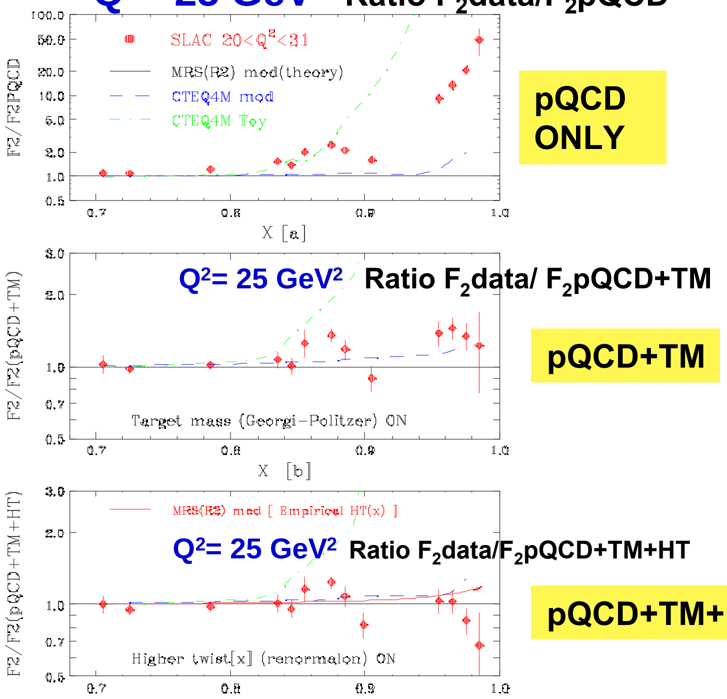
PDFs and QCD in NLO + TM + QCD Renormalon Model for Dynamic Higher Twist describe the F2 and R data reasonably well. TM Effects are LARGE



Very high x F2 proton data (DIS + resonance) (not included in the original fits Q^2=1. 5 to 25 GeV^2)

Q^2= 25 GeV^2 Ratio F_2data/F_2pQCD

F2 resonance Data versus F_2pQCD+TM+HT

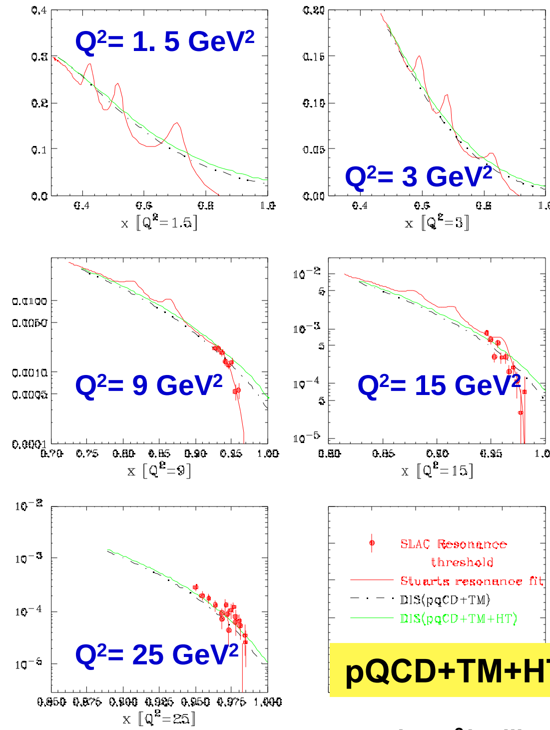


pQCD ONLY

pQCD+TM

pQCD+TM+HT

X =0.7 **X** =0.9



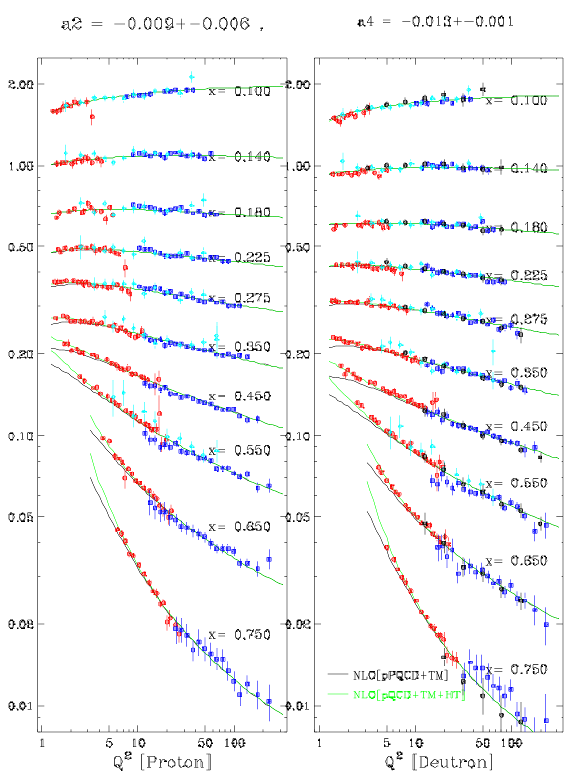
pQCD+TM+HT

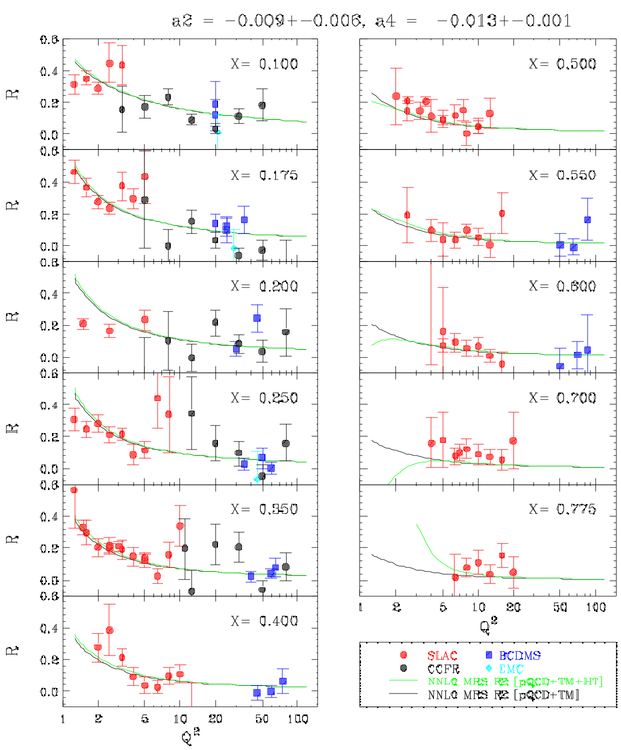
NLO pQCD + ξ TM + higher twist describes very high x DIS F_2 and resonance F_2 data well. (duality works) Q^2=1. 5 to 25 GeV^2

A_w (w, Q^2) will account for interactions with spectator quarks

F_2, R comparison with NNLO QCD+TM black => NLO HT are missing NNLO terms (Q^2>1)

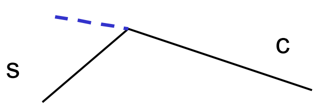
Size of the higher twist effect with NNLO analysis is really small (but not 0)
a2= -0.009 (in NNLO) versus –0.1(in NLO) - > factor of 10 smaller, a4 nonzero





"B= M* term" At LOW x, Q^2 "NNLO terms" look similar to "kinematic final state mass higher twist" or "effective final state quark mass -> "enhanced" QCD

Charm production s to c quarks in neutrino scattering-slow rescaling



Mc (final state quark mass

$(P_i + q)^2 = P_i^2 + 2q.P_i + q^2 = P_f^2 = M_c^2$

$2\xi_C\, q.P = Q^2 + M_c^2 \quad (Q^2 = -q^2)$

$2\xi_C\, M\nu = Q^2 + M_c^2$ ξ_C **– slow re-scaling**

$\xi_C = [Q^2+M_c^2] / [2M\nu]$ final state charm mass

At low Q2, the final state u and d quark effective mass is not zero

Production of pions etc
gluon emission from the Interacting quark

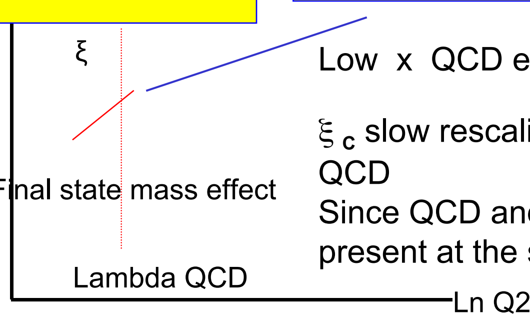
$(P_i + q)^2 = P_i^2 + 2q.P_i + q^2 = P_f^2 = M^{*2}$

$\xi_C = [Q^2+M^{*2}] / [2M\nu]$ (final state M* mass))

versus for mass-less quarks $2x\, q.P= Q^2$

$x = [Q^2] / [2M\nu]$ (M* =0 Bjorken x]

At Low x, low Q^2
$\xi_C > x$ (slow rescaling ξ_C)
(and the PDF is smaller at high x, so the low Q^2 cross section is suppressed - threshold effect.



Low x QCD evolution

ξ_C slow rescaling looks like faster evolving QCD
Since QCD and slow rescaling are both present at the same Q2

Modified LO PDFs for all Q^2 (including 0)

New Scaling Variable

1. Start with GRV98 LO (Q^2_{min}=0.8 GeV^2)
   - describe F2 data at high Q^2
2. Replace $X_{BJ} = Q^2 / (2M\nu)$

with a new scaling, ξ w

$\xi_w = [Q^2+M_F^2 + B] / [M\nu (1+(1+Q^2/\nu^2)^{1/2}) + A]$

A: initial binding/target mass effect plus NLO +NNLO terms)

B: final state mass effect (but also photo production limit)

M_F =0 for non-charm production processes

M_F =1.5 GeV for charm production processes

3. Do a fit to SLAC/NMC/BCDMS/HERA94

H, D data.- Allow the normalization of the experiments and the BCDMS major systematic error to float within errors.

A. INCLUDE DATA WITH Q2<1 if it is not in the resonance region. Do not include any resonance region data.

Photoproduction threshold

Multiply all PDFs by a factors Kvalence and Ksea for photo prod. Limit +non-perturbative effects at all Q2.

$F_2(x, Q^2) = K * F_{2QCD}(\xi_w, Q^2) * A(w, Q^2)$

Freeze the evolution at Q^2 = 0.8 GeV^2

$F_2(x, Q^2 < 0.8) = K * F_2(\xi_w, Q^2=0.8)$

For sea Quarks

K = Ksea $= Q^2 / [Q^2 + Csea]$ at all Q^2

For valence quarks (from Adler sum rule)

K = Kvalence

$= [1- G_D^2 (Q^2)] [Q^2+C2V] / [Q^2+C1V]$

$G_D^2 (Q^2) = 1/ [1+Q^2 / 0.71]^4$

= elastic nucleon dipole form factor squared

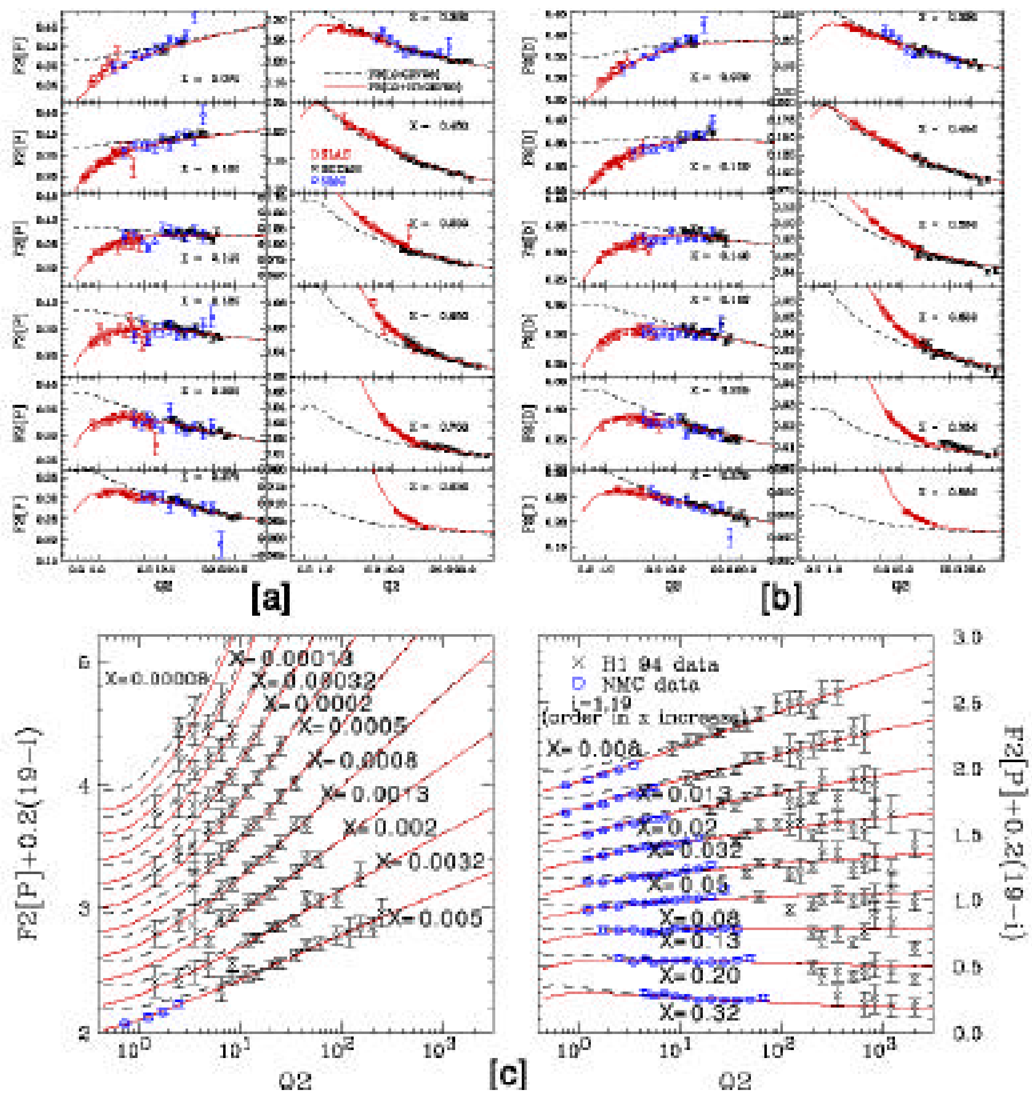
Above equivalent at low Q^2

K = Ksea $> Q^2 / [Q^2+Cvalence]$ as $Q^2 \to 0$

Resonance modulating factor

$A(w, Q^2) = 1$ for now

[Ref:Bodek and Yang [hep-ex 0210024]]



χ^2 = 1268 / 1200 DOF

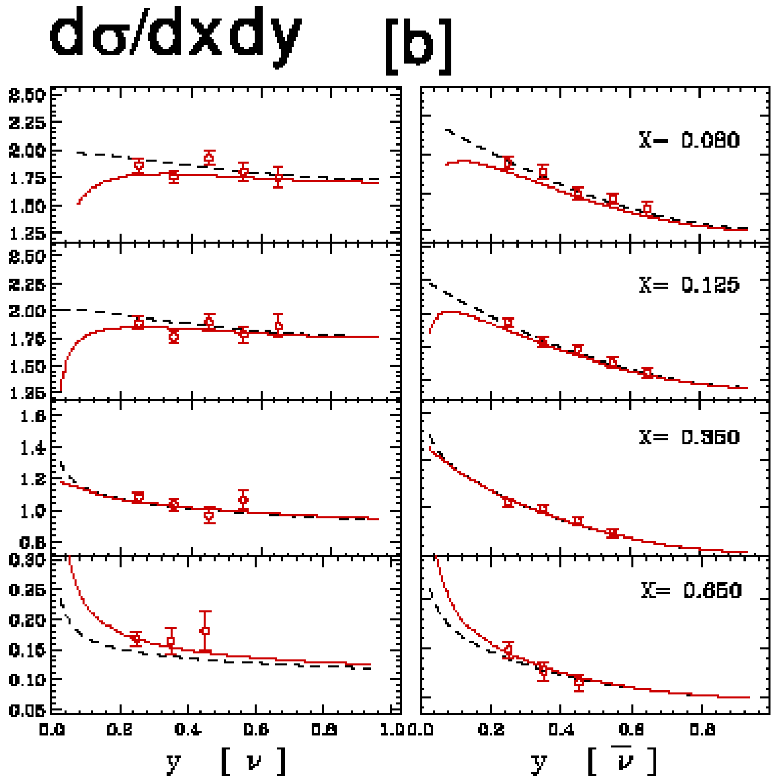
Dashed=GRV98LO QCD

$F_2 = F_{2QCD}(x,Q^2)$

Solid=modified GRV98LO QCD

$F_2 = K(Q^2) * F_{2QCD}(\xi_w, Q^2)$

Comparison of LO+HT to neutrino data on Iron [CCFR] (not used in this ξ_w fit)



Construction

- **Apply nuclear corrections using e/μ scattering data.**
- **(Next slide)**
- **Calculate F_2 and xF_3 from the modified PDFs with ξ_w**
- **Use R=Rworld fit to get $2xF_1$ from F_2**
- **Implement charm mass effect through ξ_w slow rescaling algorithm, for F_2 $2xF_1$, and XF_3**

