

***Update to the Bodek-Yang Unified Model for Electron- and
Neutrino- Nucleon Scattering Cross sections***

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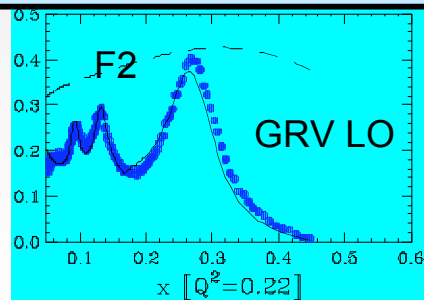
Saturday July 24 15:00-15:30

Neutrino Session

Salle Maillot, ICHEP 2010 Paris, France

Modeling neutrino cross sections

- Describe DIS, resonance, even photo-production ($Q^2=0$) in terms of **quark-parton model**. With PDFs, it is straightforward to convert charged-lepton scattering cross sections into neutrino cross section.
- **Challenge:**
 - Understanding of high x PDFs at very low Q^2 ?
 - Understanding of resonance scattering in terms of quark-parton model?
 - What happens near $Q^2=0$?



- **Bodek-Yang LO approach** (pseudo NNLO)
Use effective LO PDFs with a new scaling variable, ξ_W to absorb target mass, higher twist, missing QCD higher orders
- + add **low Q^2 K factors**

Diagram illustrating the Bodek-Yang LO approach. A vertex labeled q (quark) splits into two lines. The left line is labeled $P=M$. The right line is labeled $m_f=M^*$ (final state interaction). A red squiggly line labeled ξ connects the two lines. The text "Resonance, higher twist, and TM" is written above the right line.

$$\xi_W = \frac{Q^2 + m_f^2 + O(m_f^2 - m_i^2) + A}{M_V (1 + (1 + Q^2/V^2))^{1/2} + B} \rightarrow X_{bj} = Q^2 / 2 M_V$$

Bodek -Yang Effective LO PDF model - 2003

1. Start with GRV98 LO ($Q^2_{\min}=0.80 \text{ GeV}^2$)
- dashed line- describe F2 data at high Q^2
 2. Replace the X_{bj} with a new scaling, ξ_w
 3. Multiply all PDFs by **K factors** for photo prod. limit and higher twist

$$[\sigma(\gamma) = 4\pi\alpha/Q^2 * F_2(x, Q^2)]$$

$$K_{sea} = Q^2/[Q^2 + C_{sea}]$$

$$K_{val} = [1 - G_D^2(Q^2)] * [Q^2 + C_{2V}] / [Q^2 + C_{1V}] \text{ motivated by Adler Sum rule}$$

$$\text{where } G_D^2(Q^2) = 1/[1 + Q^2/0.71]^4$$
 4. Freeze the evolution at $Q^2 = Q^2_{\min}$
- $F_2(x, Q^2 < 0.8) = K(Q^2) * F_2(X_w, Q^2=0.8)$
- Fit to all DIS F2 P/D (with low x HERA data)
A=0.418, B=0.222

Csea = 0.381, C1V = 0.604, C2V = 0.485
 $\chi^2/\text{DOF} = 1268 / 1200$ Solid Line

A : initial state binding/TM+ higher order QCD
B : final state mass m_f^2 , Δm^2 .
K Factor: Photo-prod limit ($Q^2=0$), Adler sum-rule

For 2xF1 use R1998

Fit only precise charged lepton scattering data.
No neutrino data and
No Resonance data
included in the fit.

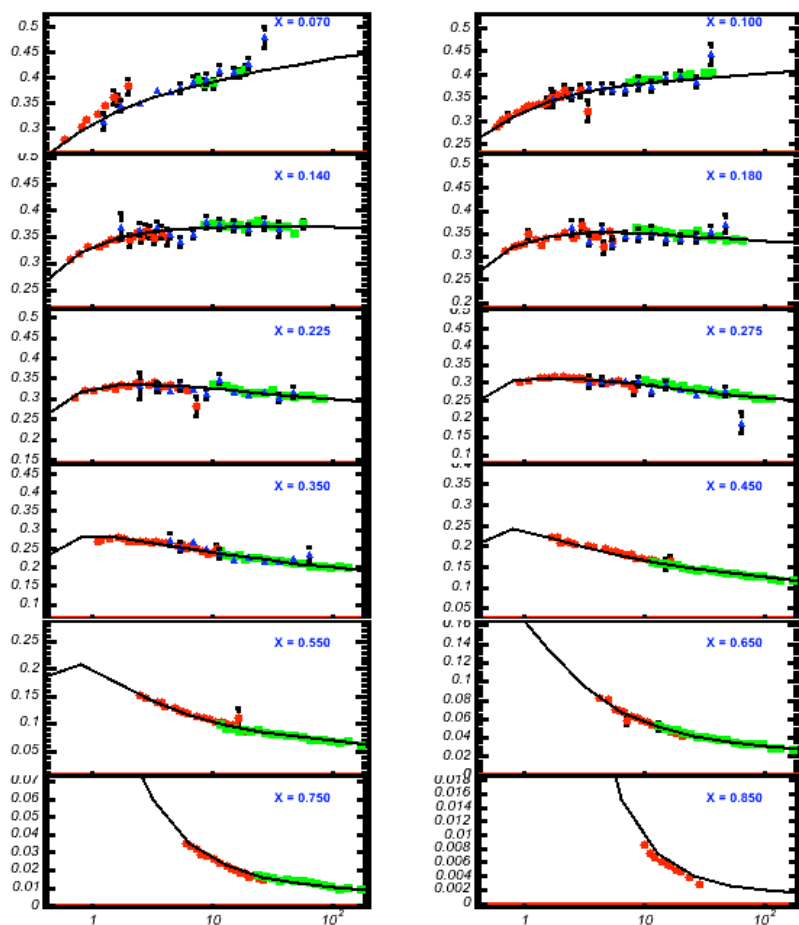
2004 update:
 Separate K factors
 for uv, dv, us, ds

For XF3 : we assumed V=A in 2003-2004

$$X_{bj} = Q^2 / 2 M_v$$

$$\xi W = \frac{Q^2 + m_f^2 + O(m_f^2 - m_i^2) + A}{M_v (1 + (1 + Q^2/v^2))^{1/2} + B}$$

Fit results GRV98 + B-Y 2004 (SLAC, BCDMS, NMC) H + D

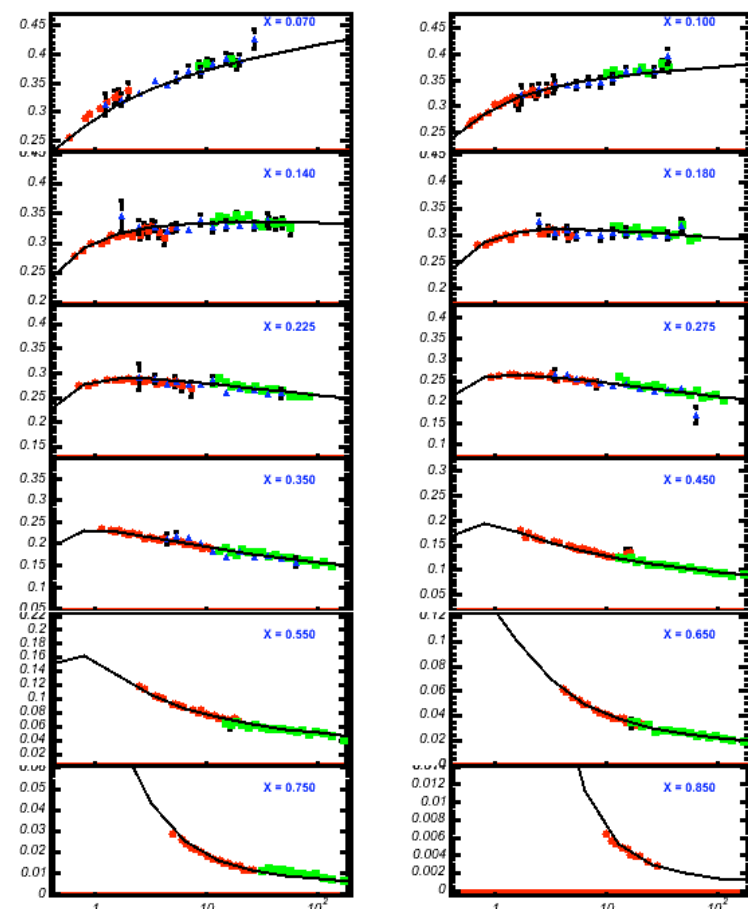


A=0.541
B=0.306
Cv1d=0.227
Cv1u=0.291
Cv2d=0.286
Cv2u=0.189
Cs1d=0.619
Cs1u=0.363

nSLACd=0.990
nBCDMSp=0.967
nBCDMSd=0.987
nNMCp=1.003
nNMCd=0.994
nH1p=0.978
aLambda=2.052
fPDF=1.014

Proton experiment data fit

SLAC
BCDMS
NMC
GRV98(LO+HT)



A=0.541
B=0.306
Cv1d=0.227
Cv1u=0.291
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Deuteron experiment data fit

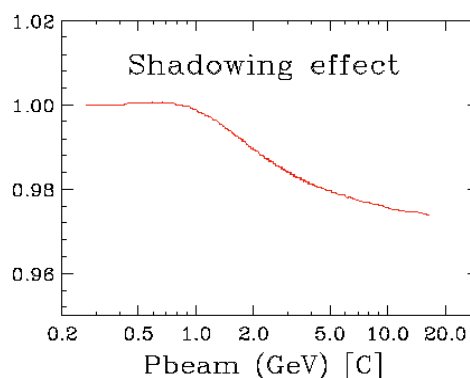
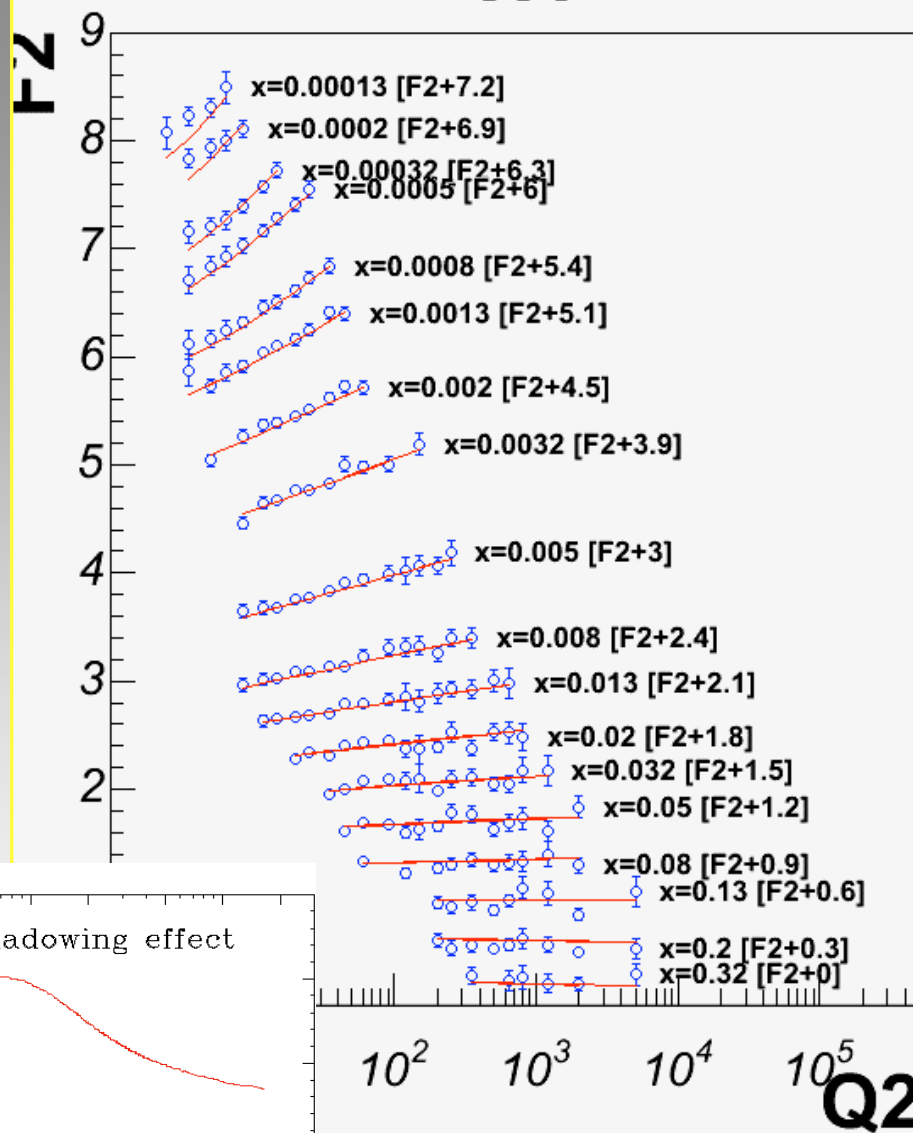
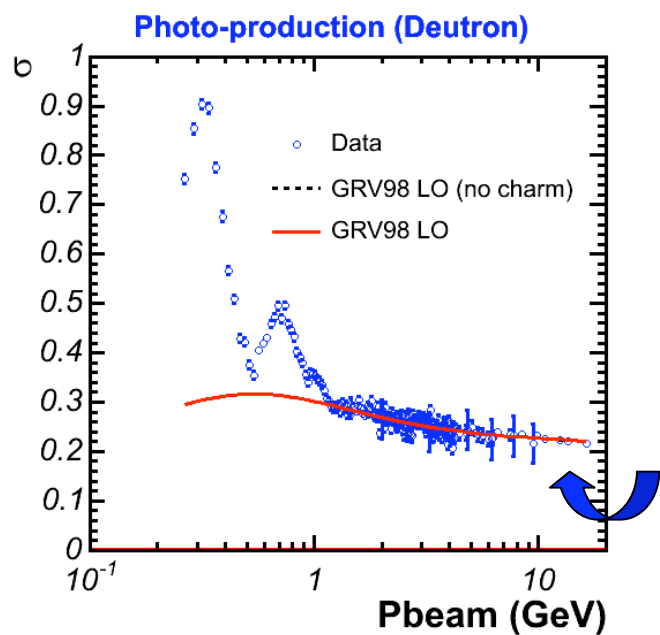
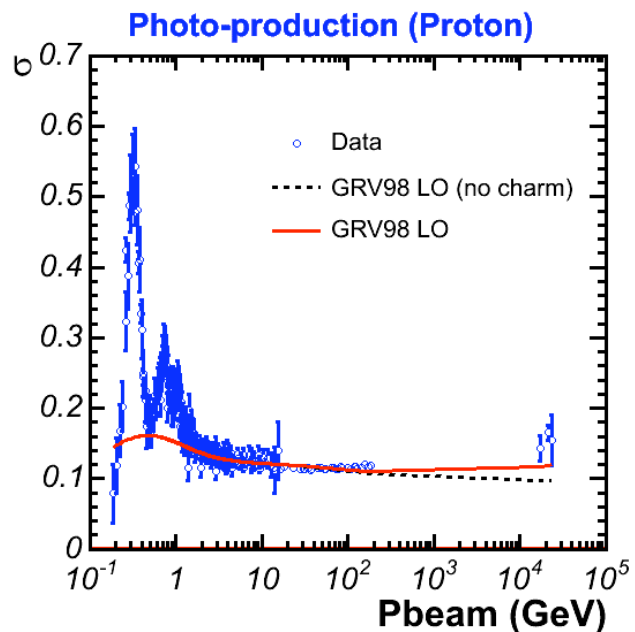
SLAC
BCDMS
NMC
GRV98(LO+HT)

F2 proton

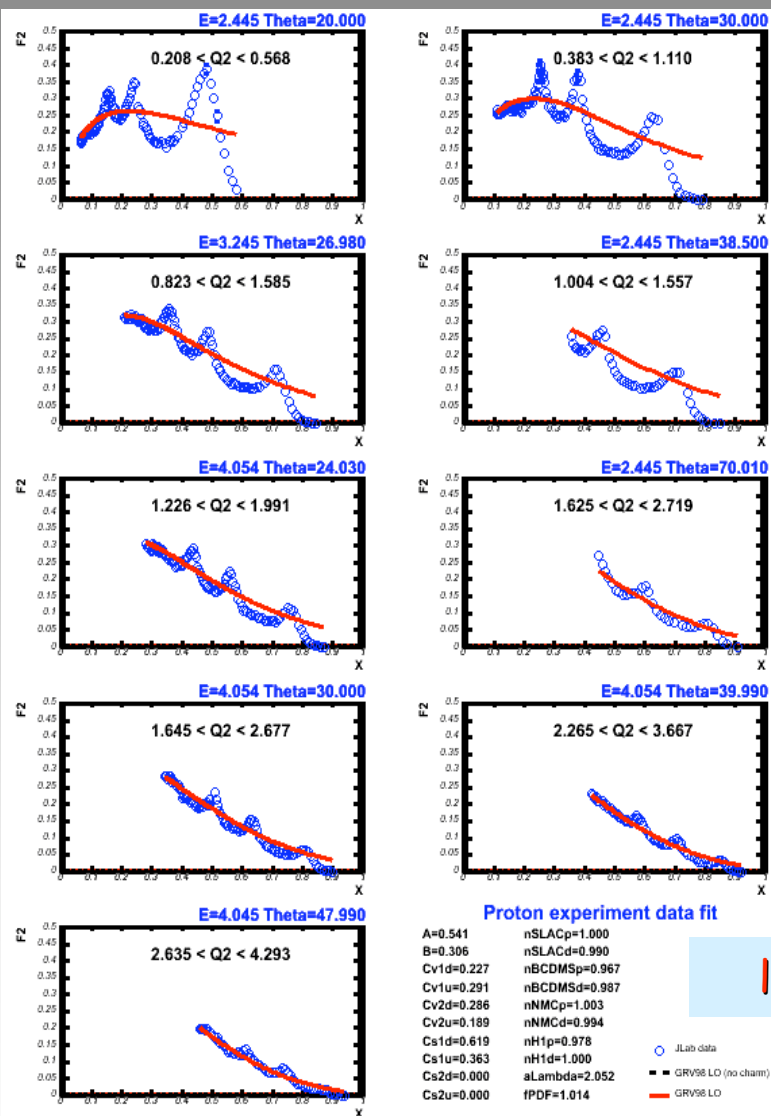
Bodek-Yang - 2010

F2 deuterium

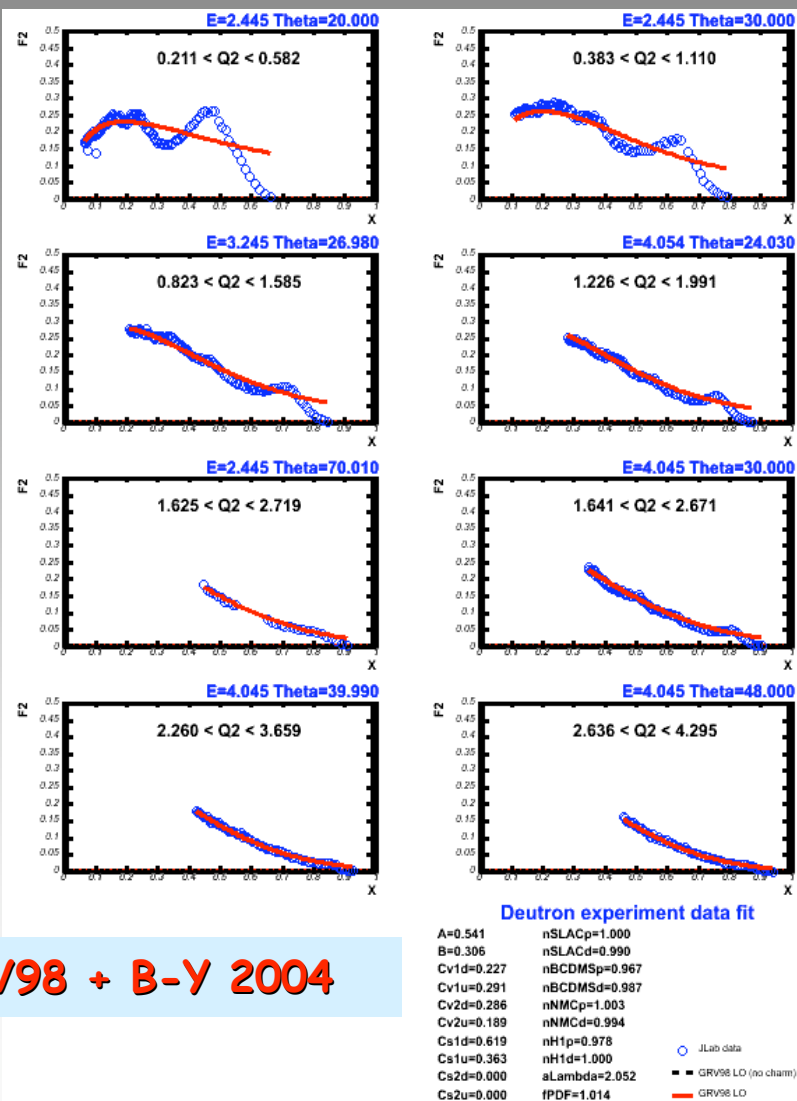
H1



Resonance F2 proton



Resonance F2 deuterium



Fit works on resonance region -Resonance data are not included in the fit!!!