—— ξ_w PDFs GRV98 modified

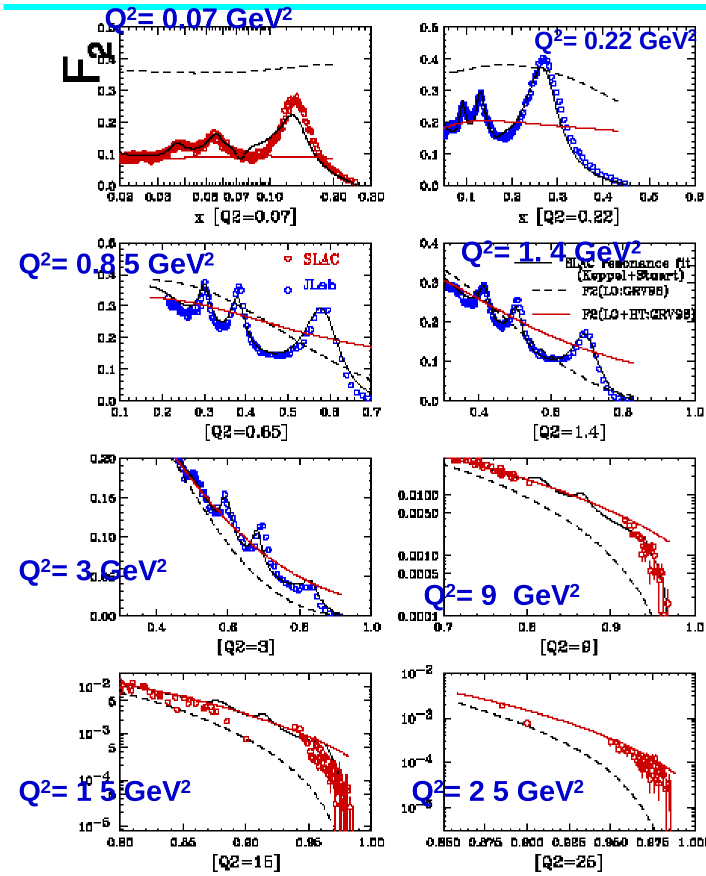
---- GRV98 (x,Q^2) unmodified

Left neutrino, Right antineutrino

The modified GRV98 LO PDFs with a new scaling variable, ξ_w describe the CCFR diff. cross section data (Eν=30–300 GeV) well. Eν= 55 GeV is shown

Comparison with F2 resonance data

[SLAC/ Jlab] (These data were not included in this ξw fit)



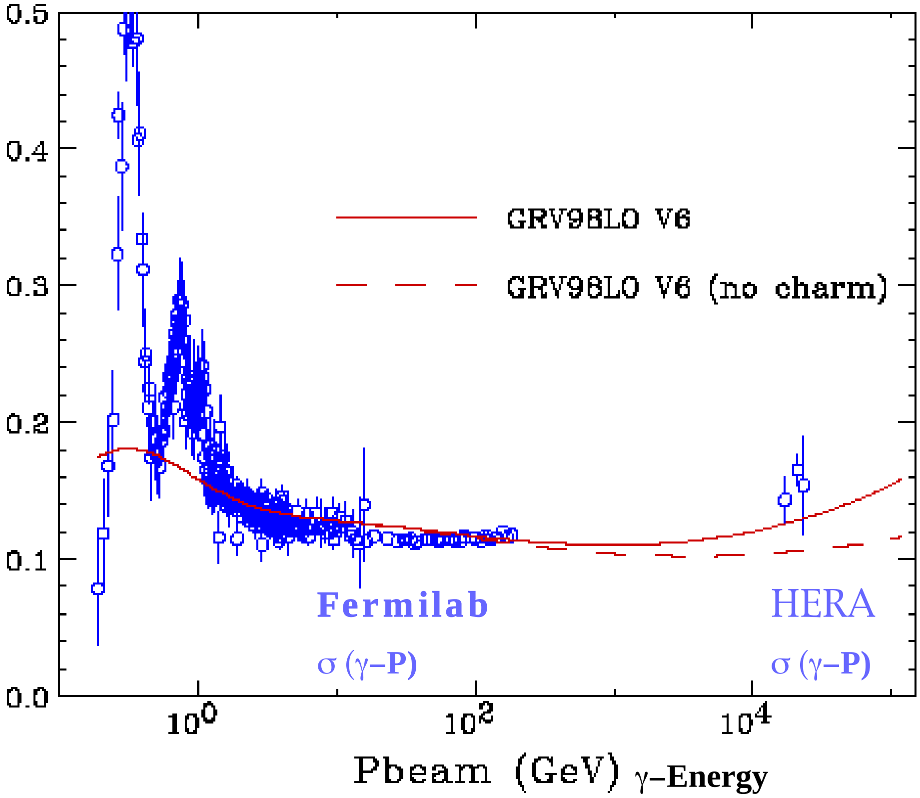
- ξw fit
- The modified LO GRV98 PDFs with a new scaling variable, ξw describe the SLAC/Jlab resonance data very well (on average).
  - Even down to $Q^2 = 0.07$ GeV^2
  - Duality works: The DIS curve describes the average over resonance region (for the First resonance works for $Q^2 > 0.8$ GeV^2)

❖ **These data and photo-production data and neutrino data can be used to get A(W,Q2).**

Comparison with photo production data

mb **(not included in this ξ w fit)** ***SLOPE of F2(Q2=0)***

- $\sigma(\gamma\text{-}P) = 0.112$ mb { $F_2(x, Q^2 = 0.8)_{valence}$ /Cvalence + $F_2(x, Q^2 = 0.8)_{sea}$/ Csea }
- = 0.112 mb { $F_2(x, Q^2 = 0.8)_{valence}$ /0.221 + $F_2(x, Q^2 = 0.8)_{sea}$/ 0.381}



The charm Sea=0 in GRV98.

Dashed line, no Charm production.

Solid line add Charm cross section above Q2=0.8 to DIS from Photon-Gluon Fusion calculation

Modified LO PDFs for all Q^2 (including 0)

Results for Scaling variable — FIT results for K photo-production threshold

$\xi_w = [Q^2+B] / [M\nu (1+(1+Q^2/\nu^2)^{1/2}) + A]$

- A=0.418 GeV^2, B=0.222 GeV^2 (from fit)
- A=initial binding/target mass effect plus NLO +NNLO terms)
- B= final state mass Δm^2 from gluons plus initial Pt.
- Very good fit with modified GRV98LO
- χ^2 = 1268 / 1200 DOF
- Next: Compare to Prediction for data not included in the fit

1. Compare with SLAC/Jlab resonance data (not used in our fit) -> $A(w, Q^2)$
2. Compare with photo production data (not used in our fit)-> check on K production threshold
3. Compare with medium energy neutrino data (not used in our fit)- except to the extent that GRV98LO originally included very high energy data on xF_3

$F_2(x, Q^2) = K * F_{2QCD}(\xi_w, Q^2) * A(w, Q^2)$

$F_2(x, Q^2 < 0.8) = K * F_2(\xi_w, Q^2=0.8)$

For sea Quarks we use

$K = Ksea = Q^2 / [Q^2+Csea]$

Csea = 0.381 GeV^2 (from fit)

For valence quarks (in order to satisfy the Adler Sum rule which is exact down to Q2=0) we use

K = Kvalence

$= [1- G_D^{\ 2}(Q^2)]\ [Q^2+C2V] / [Q^2+C1V]$

$G_D^{\ 2}(Q^2) = 1/ [1+Q^2 / 0.71]^4$

= elastic nucleon dipole form factor squared. we get from the fit

C1V = 0.604 GeV^2, C2V = 0.485 GeV^2

Which Near Q^2 =0 is equivalent to:

$Kvalence \sim Q^2 / [Q^2+Cvalence]$

With Cvalence=(0.71/4)*C1V/C2V= =0.221 GeV^2

[Ref:Bodek and Yang hep-ex/0203009]

Origin of low Q2 K factor for Valence Quarks

Adler Sum rule EXACT all the way down to $Q^2=0$ includes W_2 quasi-elastic

β- = W_2 (Anti-neutrino -Proton)
β+ = W_2 (Neutrino-Proton) **$q0=\nu$**

$$g_A(q^2)+\int_{M_\pi+(q^2+M_\pi^2)/2M_N}^{\infty} dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(8)}(q_0,q^2)]=1,$$

AXIAL Vector part of W_2

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)]=1. \quad (18)$$

If we explicitly separate out the nucleon Born term in Eq. (18), we have

$$[F_1^V(q^2)]^2+q^2\left(\frac{\mu^V}{2M_N}\right)^2[F_2^V(q^2)]^2$$
$$+\int_{M_\pi+(q^2+M_\pi^2)/2M_N}^{\infty} dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(8)}(q_0,q^2)]=1,$$

Vector Part of W2

Adler is a number sum rule at high Q^2

$$\int_0^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)] = 1 \text{ is}$$

$$\int_0^1 \frac{[F_2^-(\xi)-F_2^+(\xi)]}{\xi}d\xi = \int_0^1 [U_v(\xi)-D_v(\xi)]d\xi = 2-1$$

F_2^- = F_2 (Anti-neutrino -Proton) = νW_2

F_2^+ = F_2 (Neutrino-Proton) = νW_2

we use: d (q0) = d (ν) = (ν)d ξ / ξ

[see Bodek and Yang hep-ex/0203009] and references therein

at fixed $\mathbf{q^2 = Q^2}$

Valence Quarks Fixed $q^2=Q^2$

Adler Sum rule EXACT all the way down to $Q^2=0$ includes W_2 quasi-elastic

1= Quasielastic δ -function $(F^-_2 - F^+_2)d\xi/\xi$ Integral Separated out + Integral of Inelastic $(F^-_2 - F^+_2)d\xi/\xi$ both resonances and DIS

$$g_V(q^2) = [F_1^V(q^2)]^2 + q^2\left(\frac{\mu^V}{2M_N}\right)^2 [F_2^V(q^2)]^2$$

For Vector Part of Uv-Dv the Form below F will satisfy the Adler Number Sum rule

$$\frac{\int_0^{\xi\ pionthreshold} [\xi U_v^{QCD}(\xi_W) - \xi U_v^{QCD}(\xi_W)][1 - g_V(Q^2)] d\xi_W/\xi_W}{N(Q^2) = \int_0^{\xi\ pionthreshold} [\xi_W U_v^{QCD}(\xi_W) - \xi U_v^{QCD}(\xi_W)] d\xi_W/\xi_W} + g_V(Q^2) = 1$$

If we assume the same form for Uv and Dv --->

$$F_2^{VALENCE}(\xi_W, Q^2) = \frac{\xi V^{QCD}(\xi_W, Q^2)][1 - g_V(Q^2)]}{N(Q^2)}$$

[Ref:Bodek and Yang hep-ex/0203009]

Valence Quarks

Adler Sum rule EXACT all the way down to $Q^2=0$ includes W_2 quasi-elastic

$$F_2^{VALENCE\ Vector}(\xi_W, Q^2) = \frac{\xi_W V^{QCD}(\xi_W, Q^2)][1 - g_V(Q^2)]}{N(Q^2)}$$

This form Satisfies Adler Number sum Rule at all fixed Q^2

$$\int_0^1 \frac{[F_2^-(\xi,Q^2) - F_2^+(\xi,Q^2)]}{\xi} d\xi = \int_0^1 [U_v(\xi) - D_v(\xi)] d\xi = 1\ exact$$

$F_2^- = F_2$ (Anti-neutrino -Proton)
$F_2^+ = F_2$ (Neutrino-Proton

$$\int_0^1 [F_2^{Valence}(\xi,Q^2) + F_2^{sea}(\xi,Q^2) + xg(\xi,Q^2)] d\xi \approx 1$$

While momentum sum Rule has QCD and Non Pertu. corrections

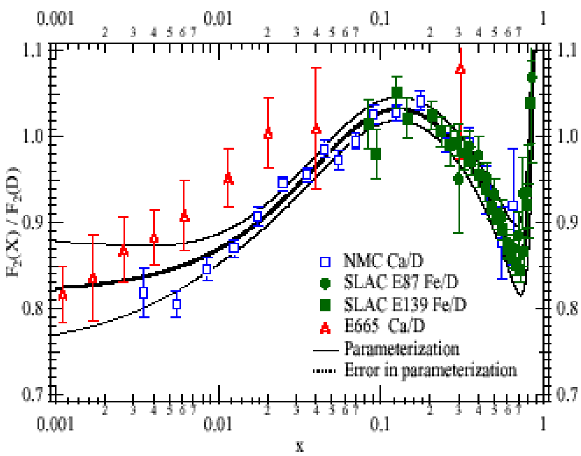
- **Use : K = Kvalence = $[1- G_D{}^2 (Q^2)]$ $[Q^2$+C2V] / $[Q^2$+C1V]**
- **Where C2V and C1V in the fit to account for both electric and magnetic terms**
- **And also account for $N(Q^2)$ which should go to 1 at high Q^2.**
- **This a form is consistent with the above expression (but is not exact since it assumes no dependence on ξ_w or W (assumes same form for resonance and DIS)**
- **Here: $G_D{}^2 (Q^2) = 1/ [1+Q^2 / 0.71]^4$ = elastic nucleon dipole form factor**

[Ref:Bodek and Yang hep-ex/0203009]

Summary

- Our modified GRV98LO PDFs with a modified scaling variable ξw and K factor for low Q2 describe all SLAC/BCDMS/NMC/HERA DIS data.
- The modified PDFs also yields the average value over the resonance region as expected from duality argument, ALL THE WAY TO $Q^2 = 0$
- Our Photo-production prediction agrees with data at all energies.
- Our prediction in good agreement with high energy neutrino data.
- Therefore, this model should also describe a low energy neutrino cross sections reasonably well -
- USE this model ONLY for W above Quasielastic and First resonance. , Quasielastic is isospin 1/2 and First resonance is both isospin 1/2 and 3/2. Best to get neutrino vector form factors from electron scattering (via Clebsch Gordon coefficients) and add axial form factors from neutrino measurments.
- We will compare to available low enegy neutrino data, Adler sum rule etc.
- This work is continuing… focus on further improvement to ξw (although very good already) and *Ai,j,k* (W, Q^2) (low W + spectator quark modulating function).
- What are the further improvement in ξ w - more theoretically motivated terms are added into the formalism (mostly intellectual curiosity, since the model is already good enough). E.g. Add Pt^2 from Drell Yan data.
- New proposed experiments at Fermilab/JHF to better measure low energy neutrino cross sections in off-axis beams. For Rochester NUMI proposal see
- http://www.pas.rochester.edu/~ksmcf/eoi.pdf

Correct for Nuclear Effects measured in e/μ expt.



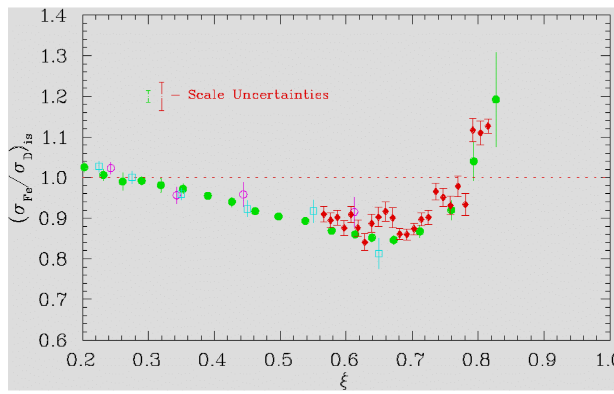


Figure 5. The ratio of F_2 data for heavy nuclear targets and deuterium as measured in charged lepton scattering experiments(SLAC,NMC, E665). The band show the uncertainty of the parametrized curve from the statistical and systematic errors in the experimental data [16].