How model uses only H and D data.

For lepton/muon cross sections on nuclear targets - need to correct for Nuclear Effects measured in e/muon expt. Use also for neutrino expt.
(Note nuclear effects can be different for neutrinos)

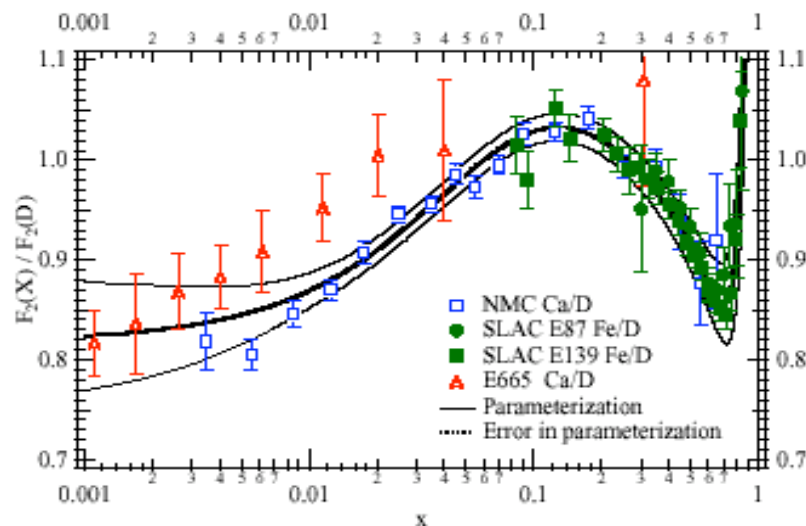
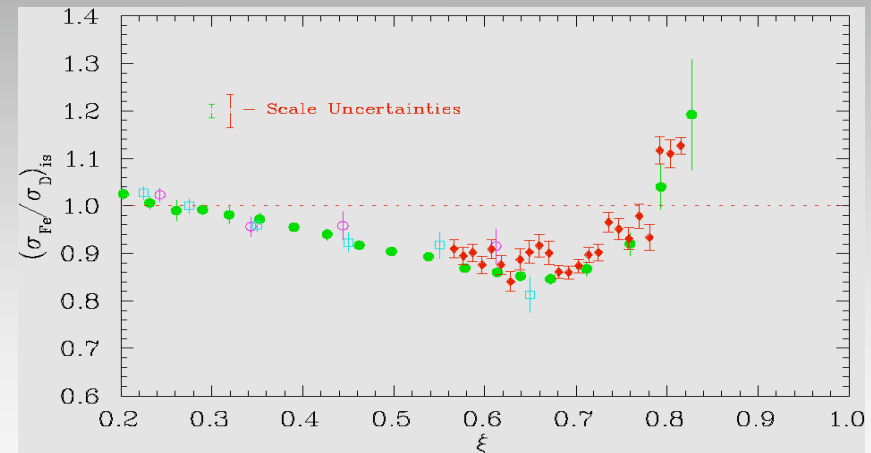


Figure 5. The ratio of F_2 data for heavy nuclear targets and deuterium as measured in charged lepton scattering experiments (SLAC, NMC, E665). The band shows the uncertainty of the parametrized curve from the statistical and systematic errors in the experimental data [16].



Comparison of Fe/D F_2 data
In resonance region (JLAB)
Versus DIS SLAC/NMC data
In ξ_{TM} (C. Keppel 2002).

The BY model is used for the inelastic part of the cross section

People use the 2004 Bodek-Yang model above $W=1.8$ and other other models for quasielastic, the delta, and the 1520 resonance region and matche to Bodek-Yang in the $W=1.8$ region .

- Find that predicted total neutrino and antineutrino cross sections are lower than high energy measurements (5%).
- The antineutrino to neutrino ratio is also a little low.

2009 updates:

1. Correct for the difference in scaling violations for F2 and XF3
2. Correct for Vector is not equal to Axial at low Q^2 .
3. Make model work down to $W=1.1$ to provide smoother matching

For higher precision:

Need to apply radiative corrections to F2

Need to include c-cbar sea high energy (no c-cbar sea in GRV98).

Need to account for the differences in nuclear effects between electrons and neutrinos

2009 Update 1: $H(x) = \text{NLO Correction to } xF_3$

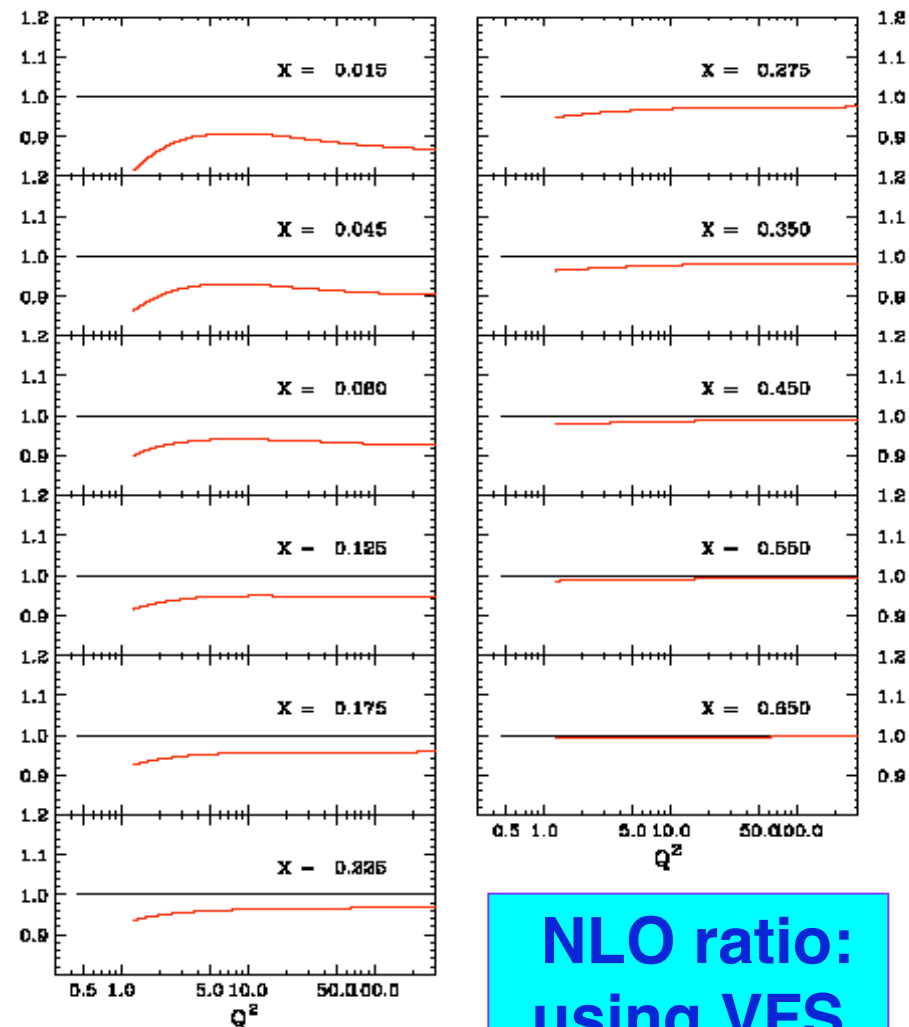
- Scaling variable, ξw absorbs higher order QCD and higher twist in F_2 , but xF_3 may be different.

(F_2 data was used in the fitting our corrections to leading order PDF)

- 1st Update: Use double ratio correction $H(x)$ from QCD

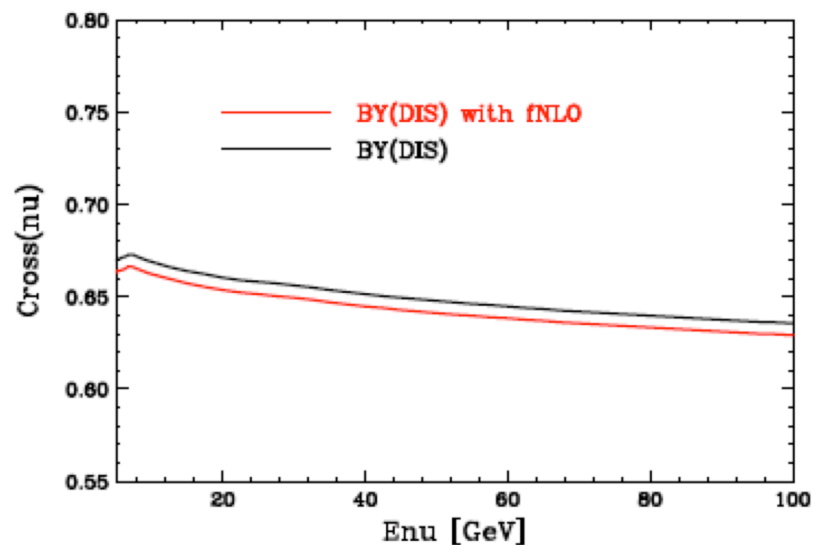
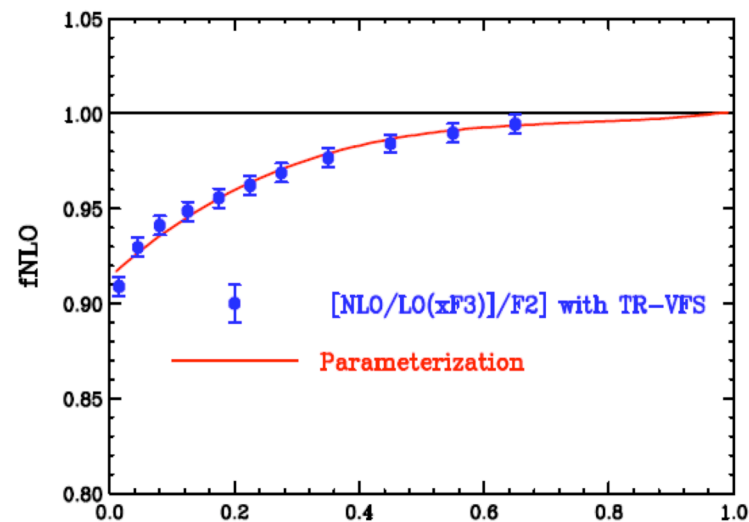
$$\frac{xF_3(\text{NLO})}{xF_3(\text{LO})} / \frac{F_2(\text{NLO})}{F_2(\text{LO})}$$

=> not 1 but indep. of Q^2

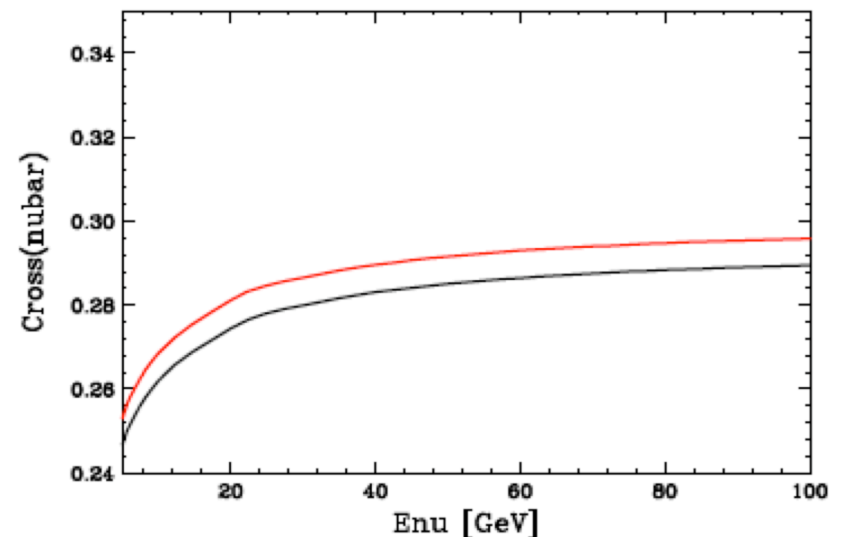


**NLO ratio:
using VFS**

Effect of $x F_3$ NLO correction $H(x)$



- Parameterized $x F_3$ correction as a function of $x = H(x)$
- Neutrino cross section down by 1%
- Anti-neutrino cross section up by 3%



2009 update 2: Axial contribution at low Q²

$$K_i^{\text{vector}}(Q^2) = \frac{Q^2}{Q^2 + C}, \quad K_i^{\text{axial}}(Q^2) = \frac{Q^2 + C_1}{Q^2 + C_2},$$

$$F_2^\nu(x, Q^2) = \sum_i \left[K_i^{\text{vector}}(Q^2) + K_i^{\text{axial}}(Q^2) \right] \times \xi_w \left[q_i(\xi_w, Q^2) + \bar{q}_i(\xi_w, Q^2) \right]$$

$$xF_3^\nu(x, Q^2) = 2 \sum_i \left[\sqrt{K_i^{\text{vector}}(Q^2) K_i^{\text{axial}}(Q^2)} \right] \\ \times H(x, Q^2) \left[x q_i(\xi_w, Q^2) - x \bar{q}_i(\xi_w, Q^2) \right],$$

- In our neutrino previous cross section model we assumed $K^{\text{axial}} = K^{\text{vector}}$.

This is only true for free quarks (which is a correct assumption for $Q^2 > 0.5 \text{ GeV}^2$)

- However: We expect that axial-vector is not suppressed at $Q^2=0$
 - 2009 Update 2 : $K^{\text{axial}} = 1$ as a first try

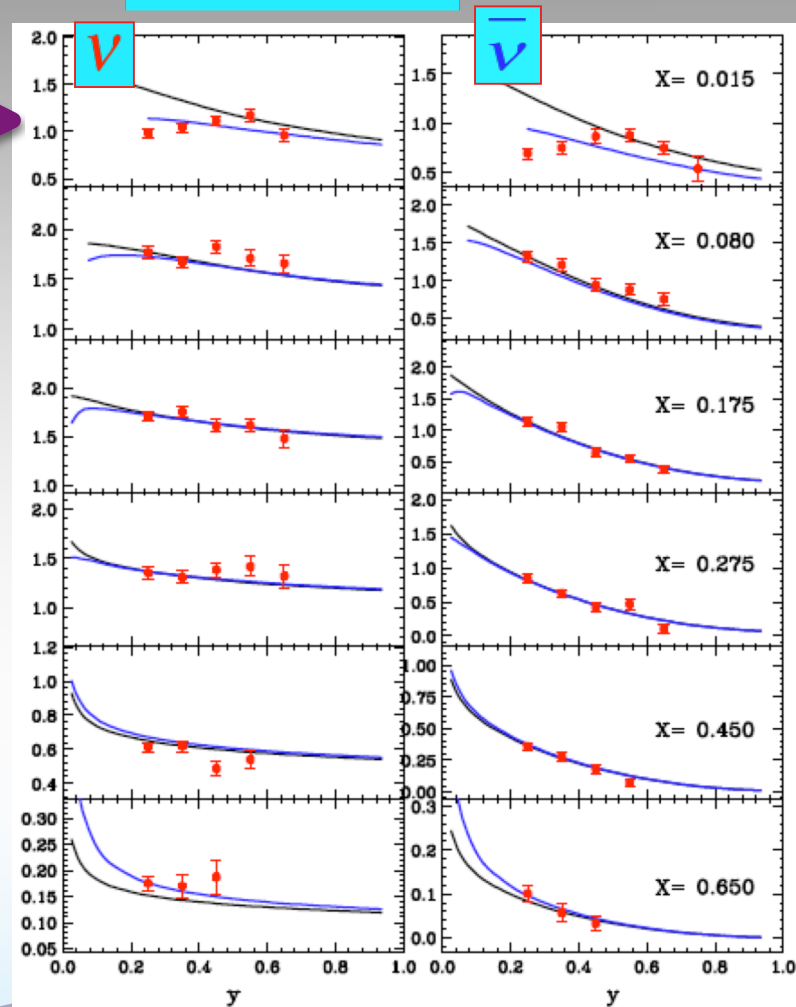
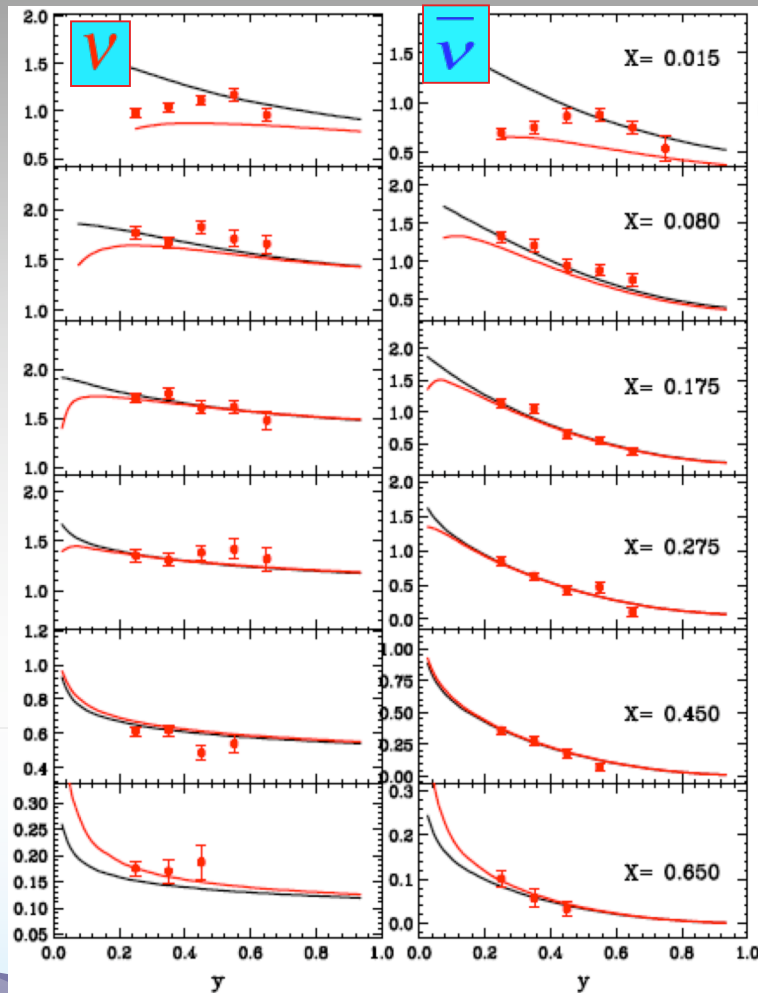
Axial-contribution

CCFR diff. cross at $E_\nu = 55 \text{ GeV}$

$$K_{\text{axial}} = Q^2/(Q^2+C)$$

$$K_{\text{axial}} = 1$$

better



Black line GRV98; red line with B-Y 2004 and $K_{\text{axial}}=K_{\text{vector}}$, blue $K_{\text{axial}}=1$

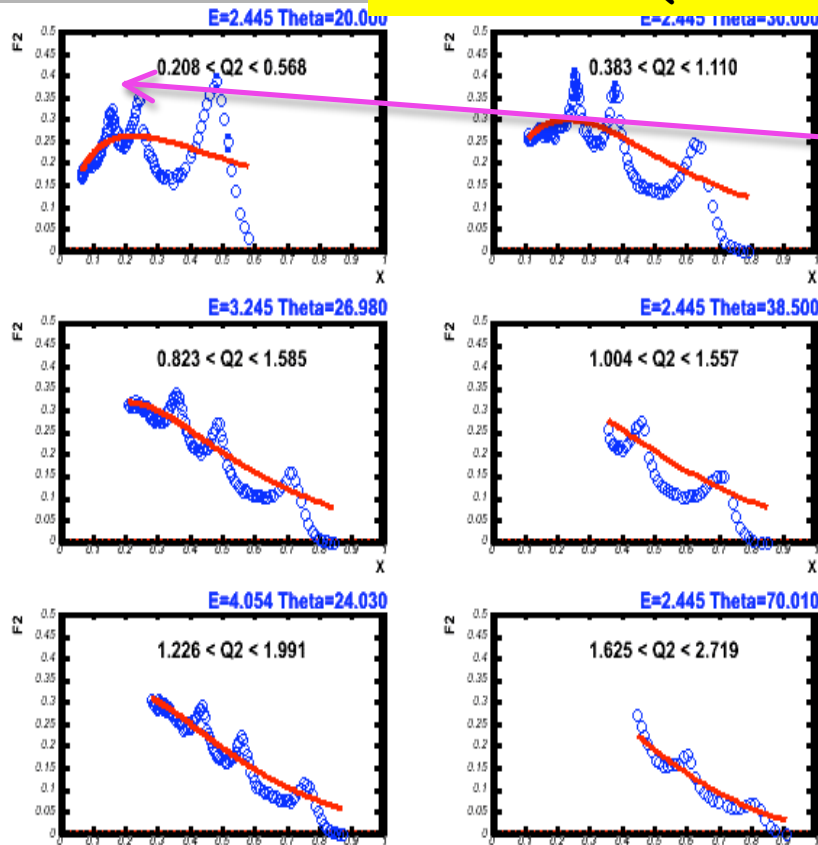
Update 3: In resonance region, duality works down to $Q^2=0.5 \text{ GeV}^2$, but breaks down at $Q^2=0$.