Comparison of Fe/D F2 data

In resonance region (JLAB)

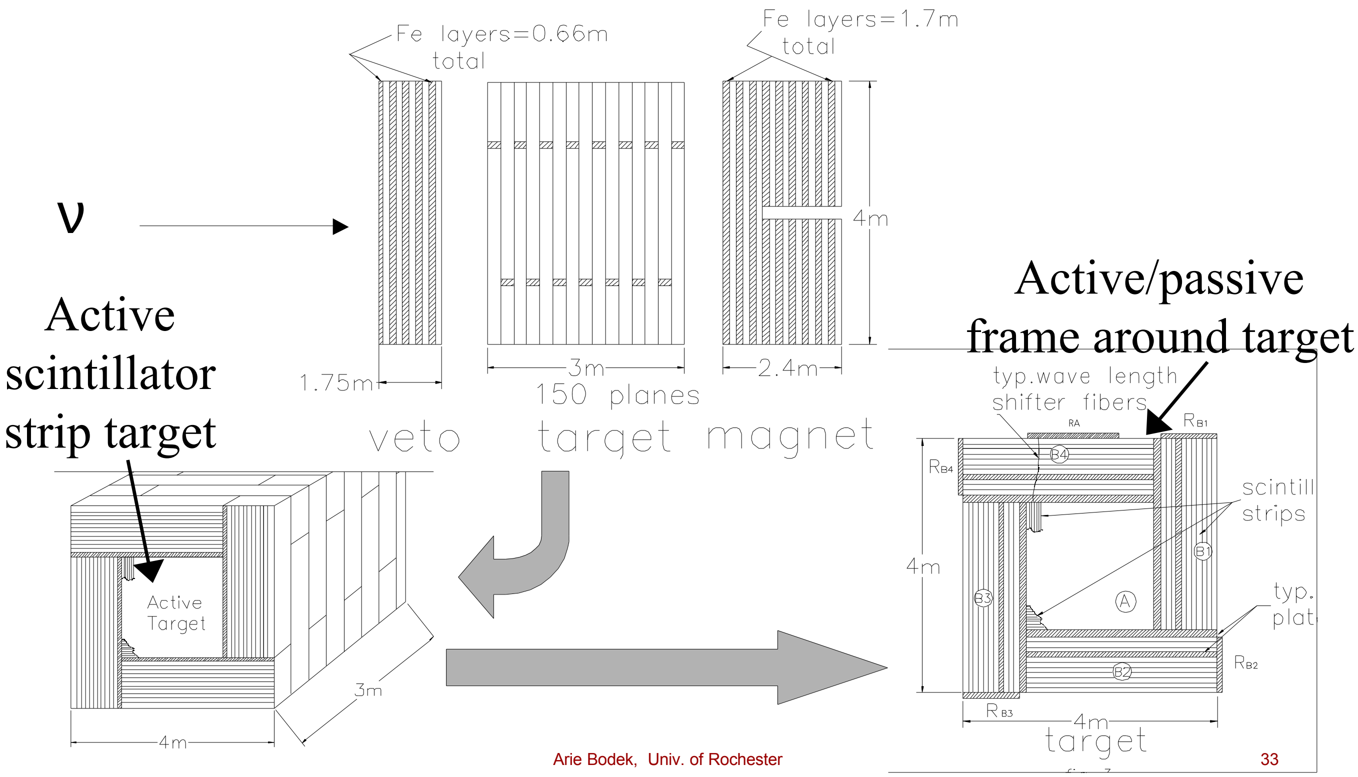
Versus DIS SLAC/NMC data

In ξ_{TM} (C. Keppel 2002).

Fully-Active Off-Axis Near Detector (Conceptual)
Rochester - NUMI EOI

http://www.pas.rochester.edu/~ksmcf/eoi.pdf

(Kevin McFarland - Spokesperson)

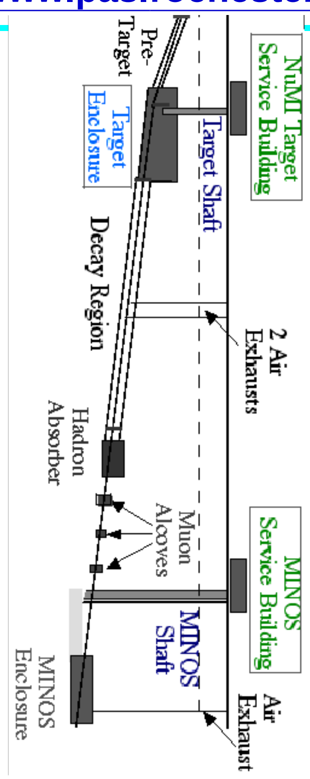


Rochester NUMI Off-Axis Near Detector

http://www.pas.rochester.edu/~ksmcf/eoi.pdf

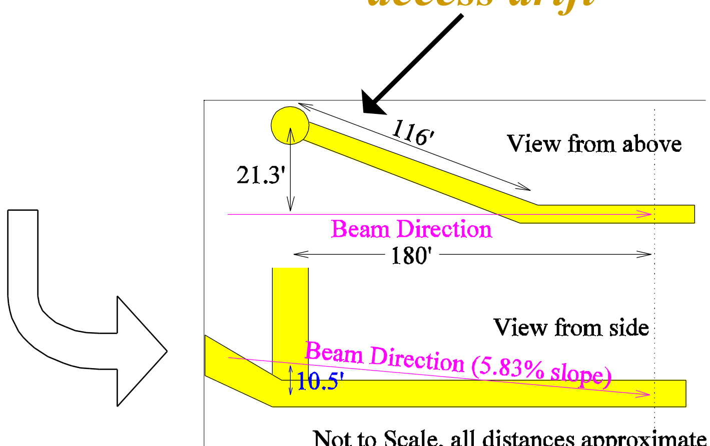
Rochester EOI to FNAL program Committee

(Collaboration to expand to include Jlab Hampton and others)



- **Narrow band beam, similar to far detector**
  - **Can study cross-sections (NBB)**
  - **Near/far for $\nu_\mu \to \nu_\mu$;**
  - **backgrounds for $\nu_\mu \to \nu_e$**

Locate off of access drift



Some of this QCD/PDF work has been published in

- HIGHER TWIST, ξ_w **scaling**, AND EFFECTIVE LO PDFS FOR LEPTON SCATTERING IN THE FEW GEV REGION. **[hep-ex 0210024] - A Bodek, U K Yang**, to be published in J. Phys. G Proc of NuFact 02-London (July 2002) - THIS TALK

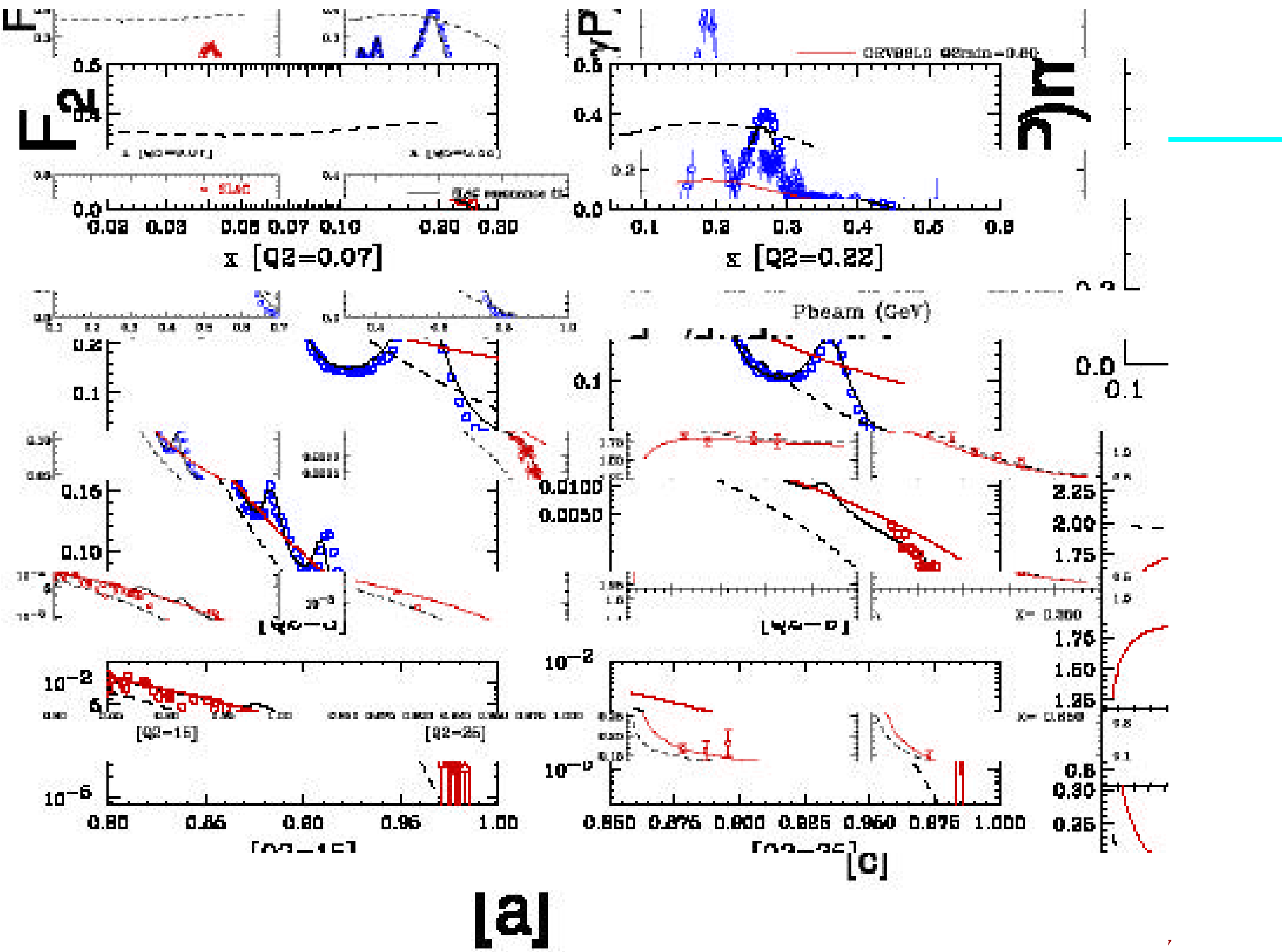
Based on Earlier work on origin of higher twist effects

1. **Studies in QCD NLO+TM+ renormalon HT - Yang, Bodek** Phys. Rev. Lett 82, 2467 (1999)
2. **Studies in QCD NNLO+TM+ renormalon HT - Yang, Bodek:** Eur. Phys. J. C13, 241 (2000)

and Earlier PDF Studies with Scaling Variable X_w

1. **0th ORDER PDF (QPM + X_w scaling) studies - A. Bodek, et al** PRD 20, 1471 (1979) **+ earlier papers in the 1970's.**
2. **LO + Modified PDFs (X_w scaling) studies -**
   Bodek, Yang: hep-ex/0203009 ***(NuInt01 Conference)***
   Nucl.Phys.Proc.Suppl.112:70-76,2002

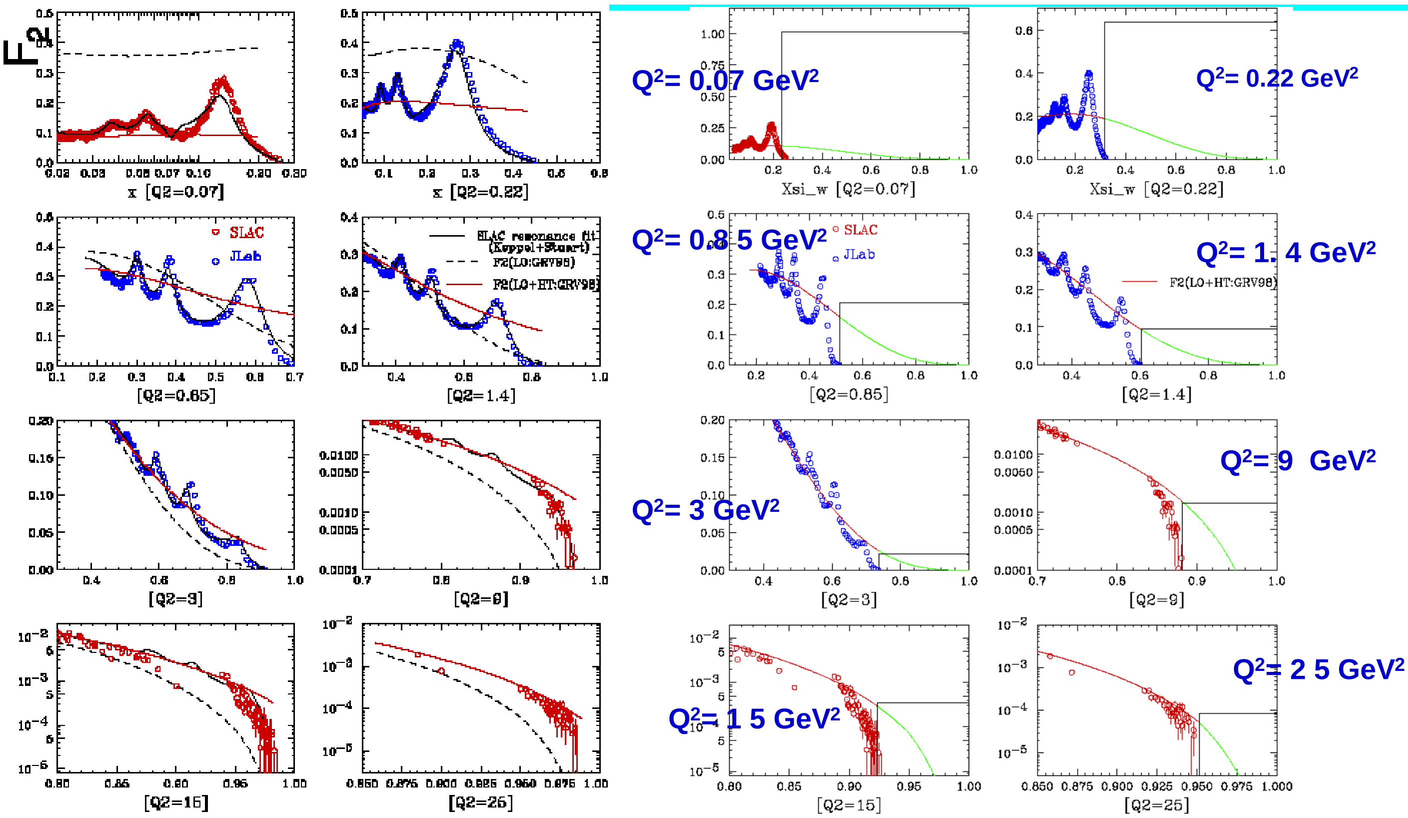
Backup Slides



F2
x [Q2=0.07]
x [Q2=0.22]
GRV98LO Q2min=0.80
SLAC
Pbeam (GeV)
[Q2=15]
[Q2=25]
[c]
[a]

GRV98 Comparison with F2 resonance data

[SLAC/ Jlab] (These data were not included in this ξw fit)

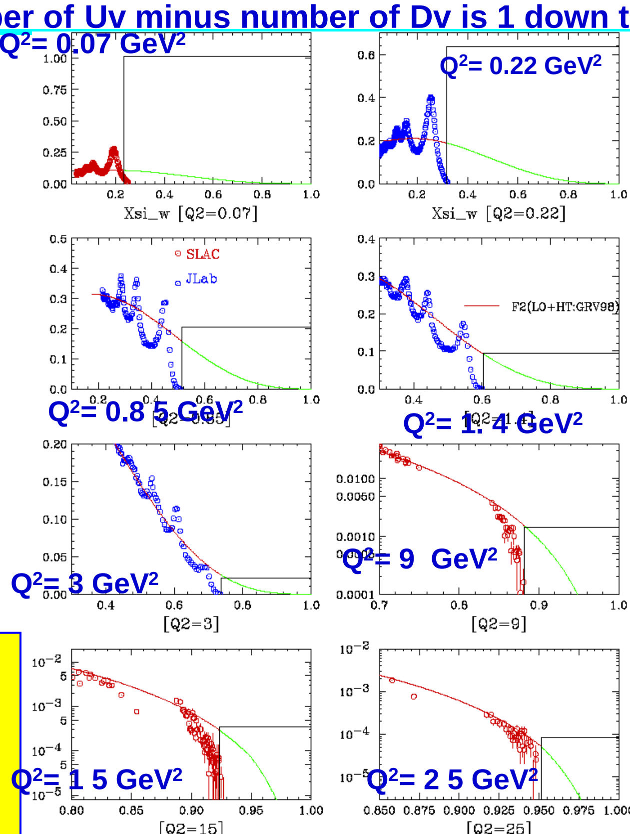


- The modified LO GRV98 PDFs with a new scaling variable, ξw describe the SLAC/Jlab resonance data very well (on average). Local duality breaks down at x=1 (elastic scattering) and Q2<0.8 in order to satisfy the Adler Sum rule).I.e. Number of Uv-Dv Valence quarks = 1.

When does duality break down

Momentum Sum Rule has QCD+non- Perturbative Corrections (breaks down at Q2=0) but ADLER sum rule is EXACT (number of Uv minus number of Dv is 1 down to Q2=0).