Not important for the Vector part, since $Q^2=0$ contributes zero to the vector part of the neutrino cross section.

$$\sigma(\gamma\text{-proton}) = 4\pi\alpha/Q^2 * F_2(\xi_w, Q^2)$$

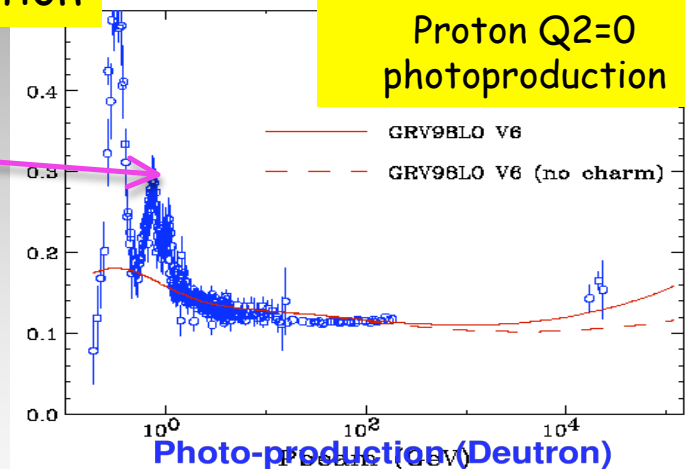
$$\text{where } F_2(\xi_w, Q^2) = Q^2 / (Q^2 + C) * F_2(\xi_w)$$

Proton low Q electroproduction

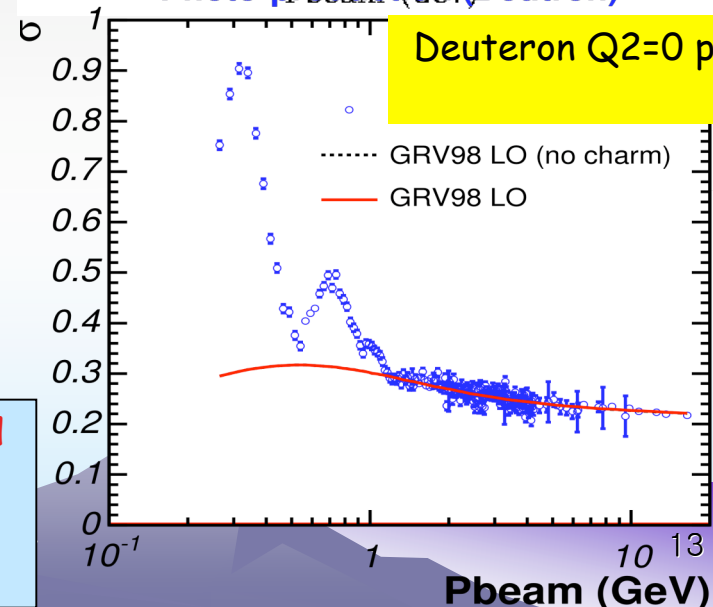


Update 3: Improve the model so that it is also valid in the resonance region at $Q^2=0$
We will fix it by applying a low Ehad K factor
Important for axial part

Proton $Q^2=0$ photoproduction



Deuteron $Q^2=0$ photo



Update (3): apply a low ν (Ehad) K factor

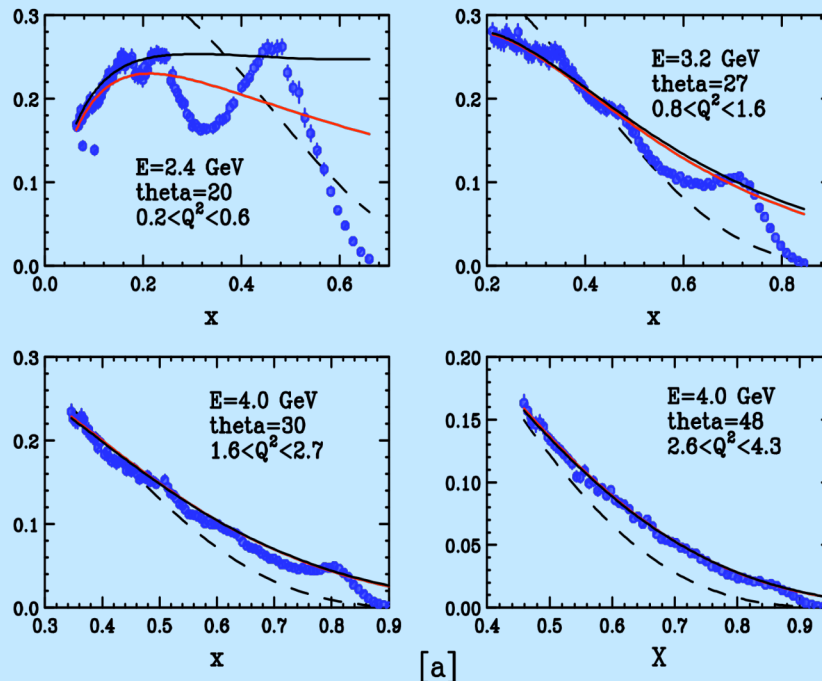
$$K(\nu) = (\nu^2 + C2\nu) / \nu^2$$

Where $C2\nu = 0.20$

It make duality work for resonance all the way to $Q^2=0$.

So vector part is now modeled everywhere including resonance region down to very low ν and very low Q^2 .

Deuteron Resonances Electroproduction

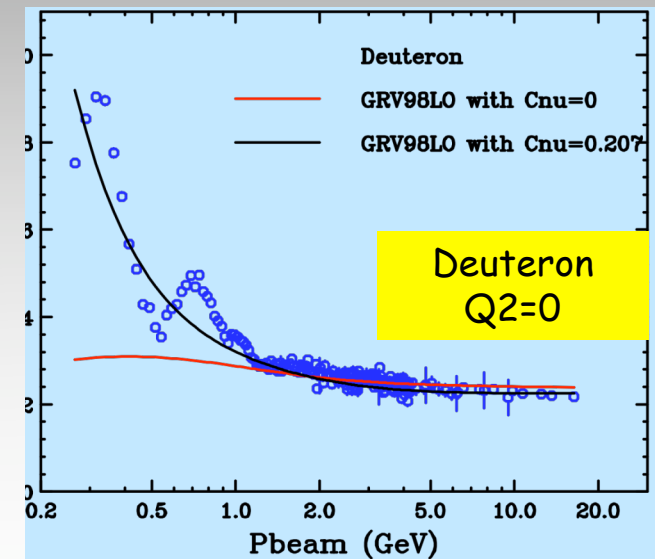


$$\sigma(\gamma\text{-deuteron}) = 4\pi\alpha/Q^2 * F_2(\xi_w, Q^2)$$

where $F_2(\xi_w, Q^2)$

$$= Q^2 / (Q^2 + C) * F_2(\xi_w)$$

Photo-production $Q^2=0$



Black line includes Low Ehad K factor

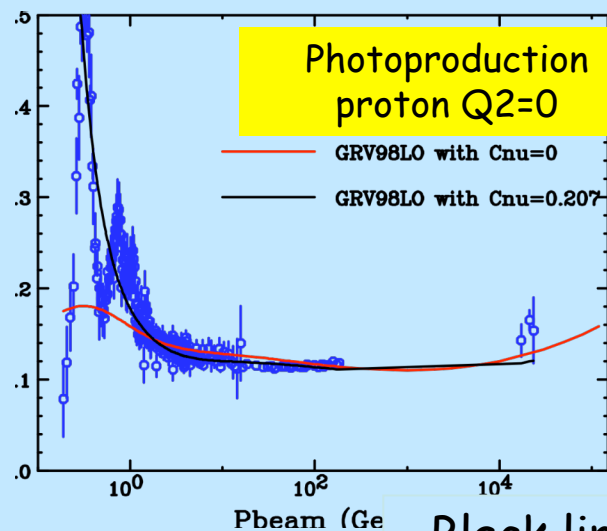
Red line - no low ν (Ehad) K factor

For a heavy nucleus, Fermi motion will smear all of the resonances

Low Ehad $K(\nu)$ factor extends the validity of the model for electron scattering in the resonance region down to $Q^2=0$

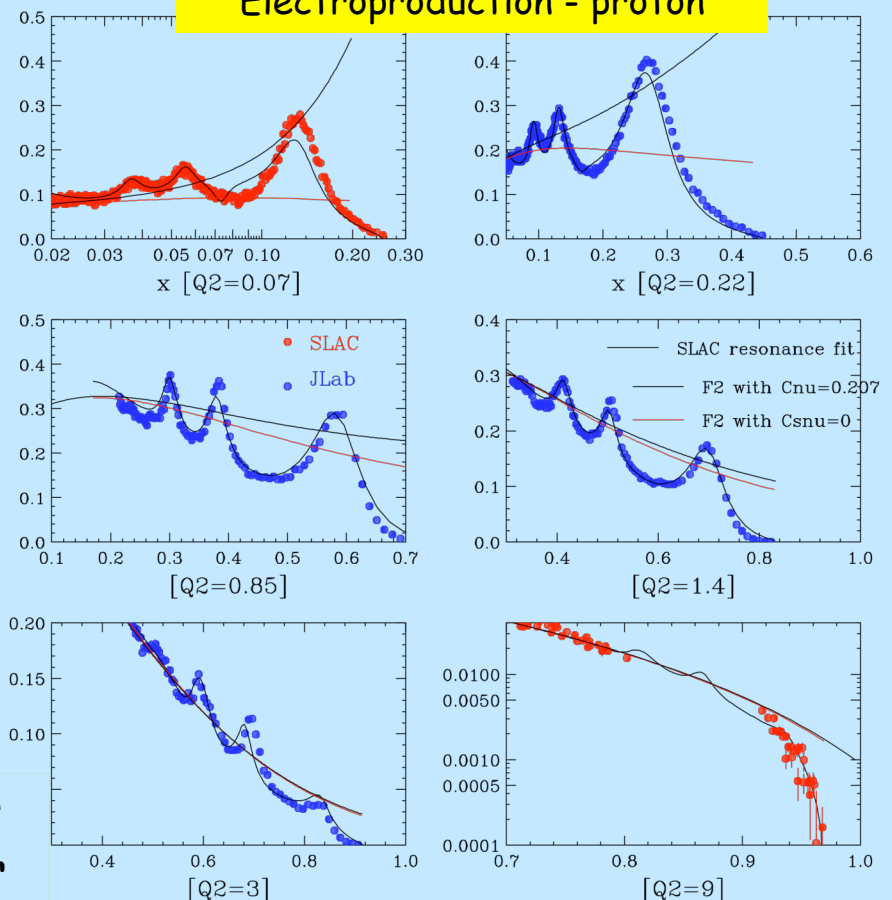
- Proton data
- (note that for nuclear targets the resonances will be smeared by Fermi Motion)

Photo-production $Q^2=0$



Black line includes
Low Ehad K factor
Red line no Low ν K
factor

Electroproduction - proton



Summary and Discussions

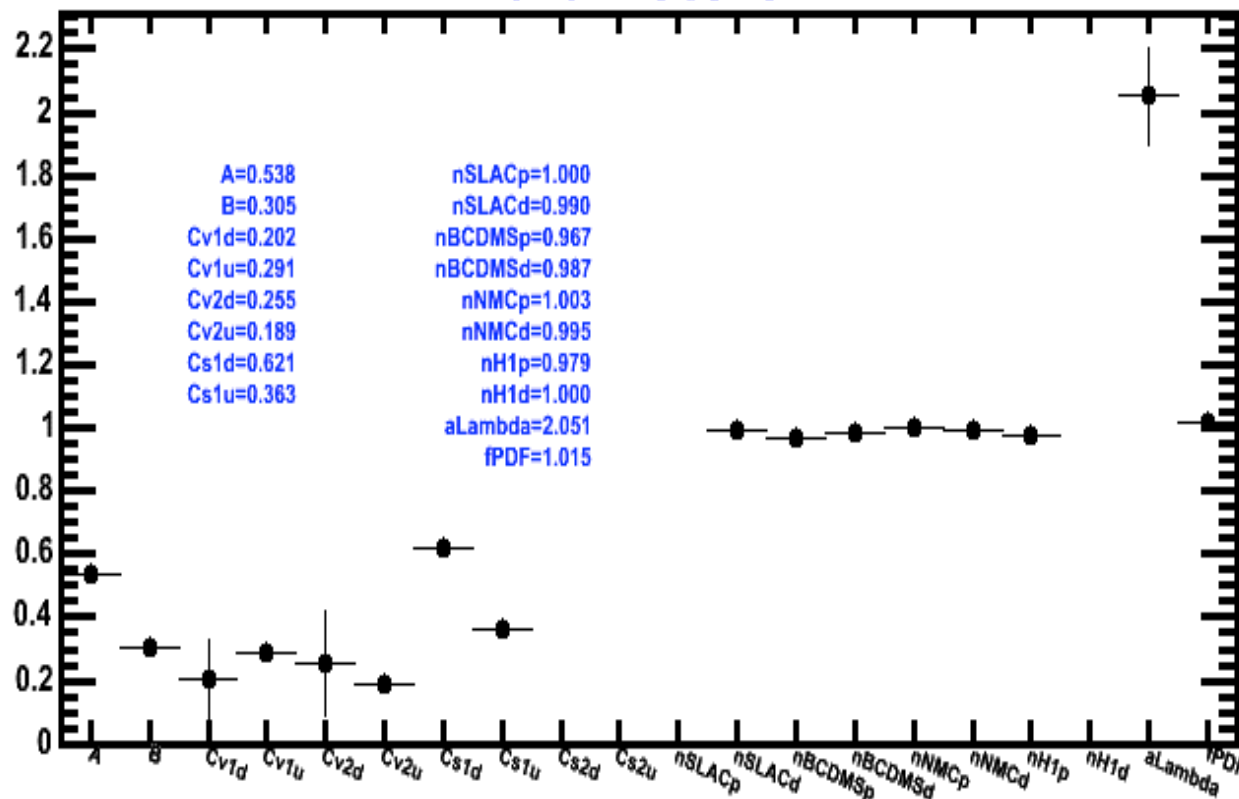
- We updated our Effective LO model with ξ_w and $K(Q^2)$ factors.
- (1) Updated to account for the difference in the higher order QCD corrections between F2 and XF3. This is accounted for with a $H(x)$ factor. Therefore, the axial part is also well described for $Q^2 > 1 \text{ GeV}^2$, where axial and vector are expected to be the same
- (2) Updated to use $K_{\text{axial}}(Q^2)=1$ for both the resonance and inelastic continuum region. This is expected since we know that neutrino quasielastic and resonance production form factor are not zero at $Q^2=0$.
- The lowest Q^2 bins in the neutrino and antineutrino measured differential cross sections favor $K_{\text{axial}}(Q^2)=1$. The total cross section as measured in high energy neutrino scattering favors $K_{\text{axial}}(Q^2)=1$. --> To be investigated in MINERvA
- (3) Updated to include a low E_{had} factor $K(\nu)$ to describe all charged lepton inelastic continuum as well as all resonance data including photo-production data down to $Q^2=0$. The vector part of the neutrino cross section is now modeled very well. Note: By Gauge Invariance, the vector structure functions must go to zero at $Q^2=0$ for both resonances and inelastic continuum.

Backup-slides

GRV98 + B-Y 2004 Fit results

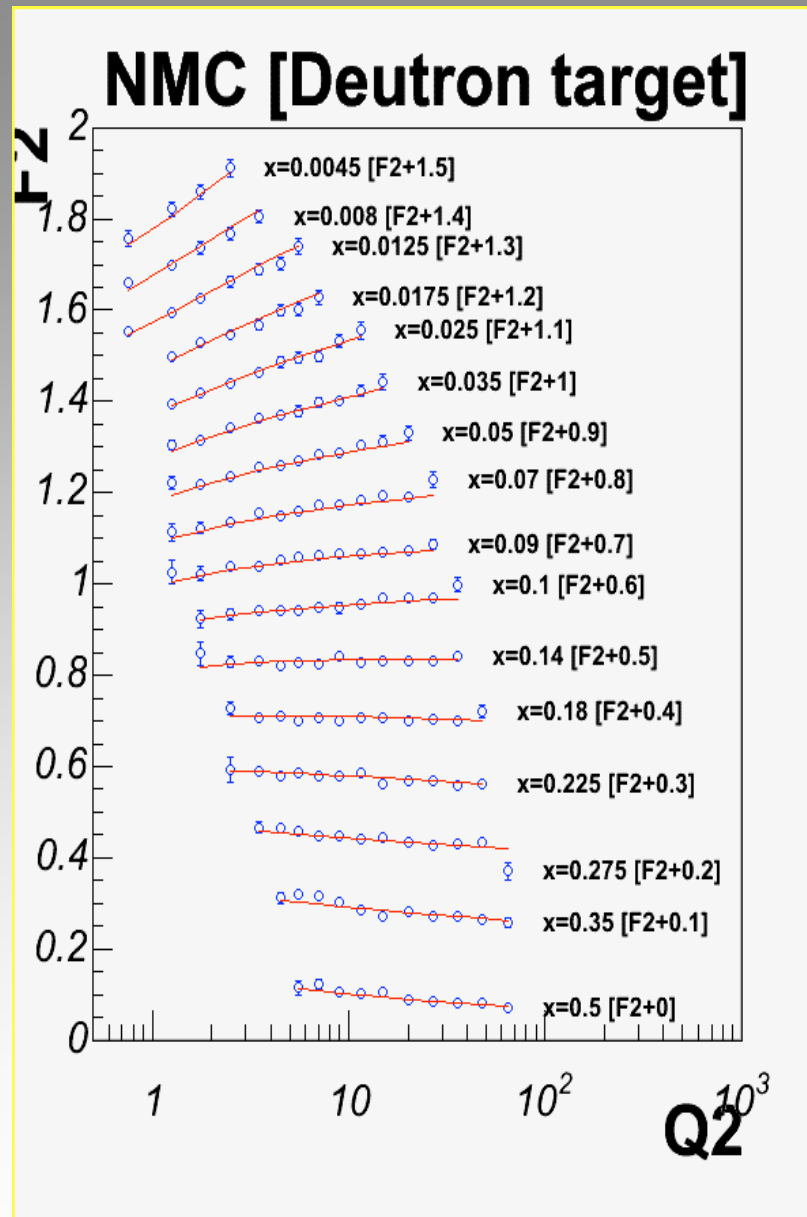
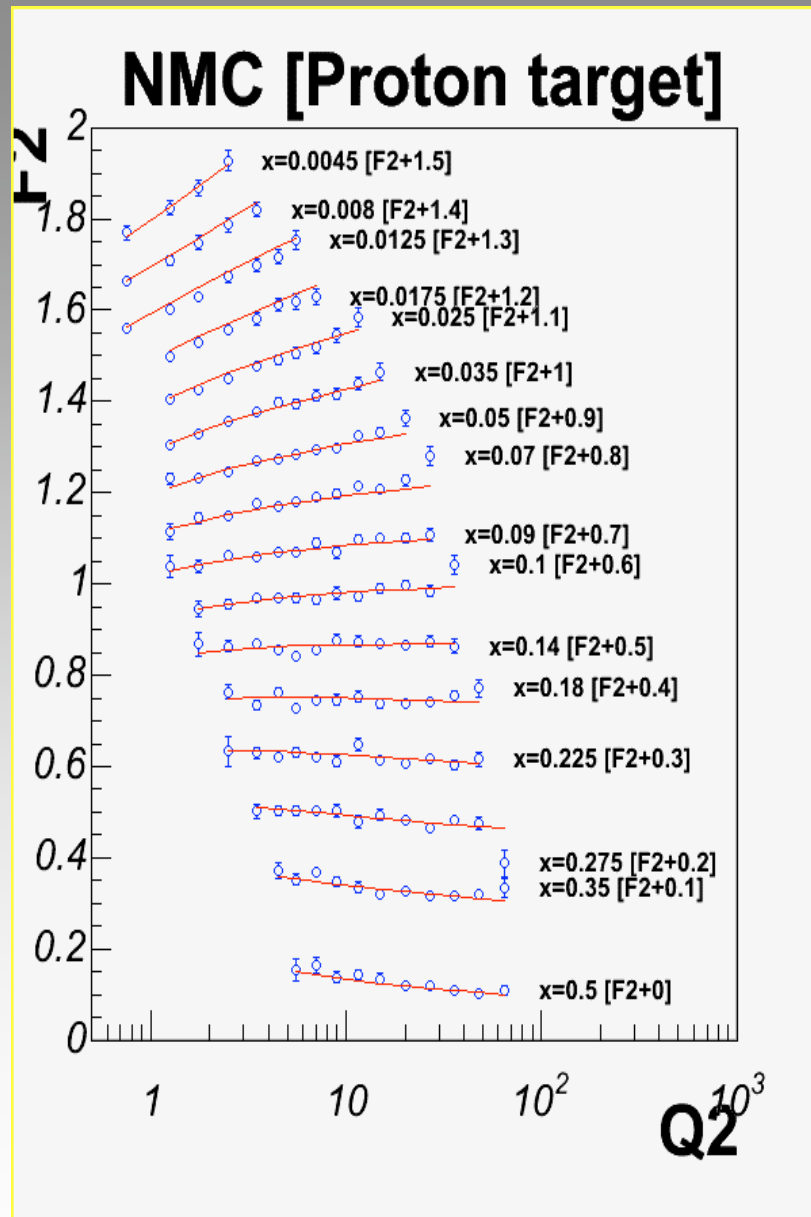
separate K factors
for uv, dv,us,ds