Int F2P	Q2
Elastic peak	
1.0000000	0
0.7775128	0.07
0.4340529	0.25
0.0996406	0.85
0.0376200	1.4
0.0055372	3
0.0001683	9
0.0000271	15
0.0000040	25
DIS high Q2 Integral F2p 0.17	

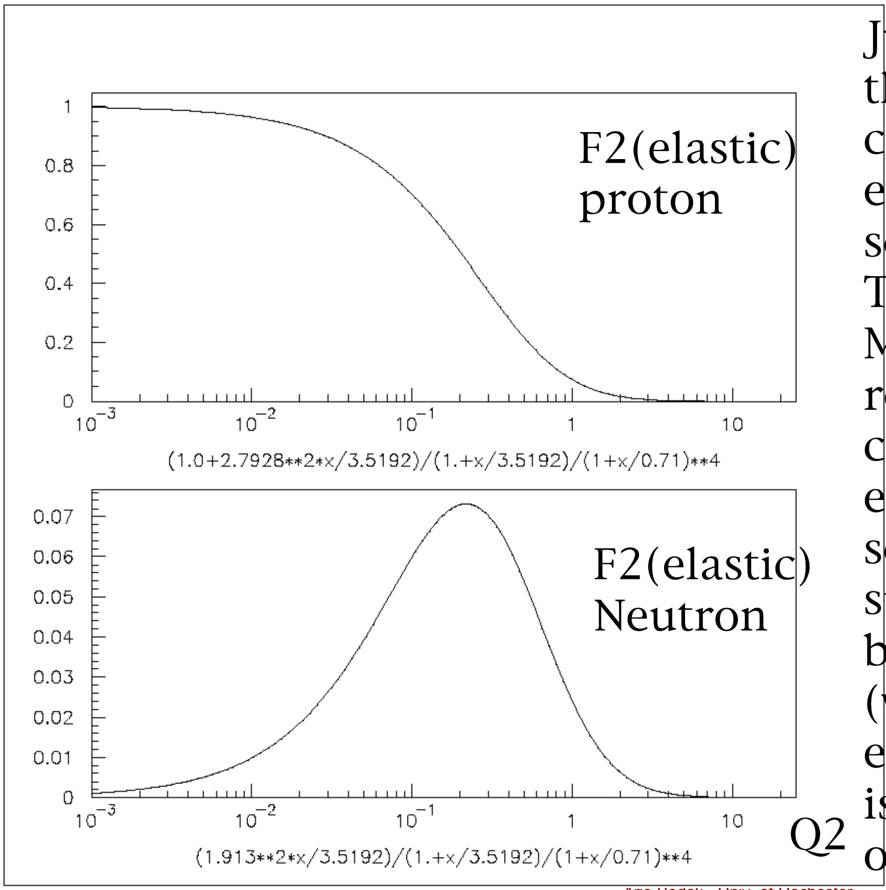


- **In proton :**
- **QPM Integral of F2p** =
- 0.17*(1/3)^2+0.34*(2/3)^2 = 0.17 (In neutron=0.11)
- Where we use the fact that
- 50% carried by gluon
- 34% u and 17% d quarks

Adler sum rule (valid to Q2=0) is the integral Of the difference of F2/x for Antineutrinos and Neutrinos on protons (including elastic)

Note that in electron scattering the quark charges remain

But at Q2=0, the neutron elastic form factor is zero)



Just like in p-p scattering there is a strong connection between elastic and inelastic scattering (Optical Theorem). Quantum Mechanics (Closure) requires a strong connection between elastic and inelastic scattering. Momentum sum rule breaks down, but the Adler sum rule (which includes the elastic part, is exact and is equal to the NUMBER of Uv-Dv = 1. (F2(x)/x)

Revenge of the Spectator Quarks

Stein et al PRD 12, 1884 (1975)-1

$$\nu W_{2p}(q^2, \nu) = [1 - W_2^{el}(q^2)] F_{2p}(\omega') , \quad (13)$$

where $F_{2p}(\omega')$ is the scaling limit structure function and

$$W_2^{el}(q^2) = \frac{G_E{}^2(q^2) + \tau G_M{}^2(q^2)}{1+\tau} , \quad \tau = \frac{q^2}{4M^2} \quad (14)$$

is the counterpart of W_2 for elastic scattering (see Appendix B), where G_E and G_M are, respectively, the elastic electric and magnetic form factors for the proton. This form satisfies the constraint that W_2 vanish at $q^2 = 0$. Integrating W_{2p} over all values of ν yields

$$\int_{\text{inelastic}} d\nu\, W_{2p}(q^2, \nu) = [1 - W_2^{el}(q^2)] \int_{\text{inelastic}} \frac{d\nu}{\nu} F_{2p}(\omega') . \quad (15)$$

But this is the Gottfried sum rule[27] for the proton,

where

$$\int_{\text{inelastic}} \frac{d\nu}{\nu} F_{2p}(\omega') = \sum_i q_i{}^2 \quad (16)$$

is the sum of the parton charges squared.

2. Application

We can now apply these results to the proton and neutron if we consider them as being made of constituents. These yield immediately

$$\int_{\text{inel}} d\nu\, W_{2p}(q^2, \nu) = \left(\sum_{i=1}^{N} e_i{}^2 \right)_p [1 - |F_{el}^p(q^2)|^2] + C_p(q^2) \left(\sum_{i \neq j}^{N} \sum e_i e_j \right)_p , \quad (B15)$$

$$\int_{\text{inel}} d\nu\, W_{2n}(q^2, \nu) = \left(\sum_{i=1}^{N} e_i{}^2 \right)_n [1 - |F_{el}^N(q^2)|^2] + C_n(q^2) \left(\sum_{i \neq j}^{N} \sum e_i e_j \right)_n . \quad (B16)$$

F_{el}^p and F_{el}^n would be equal if the momentum distributions of the constituents were the same in the proton and neutron, so if the correlation terms were negligible, one might expect W_{2n}/W_{2p} to scale to lower values of q^2 than either W_{2p} or W_{2n} alone. Gottfried noted that in the simple quark model the charge sum in the correlation contribution vanishes for the proton, but not for the neutron.[27]

For the case of particles with spin, magnetic moments, and more realistic ground states, the results get much more complicated. There are several more detailed accounts in the case of nuclear scattering in the literature.[41] However, the simple approach stated here agrees with the spirit of the more complex analyses.

Revenge of the Spectator Quarks

Stein etal PRD 12, 1884 (1975)-2

[41]For more detailed treatment of closure, see, for example O. Kofoed-Hanson and C. Wilkin, Ann. Phys. (N.Y.) 63, 309 (1971); K. W. McVoy and L. Van Hove, Phys. Rev. 125, 1034 (1962).

[27]K. Gottfried, Phys. Rev. Lett. 18, 1174 (1967).

$$G_{el}(q^2) = \left|\sum_{i=1}^{N} e_i\right|^2 |F_{el}(q^2)|^2 ,$$

$$G_{inel}(q^2) = \sum_{i=1}^{N} e_i^2[1 - |F_{el}(q^2)|^2] + C(q^2)\sum_{i\neq j}^{N}\sum e_i e_j . \qquad \text{(B14)}$$

$$\nu W_{2p}(q^2, \nu) = [1 - W_2^{el}(q^2)] F_{2p}(\omega') , \qquad (13)$$

where $F_{2p}(\omega')$ is the scaling limit structure function and

$$W_2^{el}(q^2) = \frac{G_E^2(q^2) + \tau G_M^2(q^2)}{1+\tau} , \quad \tau = \frac{q^2}{4M^2} \qquad (14)$$

$$G_{Ep} = P(q^2)/(1+q^2/0.71)^2 ,$$

P is close to 1 and gives deviations

From Dipole form factor (5%)

Note: at low Q2 (for Gep)

$[1 - W_2^{el}] = 1 - 1/(1+Q^2/0.71)^4$

$= 1-(1-4Q^2/0.71) =$

$= 1- (1-Q^2/0.178) =$

$\to Q^2/0.178$ as $Q^2 \to 0$

At low Q2 it looks the same as

$Q^2/(Q^2+C) \to Q^2/C$

Revenge of the Spectator Quarks -3 - History of Inelastic Sum rules C. H. Llewellyn Smith hep-ph/981230

Talk given at the Sid Drell Symposium
SLAC, Stanford, California, July 31st, 1998

Gottfried noted that in the '*breathtakingly crude*' naïve three-quark model the second term in the following equation vanishes for the proton (it also vanishes for the neutron, but neutrons are not mentioned):

$$\sum_{i,j} Q_i Q_j \equiv \sum_i Q_i^2 + \sum_{i \neq j} Q_i Q_j \; . \tag{5}$$

Thus for any charge-weighted, flavour-independent, one-body operator all correlations vanish, and therefore using the closure approximation the following sum rule can be derived:

$$\int_{\nu_0} W_2^{ep}(\nu, q^2) d\nu = 1 - \frac{G_E^2 - q^2 G_M^2/4m^2}{1 - q^2/4m^2} \; , \tag{6}$$

where ν_0 is the inelastic threshold (the methods used to derive this sum rule are those that have long been used to derive sum rules in atomic and nuclear physics, for example the sum rule [13] derived in 1955 by Drell and Schwarz). After observing that this sum

Revenge of the Spectator Quarks -4 - History of Inelastic Sum rules C. H. Llewellyn Smith hep-ph/981230

rule appears to be oversaturated in photoproduction (we now know that the integral is actually infinite in the deep inelastic region), Gottfried asked whether it was '*idiotic*', and stated that if, on the contrary there is some truth in it, one would want a '*derivation that a well-educated person could believe*'.

In his talk at the 1967 SLAC conference Bj quoted Gottfried's paper and stated that diffractive contributions should presumably be excluded from the integral, which could be done by taking the difference between protons and neutrons, leading to the following result, in modern notation:

$$\int \left(F_2^{ep}(x,q^2) - F_2^{en}(x,q^2) \right) \frac{dx}{x} = \frac{1}{3} \, . \qquad (7)$$

This result, which is generally known as the Gottfried sum rule, is not respected by the data which give the value [14] 0.235 ± 0.026. In parton notation, the left-hand side can be written

$$\frac{1}{3}(n_u + n_{\bar{u}} - n_d - n_{\bar{d}}) = \frac{1}{3} + \frac{2}{3}(n_{\bar{u}} - n_{\bar{d}}) \, , \qquad (8)$$

S. Adler, Phys. Rev. 143, 1144 (1966) Exact Sum rules from Current Algebra. Valid at all Q2 from zero to infinity. - 5

Strangeness-Conserving Case

The kinematic analysis of Sec. 3 shows that we may write the reaction differential cross section in the form

$$d^2\sigma\left(\binom{\nu}{\bar{\nu}}+p\rightarrow\binom{l}{\bar{l}}+\beta(S=0)\right)\Big/d\Omega_l dE_l=\frac{G^2\cos^2\theta_C}{(2\pi)^2}\frac{E_l}{E_\nu}\times\left[q^2\alpha^{(\pm)}(q^2,W)+2E_\nu E_l\cos^2(\tfrac{1}{2}\phi)\beta^{(\pm)}(q^2,W)\mp(E_\nu+E_l)q^2\gamma^{(\pm)}(q^2,W)\right]. \quad (13)$$

By measuring $d^2\sigma/d\Omega_l dE_l$ for various values of the neutrino energy E_ν, the lepton energy E_l, and the lepton-neutrino angle ϕ, we can determine the form factors $\alpha^{(\pm)}$, $\beta^{(\pm)}$, and $\gamma^{(\pm)}$ for all $q^2>0$ and for all W above threshold.

In Sec. 4 we prove that:

(i) the local commutation relations of Eq. (1a) and Eq. (1c) imply

$$2=g_A(q^2)^2+F_1^V(q^2)^2+q^2F_2^V(q^2)^2+\int_{M_N+M_\pi}^{\infty}\frac{W}{M_N}dW[\beta^{(-)}(q^2,W)-\beta^{(+)}(q^2,W)]; \quad (14)$$

Strangeness-Changing Case

$$(4,2)=\int\frac{W}{M_N}dW[\beta_{(p,n)}^{(-)}(q^2,W)-\beta_{(p,n)}^{(+)}(q^2,W)]; \quad (18)$$

The integrals of Eqs. (18)–(20) have discrete contributions at $W=M_\Lambda$ and/or M_Σ and a continuum extending from $W=M_\Lambda+M_\pi$ or from $W=M_\Sigma+M_\pi$ to $W=\infty$. We have not explicitly separated off the discrete contributions to the integrals, as was done in Eqs. (14)–(16) for the strangeness-conserving case. It would, of course, be straightforward to do this.

F. Gillman, Phys. Rev. 167, 1365 (1968)- 6
Adler like Sum rules for electron scattering.

$$\alpha = W_1/M_N\,,$$
$$\beta = W_2/M_N\,.$$

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^\infty dq_0 [\beta^{(-)}(q_0,q^2) - \beta^{(+)}(q_0,q^2)] = 1\,. \qquad (18)$$

The functions $\beta^{(\pm)}(q_0,q^2)$ are defined just as in Eq. (7) except that in place of the electromagnetic currents $J_\mu(0)$ and $J_\mu(0)$ we have put the isospin raising or lowering F-spin currents $\mathfrak{F}_{(1\pm i2)\mu}(0)$ [recall that $\mathfrak{F}_{3\mu}(0)$ is just the isovector part of the electromagnetic current]. If we explicitly separate out the nucleon Born term in Eq. (18), we have

$$[F_1{}^V(q^2)]^2 + q^2\left(\frac{\mu^V}{2M_N}\right)^2 [F_2{}^V(q^2)]^2 + \int_{M_\pi + (q^2+M_\pi{}^2)/2M_N}^\infty dq_0 [\beta^{(-)}(q_0,q^2) - \beta^{(8)}(q_0,q^2)] = 1\,, \qquad (19)$$

where the superscript V denotes the fact that we are dealing with the isovector part of the current; the isovector anomalous magnetic moment $\mu^V = \mu_p{}' - \mu_n{}' = 3.70$. As $q^2 \to 0$, we see from Eq. (10) or (17) that only the first term, $[F_1{}^V(q^2)]^2$, on the left-hand side of Eq. (19) survives, and as $q^2 \to 0$ it goes to 1, in agreement with the left-hand side.

In the derivation[3] of Eq. (18) only two assumptions enter: (1) the commutation relation Eq. (3a) of the F-spin densities, and (2) an unsubtracted dispersion relation for the forward Compton scattering amplitudes (which are the coefficients of $p_\mu p_\nu$ and $q_\mu q_\nu$ in the expansion of $T_{\mu\nu}$) corresponding to $\beta(q_0,q^2)$. It is of course the second assumption which is most open to question. However, we note the following:

(a) The fact that as $q^2 \to 0$ the left- and right-hand sides of Eq. (19) as it now stands automatically become equal rules out a q^2-independent subtraction. This just means we have done nothing grossly wrong, e.g., introduced a kinematic singularity in q^2 in one of our amplitudes.

F. Gillman, Phys. Rev. 167, 1365 (1968)- 7
Adler like Sum rules for electron scattering.

$\alpha = W_1/M_N$,
$\beta = W_2/M_N$.

Therefore the factor

$[1 - W_2^{el}] = 1 - 1/(1+Q^2/0.71)^4$

$= 1-(1-4Q^2/0.71) =$

$= 1- (1-Q^2/0.178) =$

$\to Q^2/0.178$ as $Q^2 \to 0$

For VALENCE QUARKS FROM THE ADLER SUM RULE FOR the Vector part of the interaction

As compared to the form

$Q^2/(Q^2+C) \to Q^2/C$

And C is different

for the sea quarks.

W2nu-p(vector)= d+ubar

W2nubar-p(vector) =u+dbar

1= W2nubar(p)-W2nu(p)=

= (u+dbar)-(d+ubar)

= (u-ubar)- (d-dbar) = 1

INCLUDING the

x=1 Elastic contribution

Therefore, the inelastic part is

reduced by the elastic x=1 term.

Summary continued

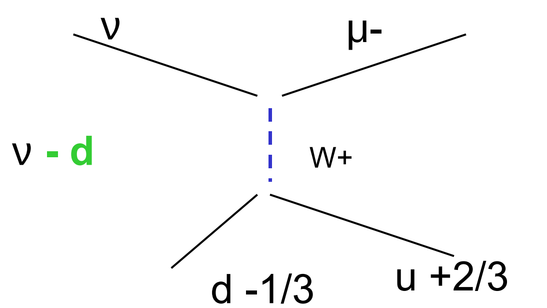
- Future studies involving both neutrino and electron scattering including new experiments are of interest.
- As x gets close to 1, local Duality is very dependent on the spectator quarks (e.g. different for Gep. Gen, Gmp, Gmn, Gaxial, Gvector neutrinos and antineutrinos
- In DIS language it is a function of Q2 and is different for W1, W2 , W3 (or transverse (--left and right, and longitudinal cross sections for neutrinos and antineutrinos on neutrons and protons.
- This is why the present model is probably good in the 2nd resonance region and above, and needs to be further studied in the region of the first resonance and quasielastic scattering region.
- Nuclear Fermi motion studies are of interest, best done at Jlab with electrons.
- Nuclear dependence of hadronic final state of interest.
- Nuclei of interest, C12, P16, Fe56. (common materials for neutrino detectors).

NEUTRINOS

On quarks

On neutrons *both quasielastic*

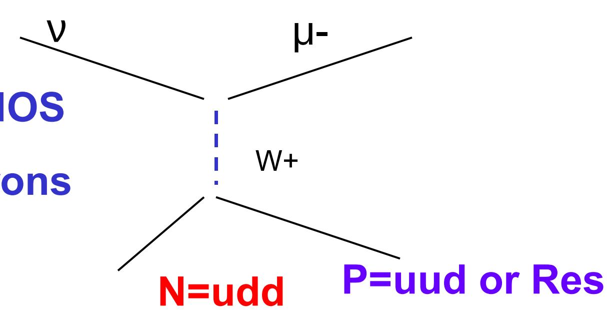
And resonance+DIS production possible.



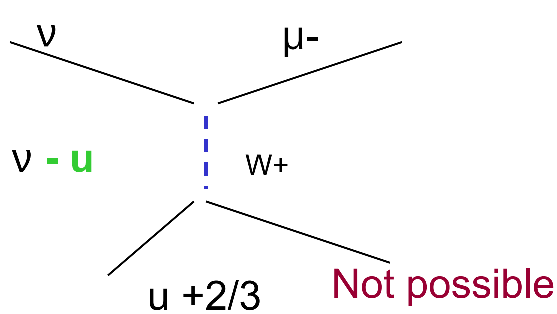
possible

NEUTRINOS

On Neutrons



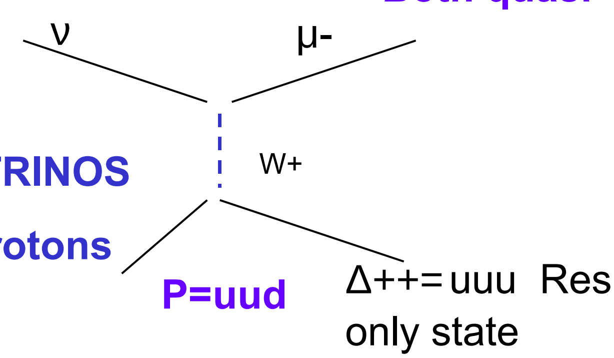
Both quasi+Res



+5/3

NEUTRINOS

On Protons

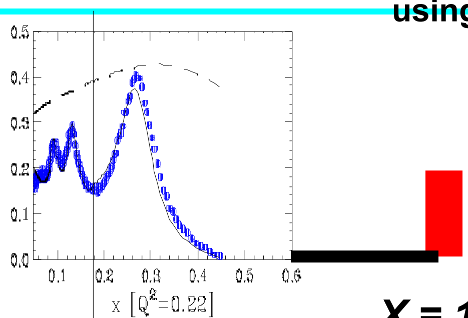


On protons only resonance+ DIS production possible.

NEUTRINOS

On nucleons

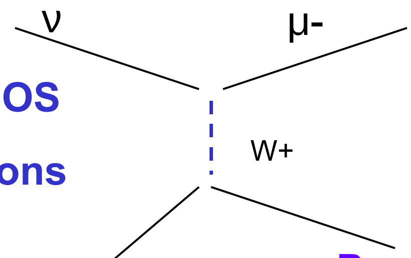
On neutrons *both quasielastic* And resonance+DIS production possible. First resonance has different mixtures of I=3/2 And I=1/2 terms. Neutrino and electron induced production are Better related using Clebsch Gordon Coeff.. (Rein Seghal model etc)



1st reson

X = 1

quasielastic



NEUTRINOS

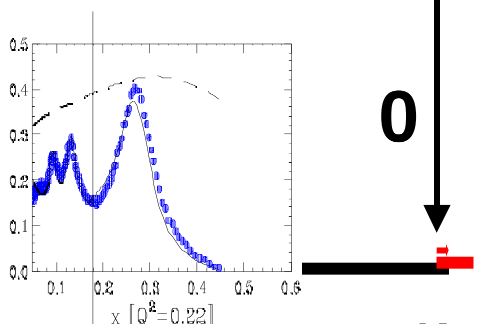
On Neutrons

N=udd

P=uud or Res

Both quasi+Res

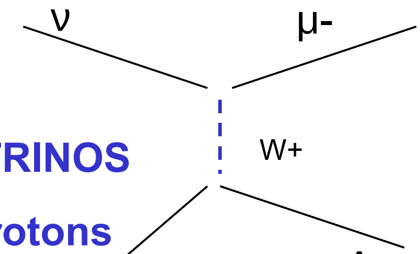
0



1st reson

X = 1

zero



NEUTRINOS

On Protons

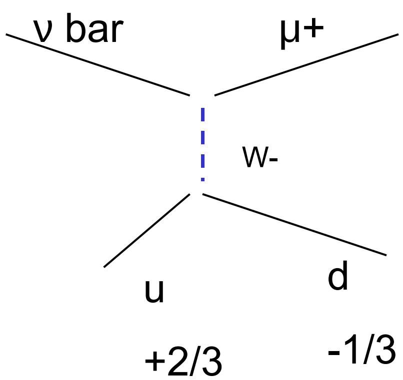
P=uud

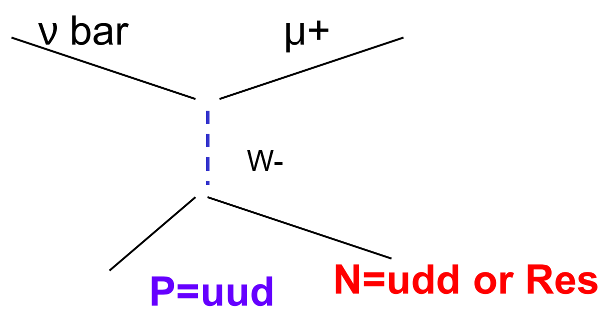
Δ++= uuu Res only state

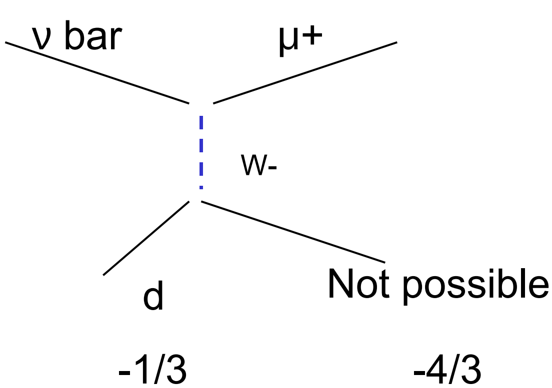
On protons only resonance+ DIS production possible.

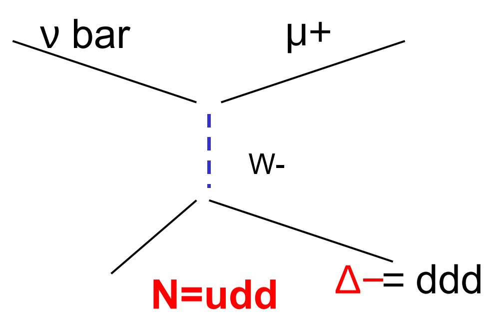
On Protons both quasielastic
And resonance+DIS production possible.

ANTI-NEUTRINOS









On Neutrons only
resonance+ DIS
production possible.

Neutrino cross sections at low energy

- Neutrino ***oscillation experiments*** (K2K, MINOS, CNGS, MiniBooNE, and future experiments with Superbeams at JHF,NUMI, CERN) are in the few GeV region
- Important to correctly model neutrino-nucleon and neutrino-nucleus reactions at 0.5 to 4 GeV (essential for precise next generation neutrino oscillation experiments with super neutrino beams) as well as at the 15-30 GeV (for future ν factories) - NuInt, Nufac
- The very high energy region in neutrino-nucleon scatterings (50-300 GeV) is well understood at the few percent level in terms QCD and Parton Distributions Functions (PDFs) within the framework of the quark-parton model (data from a series of e/μ/ν DIS experiments)
- However, neutrino differential cross sections and final states in the few GeV region are poorly understood. (especially, resonance and low Q^2 DIS contributions). *In contrast, there is enormous amount of e-N data from SLAC and Jlab in this region.*

- *Intellectually - Understanding Low Energy neutrino and electron scattering Processes is also a very way to understand quarks and QCD*. - common ground between the QCD community and the weak interaction community, and between medium and HEP physicists.

Future Progress

Next Update on this Work, NuInt02, Dec. 15,2002
At Irvine. Finalize modified PDFs and do duality
tests with electron scattering data and
Whatever neutrino data exists.
Also --> Get A(w,Q2) for electron proton and
deuteron scattering cases (collaborate with Jlab
Physicists on this next stage).

Meanwhile, Rochester and Jlab/Hampton physicists
Have formed the nucleus of a collaboration to
expand the present Rochester
EOI to a formal NUMI Near Detector off-axis
neutrino proposal (Compare Neutrino data to
existing and future data from Jlab).
--contact person, Kevin McFarland.

Tests of Local Duality at high x, How local Electron Scattering Case

- INELASTIC High Q^2 x-->1.
- QCD at High Q^2 Note d refers to d quark in the proton, which is the same as u in the neutron. d/u=0.2; x=1.
- F2 (e-P) = (4/9)u+(1/9)d = (4/9+1/45) u = (21/45) u
- F2(e-N) = (4/9)d+(1/9)u = (4/45+5/45) u = (9/45) u

- F2(e-N) /F2 (e-P) = 9/21=0.43

- Elastic/quasielastic +resonance at high Q^2 dominated by magnetic form factors which have a dipole form factor times the magnetic moment
- F2 (e-P) = A G^2mP(el) +BG^2mN(res c=+1)
- F2 (e-N) = AG^2mN(el) +BG^2mN(res c=0)
- TAKE ELASTIC TERM ONLY
- F2(e-N) /F2 (e-P) (elastic) =

μ^2(N)/ μ^2(P) = $(1.913/2.793)^2$ =0.47

Close if we just take the elastic/quasielastic x=1 term.

Different at low Q2, where Gep,Gen dominate.

Since Gep=0.

Tests of Local Duality at high x, How local Neutrino Charged current Scattering Case

- INELASTIC High Q2, x-->1. QCD at High Q2: Note d refers to d quark in the proton, which is the same as u in the neutron. d/u=0.2; x=1.
- F2 (ν -P) = 2x*d
- F2(ν -N) = 2x*u
- F2 (ν bar -P) = 2x*u
- F2(ν bar-N) = 2x*d
- F2(ν -P) /F2 (ν -N) =d/u= 0.2
- F2(ν -P) /F2 (ν bar-P) =d/u=0.2
- F2(ν -P) / F2(ν bar-N) =1
- F2(ν -N) /F2 (ν bar-P) =1

- Elastic/quasielastic +resonance at high Q^2 dominated by magnetic form factors which have a dipole form factor times the magnetic moment
- F2 (ν -P) -> A= 0 (no quasiel) + B(Resonance c=+2)
- F2(ν -N) -> A Gm (ν quasiel) + B(Resonance c=+1)
- F2 (ν bar -P) -> A Gm (ν quasiel) + B(Resonance c=0)
- F2(ν bar-N) -> A= 0(no quasiel) + B(Resonance c=-1)

<u>TAKE quasi ELASTIC TERM ONLY</u>

- F2(ν -P) /F2 (ν -N) =0
- F2(ν -P) /F2 (ν bar-P) =0
- F2(ν -P) / F2(ν bar-N) =0/0
- F2(ν -N) /F2 (ν bar-P) =1

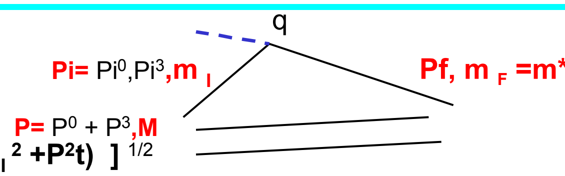
FAILS TEST MUST TRY TO COMBINE Quasielastic and first resonance)

Pseudo Next to Leading Order Calculations

Use LO : Look at PDFs(Xw) times (Q^2/Q^2+C) And PDFs (ξ w) times (Q^2/Q^2+C)

Xw= [Q +B] / [2Mν +A]

ξ w= [Q'^{2} +B] / [Mν (1+(1+Q^2/ν^2) $^{1/2}$) +A]



Where $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + [(Q^2 + m_F^2 - m_I^2)^2 + 4Q^2 (m_I^2 + P^2t)]^{1/2}$

(for now set P^2t =0, masses =0 excerpt for charm.

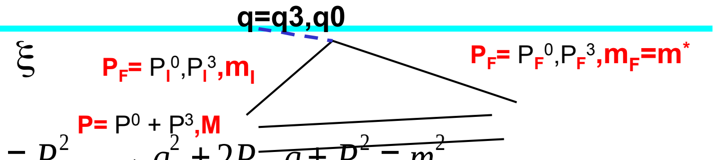
Add B and A account for effects of additional Δm^2 from NLO and NNLO effects.

There are many examples of taking Leading Order Calculations and correcting them for NLO and NNLO effects using external inputs from measurements or additional calculations: e.g.

2. **Direct Photon Production - account for initial quark intrinsic Pt and Pt due to initial state gluon emission in NLO and NNLO processes by smearing the calculation with the MEASURED Pt extracted from the Pt spectrum of Drell Yan dileptons as a function of Q2 (mass).**
3. **W and Z production in hadron colliders. Calculate from LO, multiply by K factor to get NLO, smear the final state W Pt from fits to Z Pt data (within gluon resummation model parameters) to account for initial state multi-gluon emission.**
4. **K factors to convert Drell-Yan LO calculations to NLO cross sections. Measure final state Pt.**
3. **K factors to convert NLO PDFs to NNLO PDFs**
4. **Prediction of 2xF1 from leading order fits to F2 data , and imputing an empirical parametrization of R (since R=0 in QCD leading order).**
5. **THIS IS THE APPROACH TAKEN HERE. i.e. a Leading Order Calculation with input of effective initial quark masses and Pt and final quark masses, all from gluon emission.**

Initial quark mass m_I and final mass $m_F = m^*$ bound in a proton of mass M -- Page 1 INCLUDE quark initial Pt) Get ξ scaling (not $x=Q^2/2M\nu$) DETAILS

ξ *Is the correct variable which is Invariant in any frame : q3 and P in opposite directions.*



$$\xi = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \qquad \xrightarrow[PI,P0]{quark} \quad \xleftarrow[q3,q0]{photon}$$

$$In-LAB-Frame: \rightarrow \quad P_P^0 = M, P_P^3 = 0$$

$$\xi = \frac{P_{I-LAB}^0 + P_{I-LAB}^3}{M} \rightarrow P_{I-LAB}^0 + P_{I-LAB}^3 = \xi M$$

$$\xi = \frac{(P_I^0 + P_I^3)(P_I^0 - P_I^3)}{M(P_I^0 - P_I^3)} = \frac{(P_I^0)^2 - (P_I^3)^2}{M(P_I^0 - P_I^3)}$$

$$\xi M (P_I^0 - P_I^3) = (m_I^2 + Pt^2)$$

$$\rightarrow P_I^0 - P_I^3 = (m_I^2 + Pt^2)/(\xi M)$$

$$(1): \quad P_I^0 - P_I^3 = (m_I^2 + Pt^2)/(\xi M)$$

$$(2): \quad P_I^0 + P_I^3 = \xi M$$

$$2P_I^0 = \xi M + (m_I^2 + Pt^2)/(\xi M) \xrightarrow{m, Pt=0} \xi M$$

$$2P_I^3 = \xi M - (m_I^2 + Pt^2)/(\xi M) \xrightarrow{m, Pt=0} \xi M$$

$$(q + P_I)^2 = P_F^2 \quad \rightarrow q^2 + 2P_I q + P_I^2 = m_F^2$$

$$2(P_I^0 q^0 + P_I^3 q^3) = Q^2 + m_F^2 - m_I^2 \qquad Q^2 = -q^2 = (q^3)^2 - (q^0)^2$$

$$In-LAB-Frame: \rightarrow \qquad Q^2 = -q^2 = (q^3)^2 - \nu^2$$

$$[\xi M + (m_I^2 + Pt^2)/(\xi M)]\nu + [\xi M - (m_I^2 + Pt^2)/(\xi M)]q^3$$

$$= Q^2 + m_F^2 - m_I^2 \quad : General$$

$$Set: m_I^2, Pt = 0 \quad (for\ now)$$

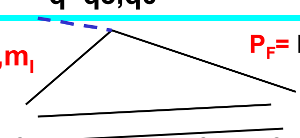
$$\xi M\nu + \xi M q^3 = Q^2 + m_F^2$$

$$\xi = \frac{Q^2 + m_F^2}{M(\nu + q^3)} = \frac{Q^2 + m_F^2}{M\nu(1 + q^3/\nu)} \quad for\ m_I^2, Pt = 0$$

$$\xi = \frac{Q^2 + m_F^2}{M\nu[1 + \sqrt{(1 + Q^2/\nu^2)}]} \qquad for\ m_I^2, Pt = 0$$

Special cases : Denom – TM term, Num – Slow rescaling

initial quark mass m_I and final mass $m_F = m^*$ bound in a proton of mass M -- Page 2 INCLUDE quark initial Pt) DETAILS



ξ *For the case of non zero* m_I, P_t

(note P and q3 are opposite)

$$\xi = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \qquad \underset{quark}{PI, P0} \longrightarrow \longleftarrow \underset{photon}{q3, q0}$$

$$In - LAB - Frame: \rightarrow \quad P_P^0 = M, P_P^3 = 0$$

$$(1):\ 2P_I^0 = \xi M + (m_I^2 + Pt^2)/(\xi M) \quad \rightarrow \rightarrow \rightarrow \rightarrow$$

$$(1):\ 2P_I^3 = \xi M - (m_I^2 + Pt^2)/(\xi M) \quad \rightarrow \rightarrow \rightarrow \rightarrow$$

$$(q + P_I)^2 = P_F^2 \quad \rightarrow q^2 + 2P_I\ q + P_I^2 = m_F^2$$

$$Q^2 = -q^2 = (q^3)^2 - \nu^2$$

$$[\xi M + (m_I^2 + Pt^2)/(\xi M)]\nu + [\xi M - (m_I^2 + Pt^2)/(\xi M)]q^3$$

$$= Q^2 + m_F^2 - m_I^2$$

Keep all terms here and : multiply by ξ **M** and group terms in ξ qnd ξ **2**

$\xi^2\ M^2 (\nu + q3) - \xi\ M [Q^2 + m_F^2 - m_I^2] + [m_I^2 + Pt^2 (\nu - q3)^2] = 0$ General Equation

a (under $\xi^2 M^2(\nu+q3)$), **b**, **c**

=> solution **of quadratic equation** $\xi = [-b + (b^2 - 4ac)^{1/2}] / 2a$

use $(\nu^2 - q3^2) = q^2 = -Q^2$ **and** $(\nu + q3) = \nu + \nu [1 + Q^2/\nu^2]^{1/2} = \nu + \nu [1 + 4M^2 x^2/Q^2]^{1/2}$