Parameters



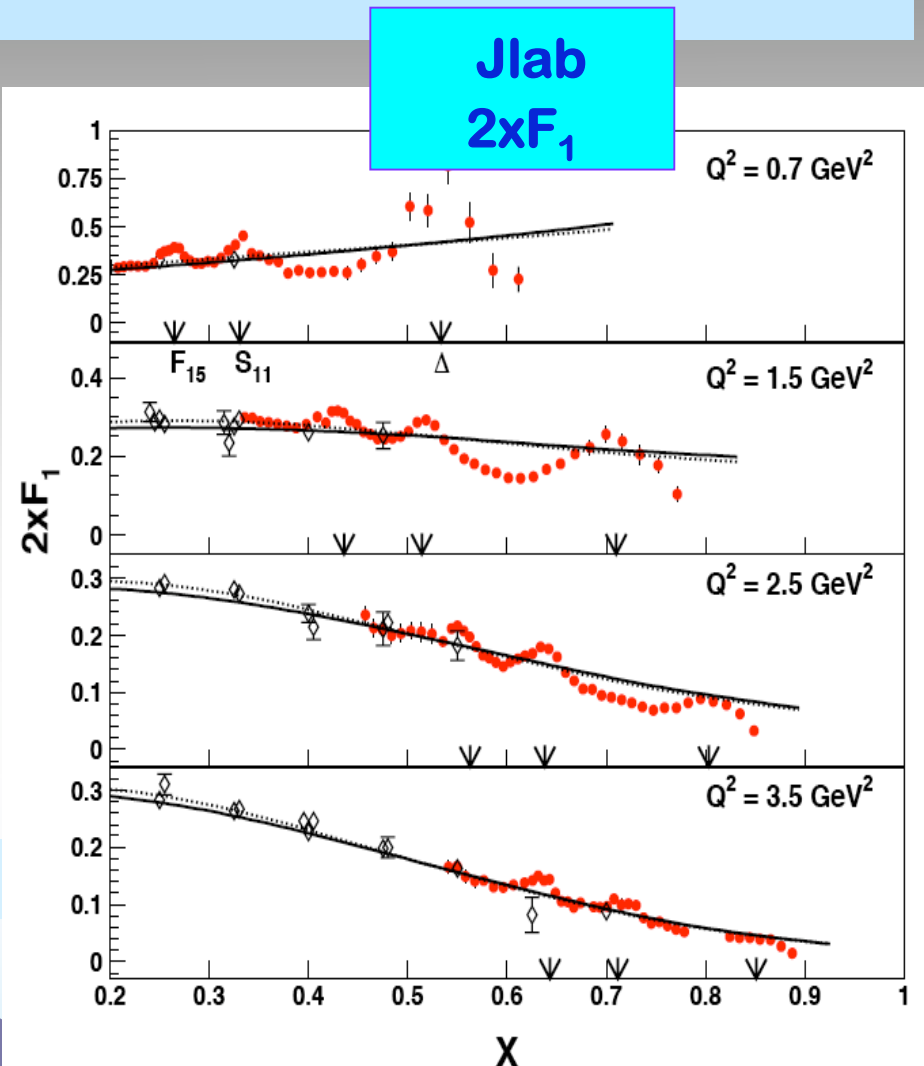
Separate K factors
for uv, dv,us,ds
provided additional
parameters. They
provide separate
tuning for H
and D data, but
are not important for
Heavy nuclei.

Fit results GRV98 + B-Y 2004 muon scattering

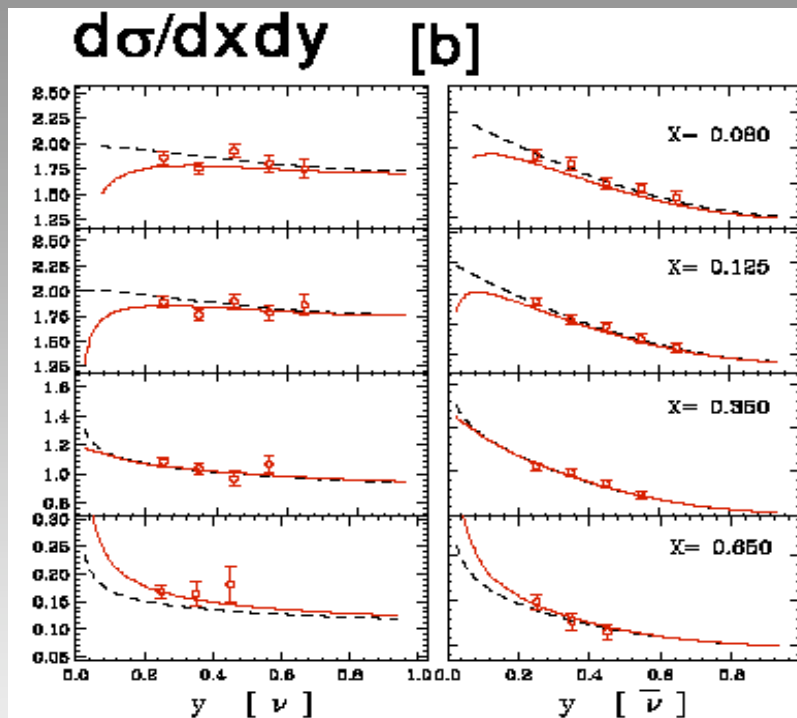


$2xF_1$ data

- All DIS e/μ F_2 data are well described
- Photo-production data ($Q^2=0$) also work: thus included in the latest fit
- $2xF_1$ data (Jlab/SLAC) also work:
using $F_2(\xi_w) + R1998$
line GRV98 + B-Y 2004



Comparison with CCFR neutrino data (Fe) (assume $V=A$) apply nuclear corrections



-- ξ_w PDFs GRV98 modified (red line)

---- GRV98 (x, Q^2) unmodified (black)

Left: CCFR neutrino data -55 GeV

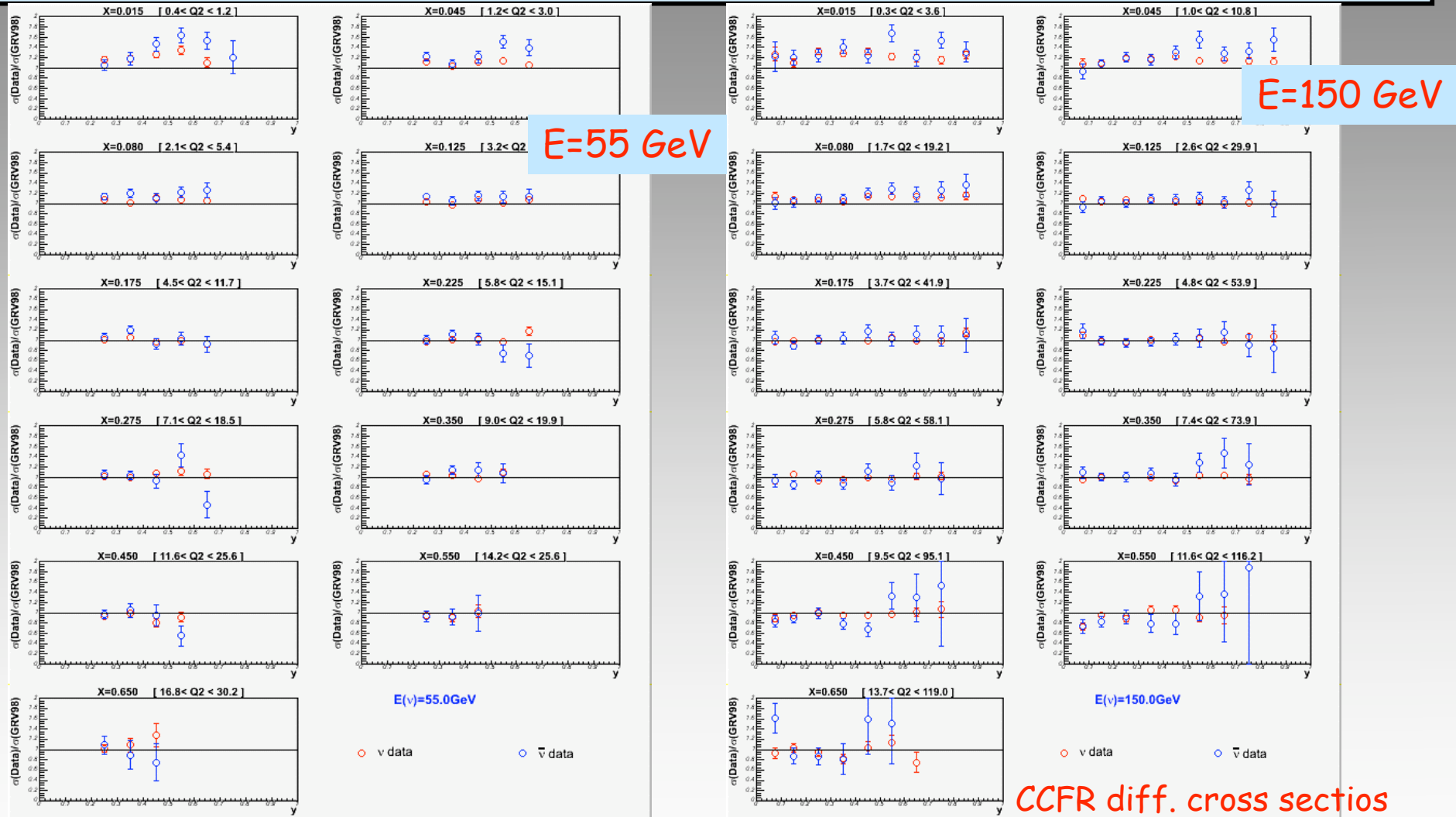
Right : CCFR anti-neutrino data , -55 GeV
(NuFact03 version)