$\xi'_w = [Q'^2 + B] / [M\nu (1 + (1 + Q^2/\nu^2))^{1/2} + A]$

where $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + [(Q^2 + m_F^2 - m_I^2)^2 + 4Q^2 (m_I^2 + P^2t)]^{1/2}$

Add B and A to account for effects of additional Δm^2 **from NLO and NNLO effects.**

or $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + [Q^4 + 2Q^2(m_F^2 + m_I^2 + 2P^2t) + (m_F^2 - m_I^2)^2]^{1/2}$

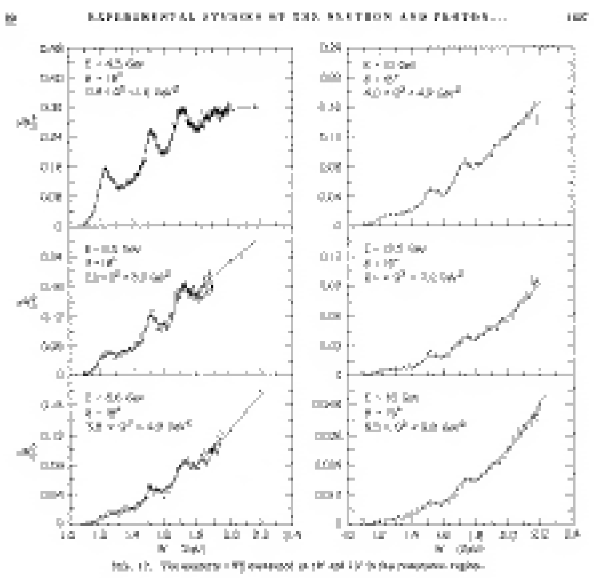
$\xi_w = [Q'^2 + B] / [M\nu (1 + [1 + 4M^2 x^2/Q^2]^{1/2}) + A]$ **(equivalent form)**

$\xi_w = x [2Q'^2 + 2B] / [Q^2 + (Q^4 + 4x^2 M^2 Q^2)^{1/2} + 2Ax]$ **(equivalent form)**

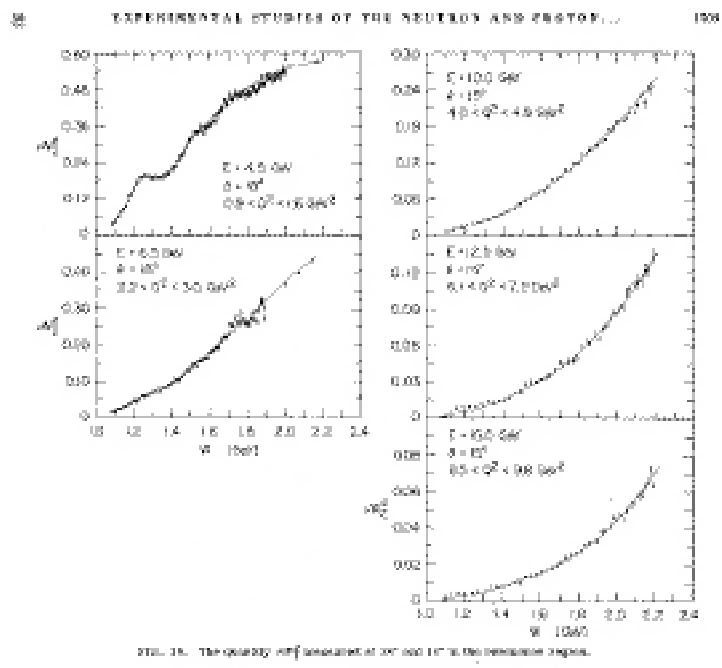
Model	χ^2 / DOF	Data Fit	PDF used	Scaling Variable	Power Param	Photo limit	A(W,Q2) Reson.	Ref.
QPM-0 Published 1979	--	e-N DIS/Res Q2>0	F2p F2d * f(x)	Xw= (Q2+B)/ (2Mν+A)	A=1.64 B=0.38	X/Xw C=B =0.38	A_P(W,Q) A_D(W,Q)	Bodek et al PRD-79
NLO-2 Published 1999	1470 /928 DOF	e/ μ --N, DIS Q2>1	MRSR2 * f(x)	ξ_{TM}=Q2/TM+ Renormalon model for 1/Q2	a2= -0.104 a4= - 0.003	Q2>1 NA	1.0- average	Yang/ Bodek PRL -99
NNLO-3 Published 2000	1406 /928 DOF	e/ μ --N, DIS Q2>1	MRSR2 * f(x)	ξ_{TM}=Q2/TM+ Renormalon model for 1/Q2	a2= -0.009 a4= -0.013	Q2>1 NA	1.0- average	Yang/ Bodek EPJC -00
LO-1 published 2001	1555 /958 DOF	e/μ--N, DIS Q2>0	GRV94 f(x)=1	Xw= (Q2+B)/ (2Mν+A)	A=1.74 B=0.62	Q2/ (Q2+C) C=0.19	1.0- average	Bodek/ Yang NuInt01
LO-1- published / 2002	1268 /1200 DOF	e/ μ --N, DIS HERA Q2>0	GRV98 f(x)=1	ξ w= (Q2+B) / (TM+A)	A=.418 B=.222	comple x	1.0- average	Bodek/ Yang NuFac02
LO-1- Future work 2002-3	TBA	e/ μ --N, γ--N, ν--N, DIS/Res Q2>0	GRV? or other * f(x)	ξ 'w= (Q2+B..Pt^2)/ (TM+A)	A=TBA B=TBA Pt^2 = TBA	comple x	Au(W,Q) Ad(W,Q) ? Spect. Quark dependent	Bodek/ Yang Nutin02 +PRD

e-P, e-D: Xw scaling MIT SLAC DATA 1972 Low Q2
QUARK PARTON MODEL 0TH order (Q^2>0.5)

e-P scattering Bodek PhD thesis 1972
[PRD 20, 1471(1979)] Proton Data
Q^2 from 1.2 to 9 GeV^2 versus
$\nu W2 = (x/x_w)^* F_2(X_w)^* A_P (W,Q^2)$-- QPM fit.

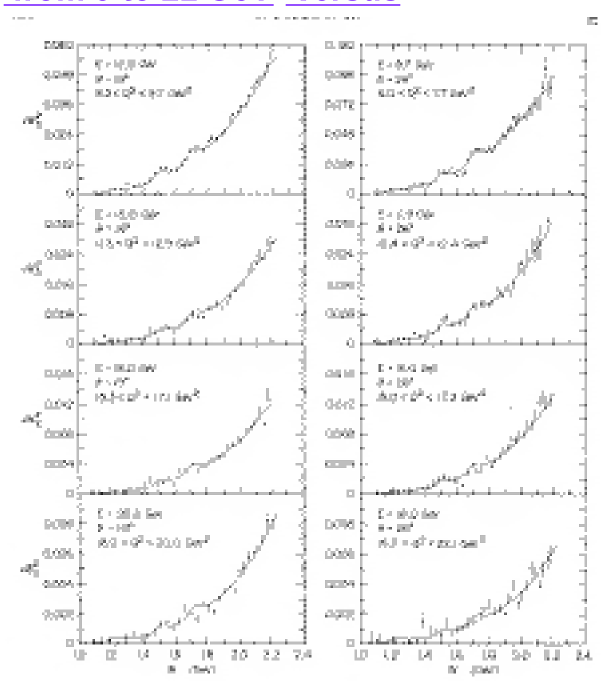


e-D scattering from same publication.
NOTE Deuterium Fermi Motion
Q^2 from 1.2 to 9 GeV^2 versus
$\nu W2 = (x/x_w)^* F_2(X_w)^* A_D(W,Q^2)$ --QPM fit.

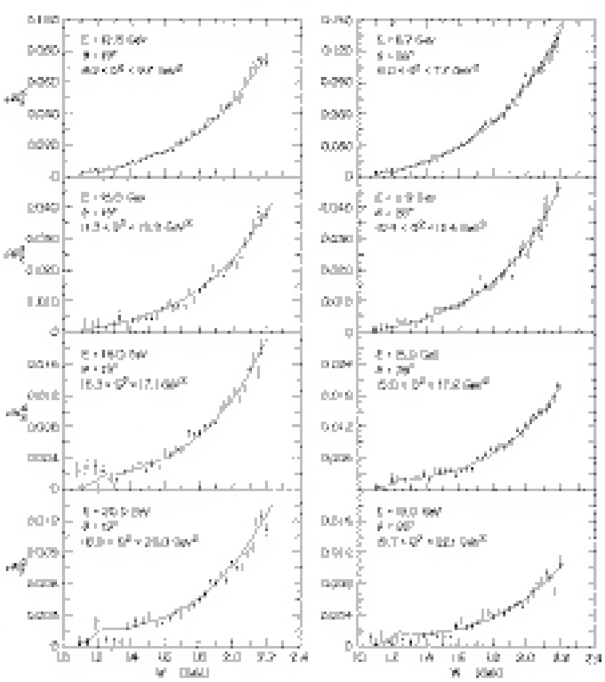


e-P, e-D: Xw scaling MIT SLAC DATA 1972 High Q2
QUARK PARTON MODEL 0TH order (Q^2>0.5)

e-P scattering Bodek PhD thesis 1972
[PRD 20, 1471(1979)] Proton Data
$\nu W2= (x/x_w)^* F_2(X_w)^* A_P$ (W,Q^2)-- QPM fit
Q^2 from 9 to 21 GeV^2 versus

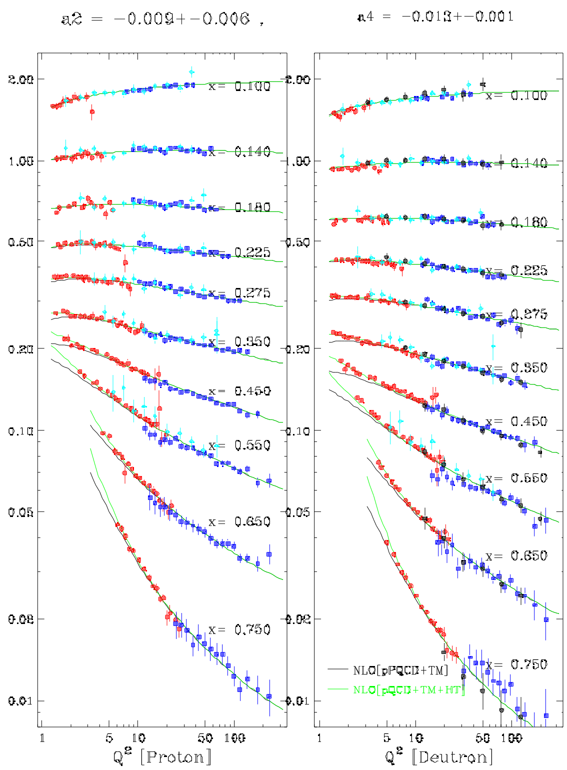


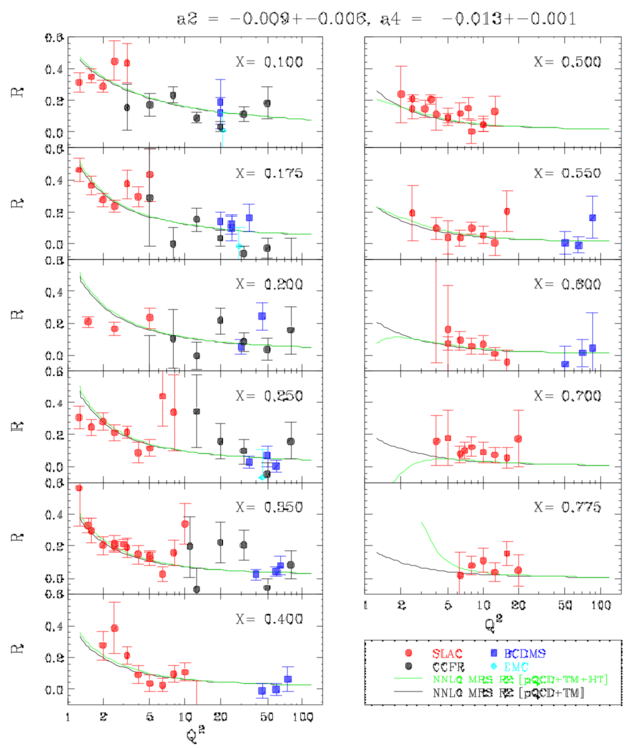
e-D scattering from same publication.
NOTE Deuterium Fermi Motion
$\nu W2= (x/x_w)^* F_2(X_w)^* A_D(W,Q^2)$ --QPM fit.
Q^2 from 9 to 21 GeV^2 versus



F_2, R comparison with NNLO QCD-works => NLO HT are missing NNLO terms (Q^2>1)

Size of the higher twist effect with NNLO analysis is really small (but not 0)
a2= -0.009 (in NNLO) versus –0.1(in NLO) - > factor of 10 smaller, a4 nonzero





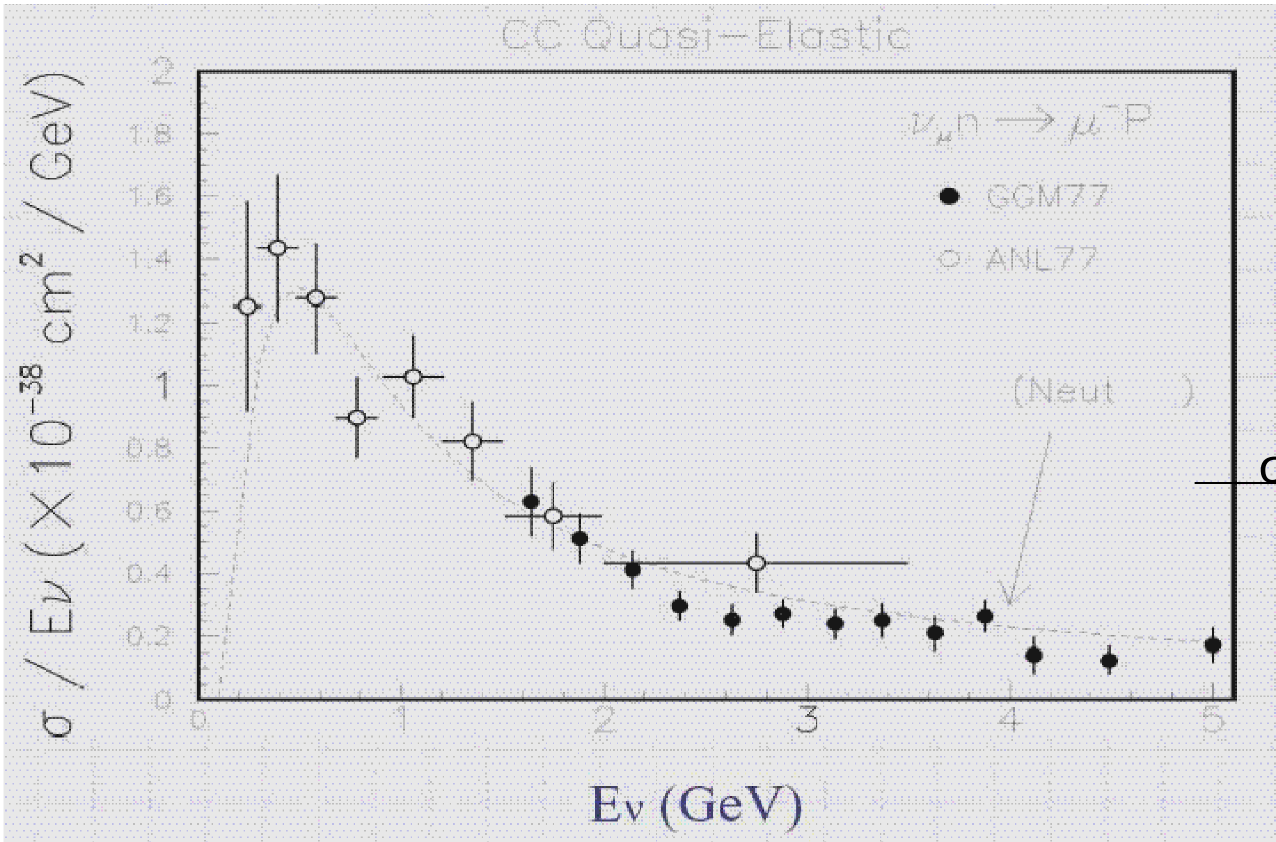
Future Work - part 1

- Implement $A_{e/\mu}$ (W,Q^2) resonances into the model for F_2 with ξ_w scaling.
- For this need to fit all DIS and SLAC and JLAB resonance date and Photo-production H and D data and CCFR neutrino data.
- Check for local duality between ξ_w scaling curve and elastic form factors Ge, Gm in electron scattering. - Check method where its applicability will break down.
- Check for local duality of ξ_w scaling curve and quasielastic form factors Gm. Ge, G_A, G_V in quasielastic electron and neutrino and antineutrino scattering.- Good check on the applicability of the method in predicting exclusive production of strange and charm hyperons
- Compare our model prediction with the Rein and Seghal model for the 1st resonance (in neutrino scattering).
- Implement differences between ν and e/μ final state resonance masses in terms of
- A (i,j, k) (W,Q^2) (i is the interacting quark, and j,k are spectator quarks).
- Look at Jlab and SLAC heavy target data for possible Q^2 dependence of nuclear dependence on Iron.
- Implementation for R (and $2xF_1$) is done exactly - use empirical fits to R (agrees with NNLO+GP tgt mass for $Q^2>1$); Need to update Rw $Q^2<1$ to include Jlab R data in resonance region.
- Compare to low-energy neutrino data (only low statistics data, thus new measurements of neutrino differential cross sections at low energy are important).
- Check other forms of scaling e.g. $F_2=(1+ Q^2/ \nu^2)^{1/2}\nu W_2$ (for low energies)

Future Work - part 2

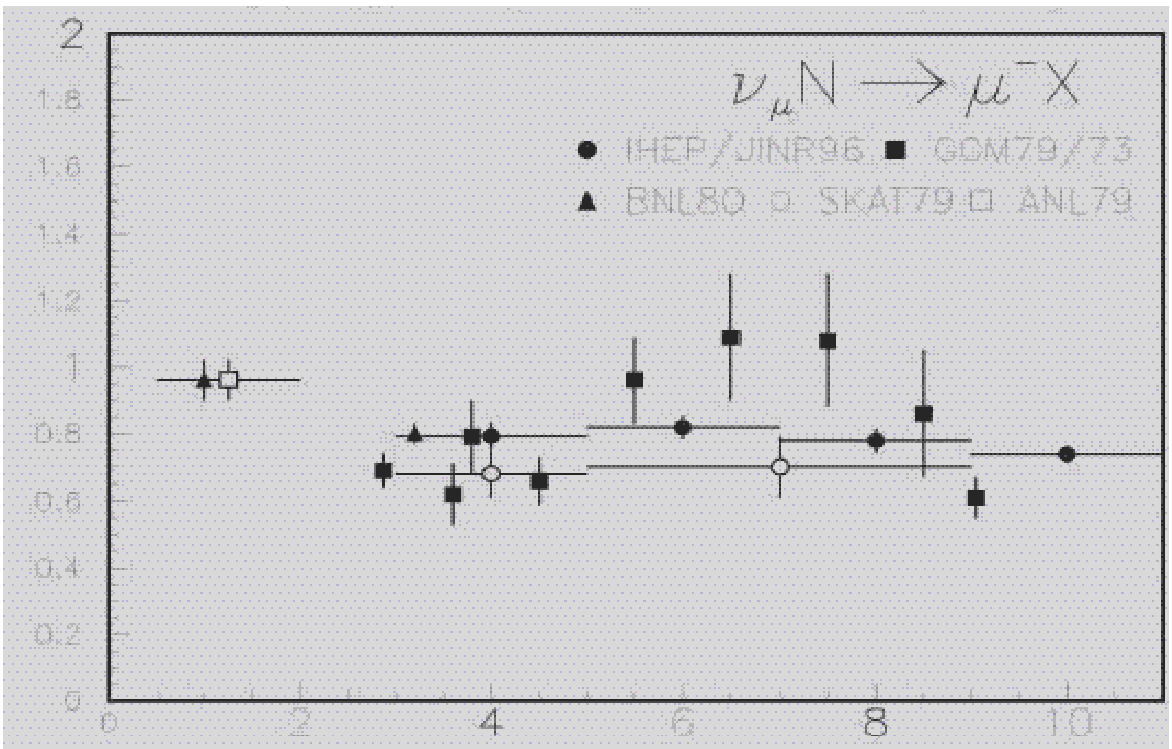
- Investigate different scaling variable parameters for different flavor quark masses (u, d, s, u_v, d_v, u_{sea}, d_{sea} in initial and final state) for F_2. ,
- **Note:** $\xi_w = [Q^2+B] / [M\nu (1+ (1+Q^2/\nu^2)^{1/2}) +A]$ **assumes** $m_F = m_i = 0$, $P^2t=0$
- **More sophisticated General expression (see derivation in Appendix):**
- $\xi_w' = [Q'^2+B] / [M\nu (1+ (1+Q^2/\nu^2)^{1/2}) +A]$ **with**
- $2Q'^2 = [Q^2+ m_F^2 - m_I^2] + [(Q^2+ m_F^2 - m_I^2)^2 + 4Q^2 (m_I^2 +P^2t)]^{1/2}$
- **or** $2Q'^2 = [Q^2+ m_F^2 - m_I^2] + [Q^4 +2 Q^2(m_F^2 + m_I^2 +2P^2t) + (m_F^2 - m_I^2)^2]^{1/2}$
Here B and A account for effects of additional Δm^2 from NLO and NNLO effects. However, one can include P^2t, as well as m_F, m_i as the current quark masses (e.g. Charm, production in neutrino scattering, strange particle production etc.). In ξ_w, B and A account for effective masses+initial Pt. When including Pt in the fits, we can constrain Pt to agree with the measured mean Pt of Drell Yan data..
- Include a floating factor f(x) to change the x dependence of the GRV94 PDFs such that they provide a good fit to all high energy DIS, HERA, Drell-Yan, W-asymmetry, CDF Jets etc, for a global PDF QCD LO fit to include Pt, quark masses A, B for ξ_w **scaling and the $Q^2/(Q^2+C)$ factor, and** $A_{e/\mu}(W,Q^2)$ as a first step towards modern PDFs. (but need to conserve sum rules).
- Put in fragmentation functions versus W, Q2, quark type and nuclear target

Examples of Current Low Energy Neutrino Data: Quasi-elastic cross section



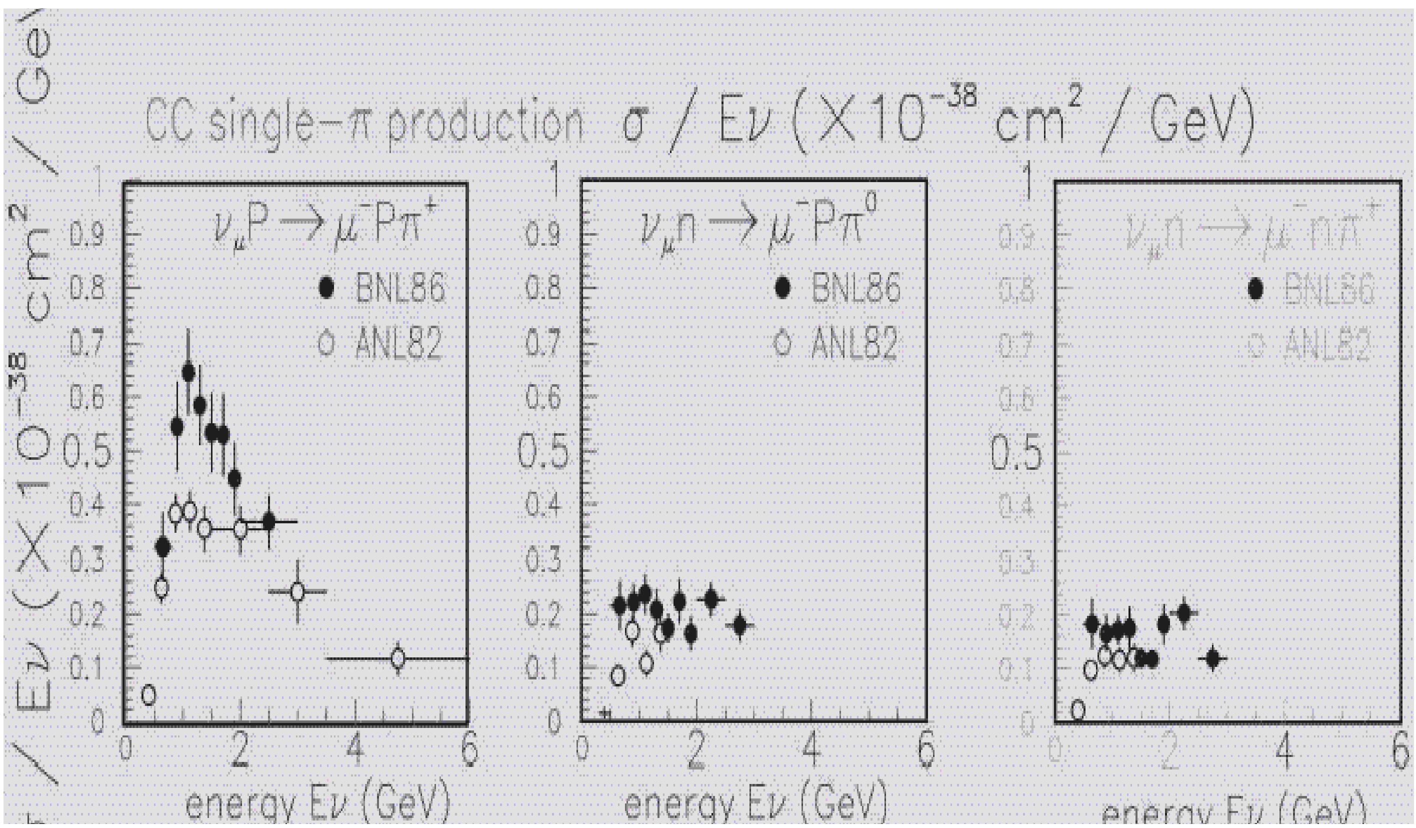
σ_{tot}/E

Examples of Low Energy Neutrino Data: Total (inelastic and quasielastic) cross section



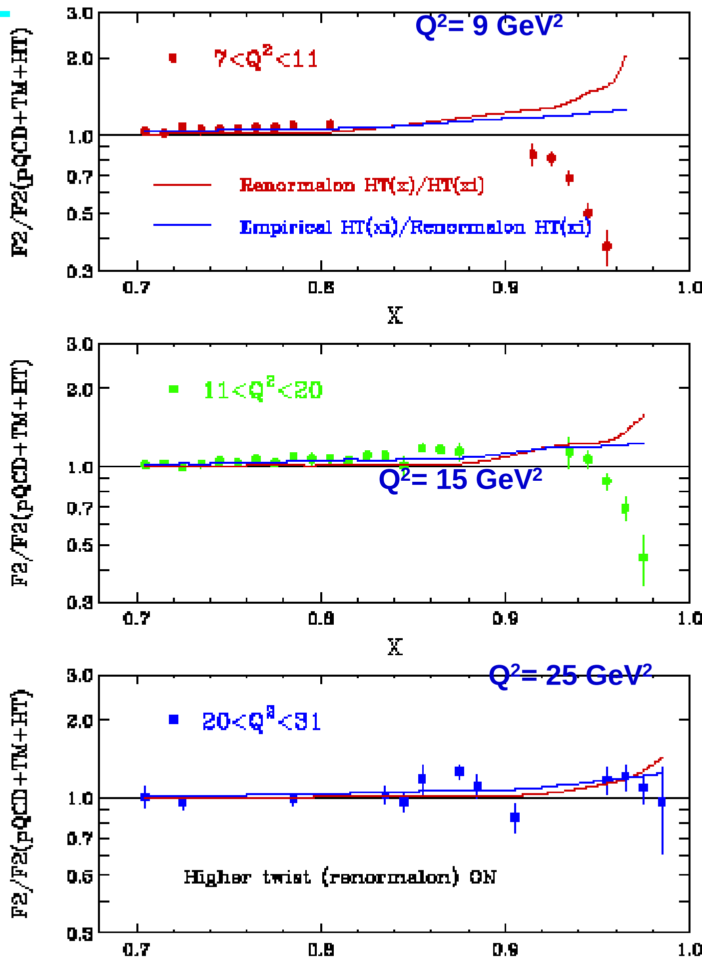
E GeV

Examples of Current Low Energy Neutrino Data: Single charged and neutral pion production



Old bubble chamber language

Look at Q^2= 8, 15, 25 GeV^2 very high x data-backup slide*



Ratio
F_2data/F_2pQCD+TM+HT

- **Pion production threshold** A_w **(w, Q^2)**
- Now Look at lower Q^2 (8,15 vs 25) DIS and resonance data for the ratio of
  F2 data/(NLO pQCD +TM +HT}
- High x ratio of F2 data to NLO pQCD +TM +HT parameters extracted from lower x data. These high x data were not included in the fit.
  - **The Very high x(=0.9**) region: It is described by NLO pQCD (if target mass and higher twist effects are included) to better than 10%

Importance of Precision Measurements of $P(\nu_\mu \to \nu_e)$ Oscillation Probability with ν_μ and $\bar{\nu}_\mu$ Superbeams

- Conventional "superbeams" of both signs (e.g. NUMI) will be our only windows into this suppressed transition
  - Analogous to $|V_{ub}|$ in quark sector (CP phase δ could be origin of matter-antimatter asymmetry in the universe)
  - (The next steps: μ sources or "β beams" are too far away)

Studying $P(\nu_\mu \to \nu_e)$ in neutrinos and anti-neutrinos gives us magnitude and phase information on $|U_{e3}|$

http://www-numi.fnal.gov/fnal_minos/new_initiatives/loi.html **A.Para-NUMI off-axis**

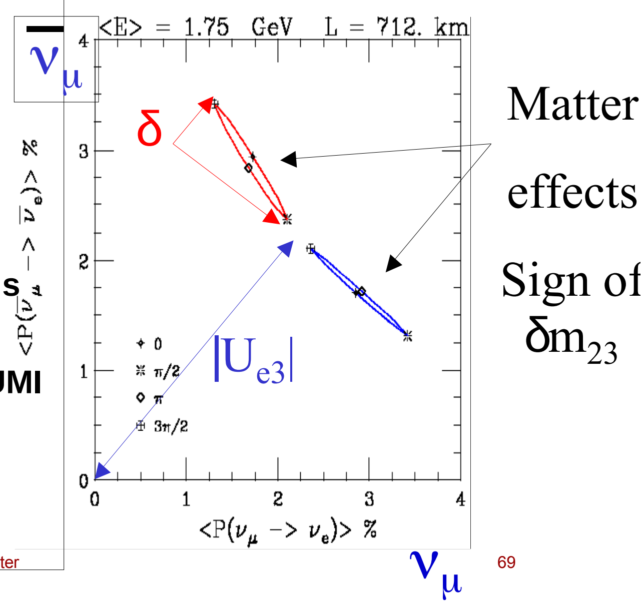
http://www-jhf.kek.jp/NP02 **K. Nishikawa JHF off-axis**

http://www.pas.rochester.edu/~ksmcf/eoi.pdf

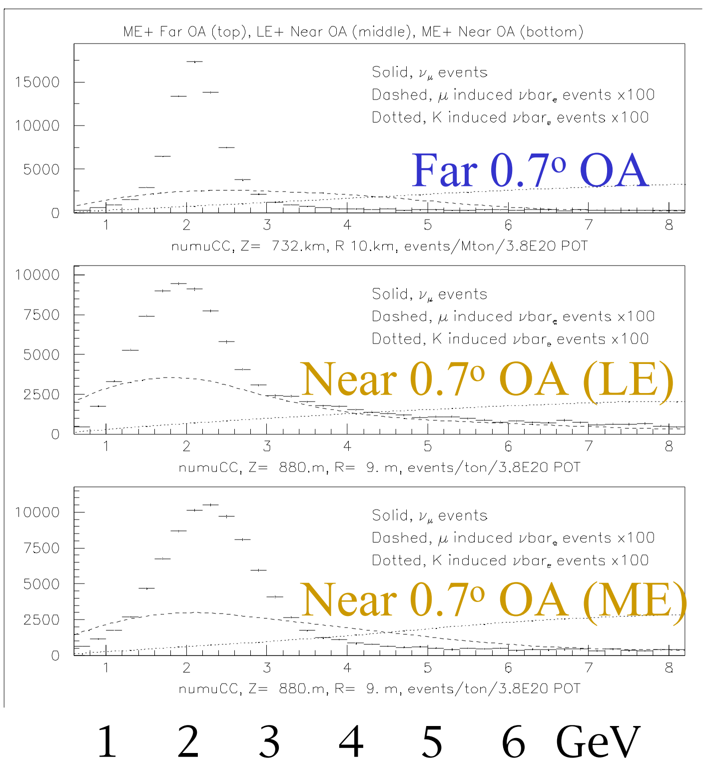
K. McFarland (Rochester) - off-axis near detector NUMI

http://home.fnal.gov/~morfin/midis/midis_eoi.pdf).

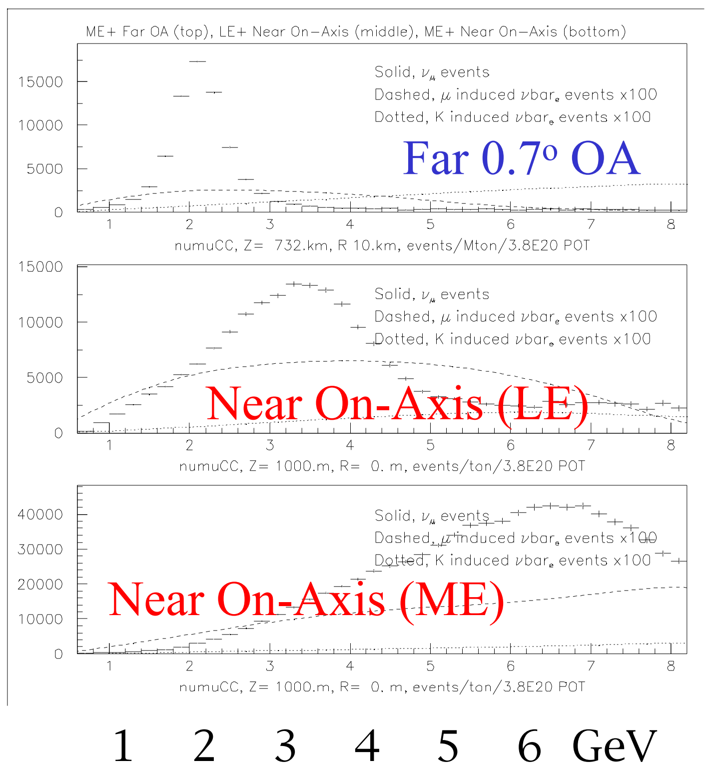
J. Morfin (FNAL-)Low E neutrino reactions in an on-axis near detector at MINOS/NUMI



Event Spectra in NUMI Near Off-Axis, Near On-Axis and Far Detectors (The miracle of the off-axis beam is a nearly mono-energetic neutrino beam making future precision neutrino oscillations experiments possible for the first time



Neutrino Energy



Neutrino Energy

http://nuint.ps.uci.edu (NuInt02)

http://neutrino.kek.jp/nuint01/ (NuInt01)

Note: 2nd conf.

NuInt02 to

Be held at

UC Irvine

Dec 12-15,2002

Needed even for the

Low statistics at K2K

Bring people of

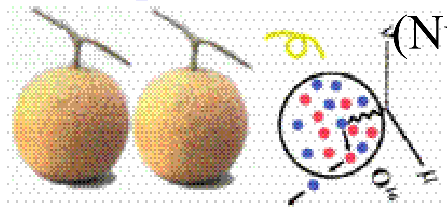
All languages

And nuclear and

Particle physicists

Together.

NuInt01 7/20/02 6:48 AM



NuInt01 : The First International Workshop on Neutrino-Nucleus Interactions in the Few GeV Region

December 13-16, 2001, KEK, Tsukuba, Japan

List of participants(PDF)

What do we want to know about low energy ν_μ reactions and why

Reasons

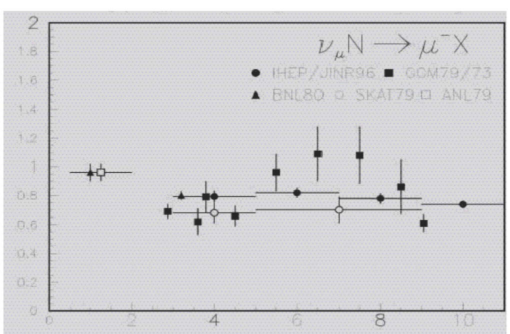
- <u>Intellectual Reasons:</u>
- **Understand how QCD works in both neutrino and electron scattering at low energies - different spectator quark effects. (*There are fascinating issues here as we will show*)**
- **How is fragmentation into final state hadrons affected by nuclear effects in electron versus neutrino reactions.**
- **Of interest to : Nuclear Physics/Medium Energy, QCD/ Jlab communities**
- ***<u>IF YOU ARE INTERESTED QCD</u>***

- <u>Practical Reasons</u>:
- **Determining the neutrino sector mass and mixing matrix precisely**
  - **requires knowledge of both Neutral Current (NC) and Charged Current(CC) differential Cross Sections and Final States**
  - **These are needed for the NUCLEAR TARGET from which the Neutrino Detector is constructed (e.g Water, Carbon, Iron).**
- **Particle Physics/ HEP/ FNAL /KEK/ Neutrino communities**
- ***<u>IF YOU ARE INTERESTED IN NEUTRINO MASS and MIXING</u>*.**

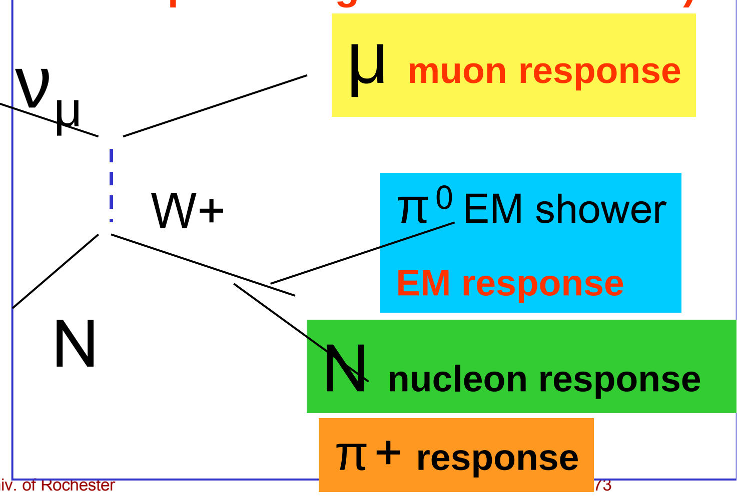
ν_μ Charged Current Processes is of Interest

Charged - Current: both differential cross sections and final states

- **<u>Neutrino mass ΔM^2: -> Charged Current Cross Sections and Final States are needed</u>:**
- **The level of neutrino charged current cross sections versus energy provide the baseline against which one measures ΔM^2 at the oscillation maximum.**



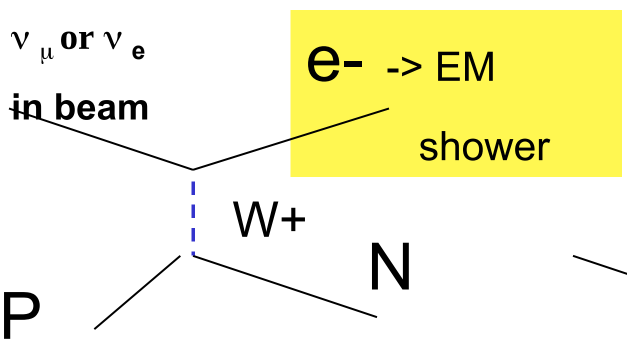
- **Measurement of the neutrino energy in a detector depends on the composition of the final states (different response to charged and neutral pions, muons and final state protons (e.g. Cerenkov threshold, non compensating calorimeters etc).**

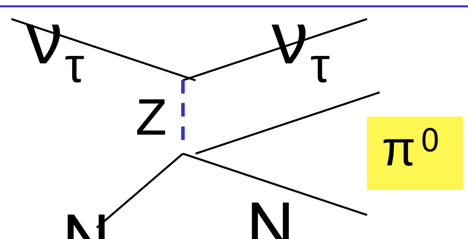


N_{μ} Neutral Current Processes is of Interest

Neutral - Current both differential cross sections and final states

- **SIGNAL** $\nu_{\mu} \rightarrow \nu_e$ **transition** ~ **0.1% oscillations probability of** $\nu_{\mu} \rightarrow \nu_e$.

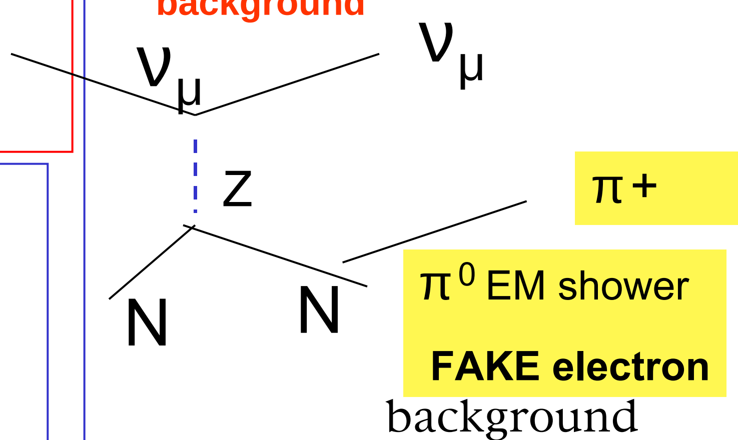




SIGNAL ν_{τ}

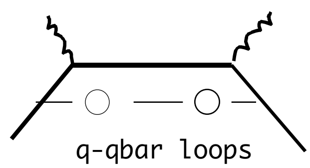
- **Backgrounds**: **<u>Neutral Current Cross Sections and Final State Composition are needed</u>**:

- **Electrons from Misidentified π^0 in NC events without a muon from higher energy neutrinos are a background**



Dynamic Higher Twist- Power Corrections- e.g. Renormalon Model

- Use: Renormalon QCD model of Webber&Dasgupta- Phys. Lett. B382, 272 (1996), Two parameters a_2 and a_4. This model includes the (1/ Q^2) and (1/ Q^4) terms from gluon radiation turning into virtual quark antiquark fermion loops (from the interacting quark only, the spectator quarks are not involved).



- $F_2^{\text{theory}}(x,Q^2) = F_2^{\text{PQCD+TM}}\ [1+ D_2(x,Q^2) + D_4(x,Q^2)]$

$D_2(x,Q^2) = (1/Q^2)\ [\, a_2 / q(x,Q^2)\,] \circ (dz/z)\, c_2(z)\, q(x/z, Q^2)$

$D_4(x,Q^2) = (1/Q^4)\ [\, a_4$ times function of x)]

In this model, the higher twist effects are different for $2xF_1$, xF_3, F_2. With complicated x dependences which are defined by only two parameters a_2 and a_4. (the $D_2(x,Q^2)$ term is the same for $2xF_1$ and , xF_3)

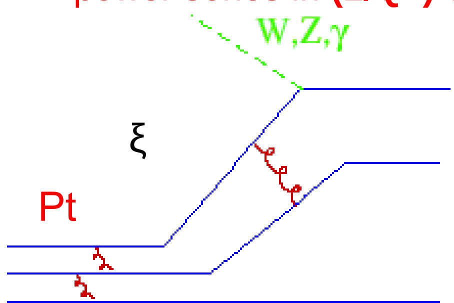
Fit a_2 and a_4 to experimental data for F_2 and $R=F_L/2xF_1$.

$F_2^{\text{data}}(x,Q^2) = [\, F_2^{\text{measured}} + \lambda\, \delta F_2^{\text{syst}}\ ]\,(1+ \mathbf{N})$: χ^2 weighted by errors

where **N** is the fitted normalization (within errors) and δF_2^{syst} is the is the fitted correlated systematic error BCDMS (within errors).

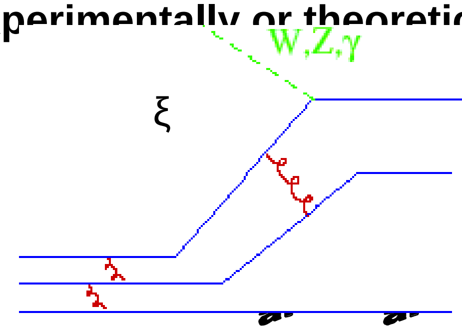
What are $1/Q^2$ Higher Twist Effects- page 1

- **Higher Twist Effects** are terms in the structure functions that behave like a power series in **$(1/Q^2)$ or $[Q^2/(Q^4+A)]$,... $(1/Q^4)$** etc....



(a)**Higher Twist: Interaction between Interacting and Spectator quarks via gluon exchange at Low Q2-at low W**
(b) **Interacting quark TM binding, initial Pt and Missing Higher Order QCD terms DIS region. ->$(1/Q^2)$ or $[Q^2/(Q^4+A)]$,... $(1/Q^4)$.**

- **While pQCD** predicts terms in α_s^2 **$(\sim 1/[\ln(Q^2/\Lambda^2)])$...** α_s^4 etc...
- (i.e. LO, NLO, NNLO etc.) **In the few GeV region, the terms of the two power series cannot be distinguished, experimentally or theoretically**



high Q2 impulse
ion, the interacting
spectator quarks
ved and do not
ct each other.

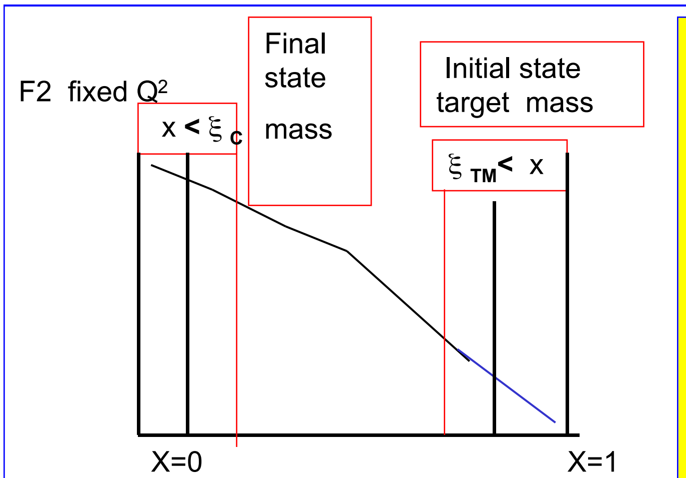
In NNLO p-QCD additional gluons emission: terms like α_s^2 **$(\sim 1/[\ln(Q^2/\Lambda^2)])$...** α_s^4
Spectator quarks are not Involved.

Modified LO PDFs for all Q^2 region?

Philosophy

1. We find that NNLO QCD+tgt mass works very well for $Q^2 > 1$ GeV^2.
2. That target mass and missing NNLO terms "explain" what we extract as higher twists in a NLO analysis. i.e. SPECTATOR QUARKS ONLY MODULATE THE CROSS SECTION AT LOW W. THEY DO NOT CONTRIBUTE TO DIS HT.
2. However, we want to go down all the way to $Q^2=0$. All NNLO and NLO terms blow up. However, higher twist formalism in terms of initial state target mass binding and Pt, and final state mass are valid below $Q^2=1$, and mimic the higher order QCD terms for $Q^2>1$ (in terms of effective masses, Pt due to gluon emission).
3. While the original approach was to explain the "empirical higher twists" in terms of NNLO QCD at low Q^2 (and extract NNLO PDFs), we can reverse the approach and have "higher twists" Model non-perturbative QCD, down to $Q^2=0$, by using LO PDFs and "effective target mass and final state masses" to account for initial target mass, final target mass, and missing NLO and NNLO terms. I.e. Do a fit with:
4. $F_2(x, Q^2) = K(Q^2)\, F_{2QCD}(\xi_w, Q^2)\, A(w, Q^2)$ (set $A_w(w, Q^2)=1$ for now - spectator quarks) $K(Q^2)$ is the photo-production limit Non-perturbative term.
5. $\xi_w = [Q^2+B] / [M\nu (1+(1+Q^2/\nu^2)^{1/2}) + A]$
6. B=effective final state quark mass. A=enhanced TM term, [Ref:Bodek and Yang hep-ex/0203009] previously used $X_w = [Q^2+B] / [2M\nu + A]$

"A term" At High x, "NNLO QCD terms" have a similar form to the "kinematic -Georgi-Politzer ξ_{TM} TM effects" -> look like "enhanced" QCD evolution at low Q



At high x, Mi,Pt from multi gluon emission by initial state quark -> look like enhanced QCD evolution or enhance target mass effect. Add a term A

$\xi_{TM} = Q^2 / [M\nu\,(1 + (1 + Q^2/\nu^2)^{1/2}) + A]$ proton target mass effect in Denominator plus enhancement)

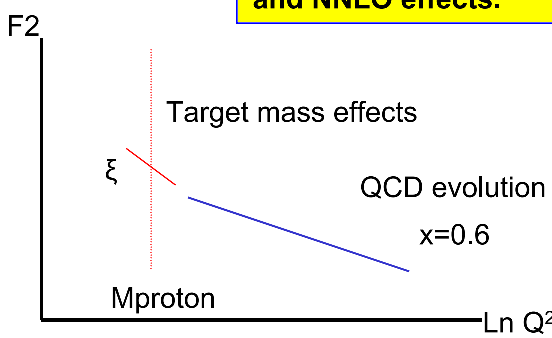
$\xi_C = [Q^2 + M^{*2}] / [2M\nu]$ (final state M* mass)

Combine both target mass and final state mass:

$\xi_{C+TM} = [Q^2 + M^{*2} + B] / [M\nu\,(1 + (1 + Q^2/\nu^2)^{1/2}) + A]$

- includes both initial state target proton mass and final state M* mass effect) - Exact derivation in Appendix. Add B and A account for additional Δm^2 from NLO and NNLO effects.

At high x, low Q^2 $\xi_{TM} < x$ (tgt mass ξ) (and the PDF is higher at lower x, so the low Q^2 cross section is enhanced .



[Ref:Georgi and Politzer Phys. Rev. D14, 1829 (1976)]]