- Apply nuclear corrections using e/m scattering data.
- Calculate F_2 and xF_3 from the modified PDFs with ξ_w
- Use $R=R_{\text{world}}$ fit to get $2xF_1$ from F_2
- Implement charm mass effect through ξ_w slow rescaling algorithm, for F_2 , $2xF_1$, and xF_3

Red line GRV98 + B-Y 2004

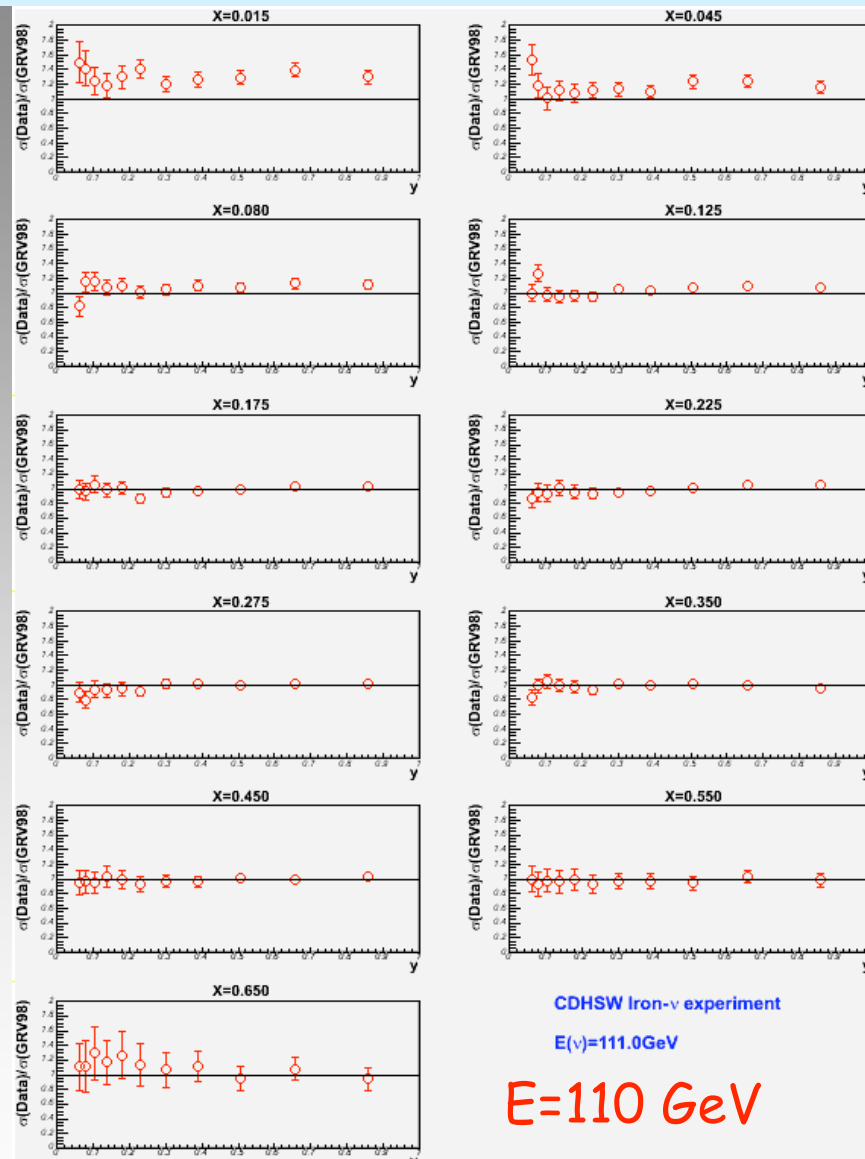
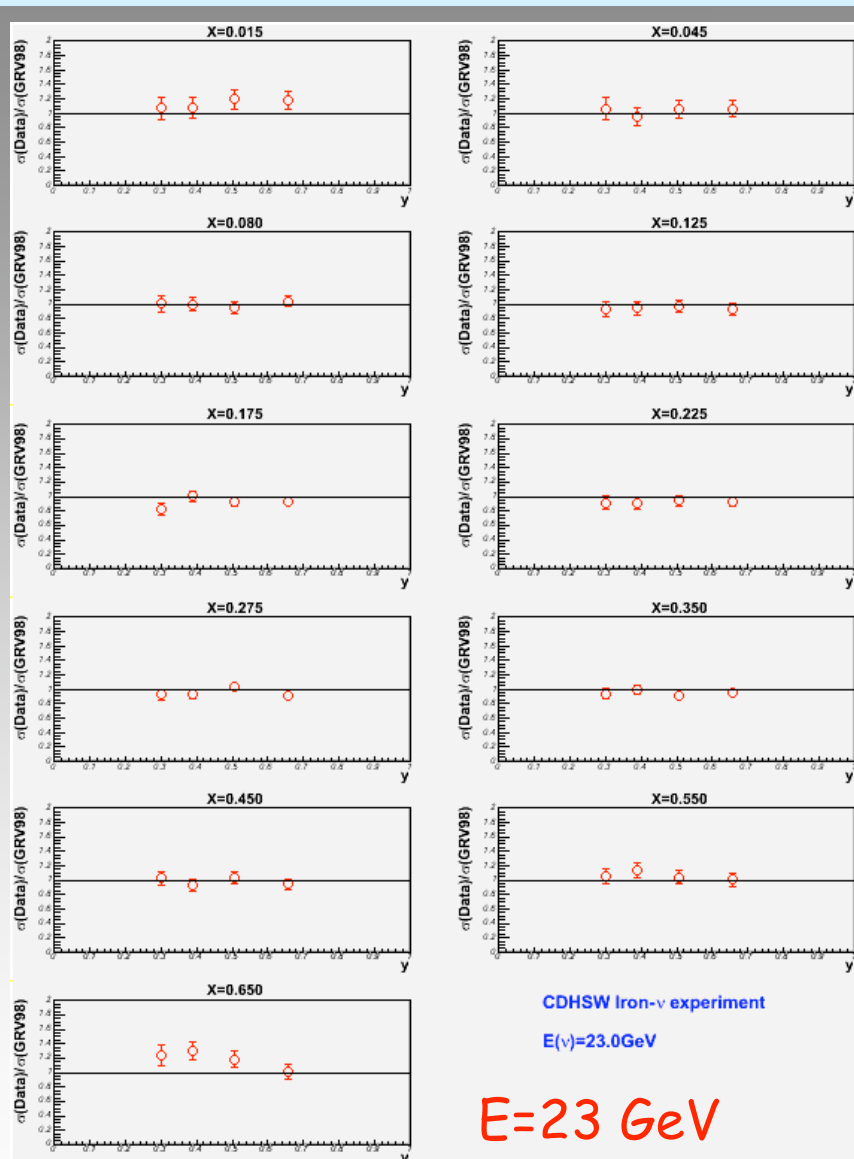
Our model describes **CCFR** diff. cross sect. ($E_\nu=30-300$ GeV) well
Note that no neutrino data was included in fit. (However, Lets look in more detail).

Comparison with updated model (assume V=A) CCFR Fe data/ (GRV98 + B-Y 2004)



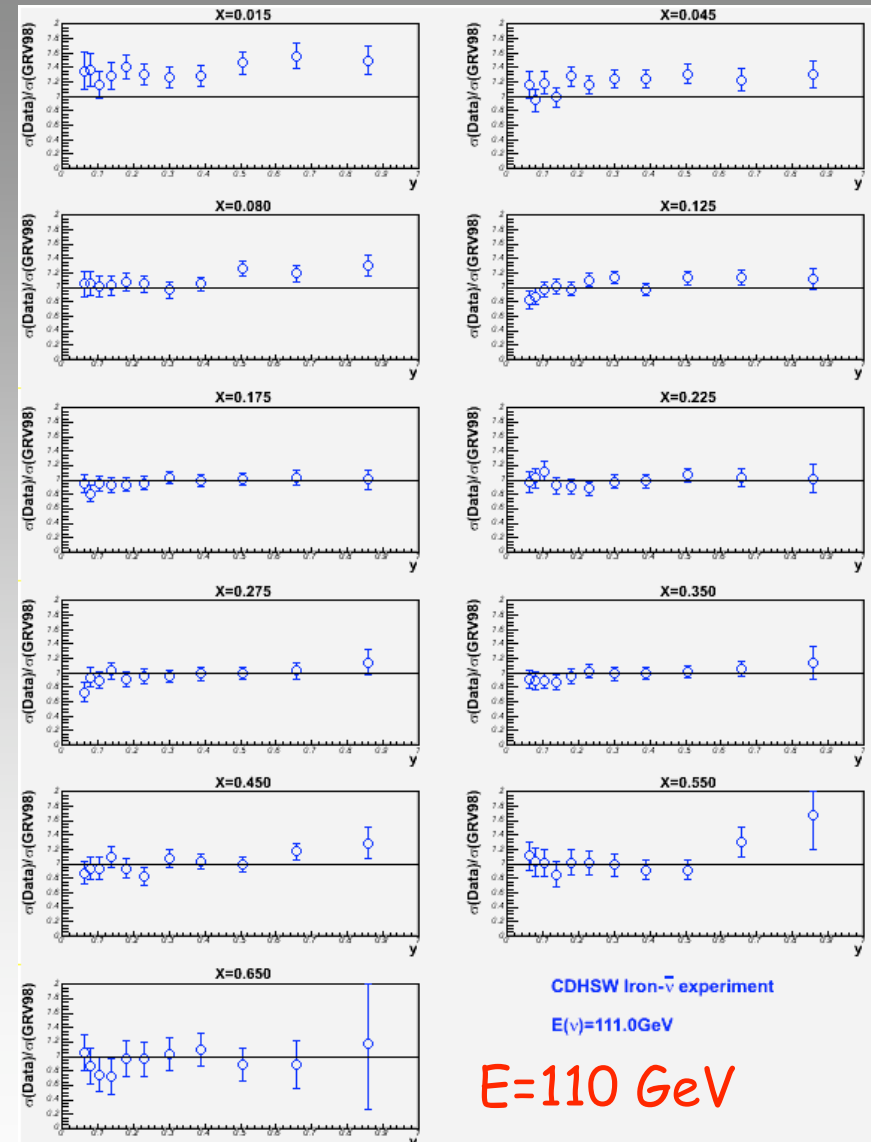
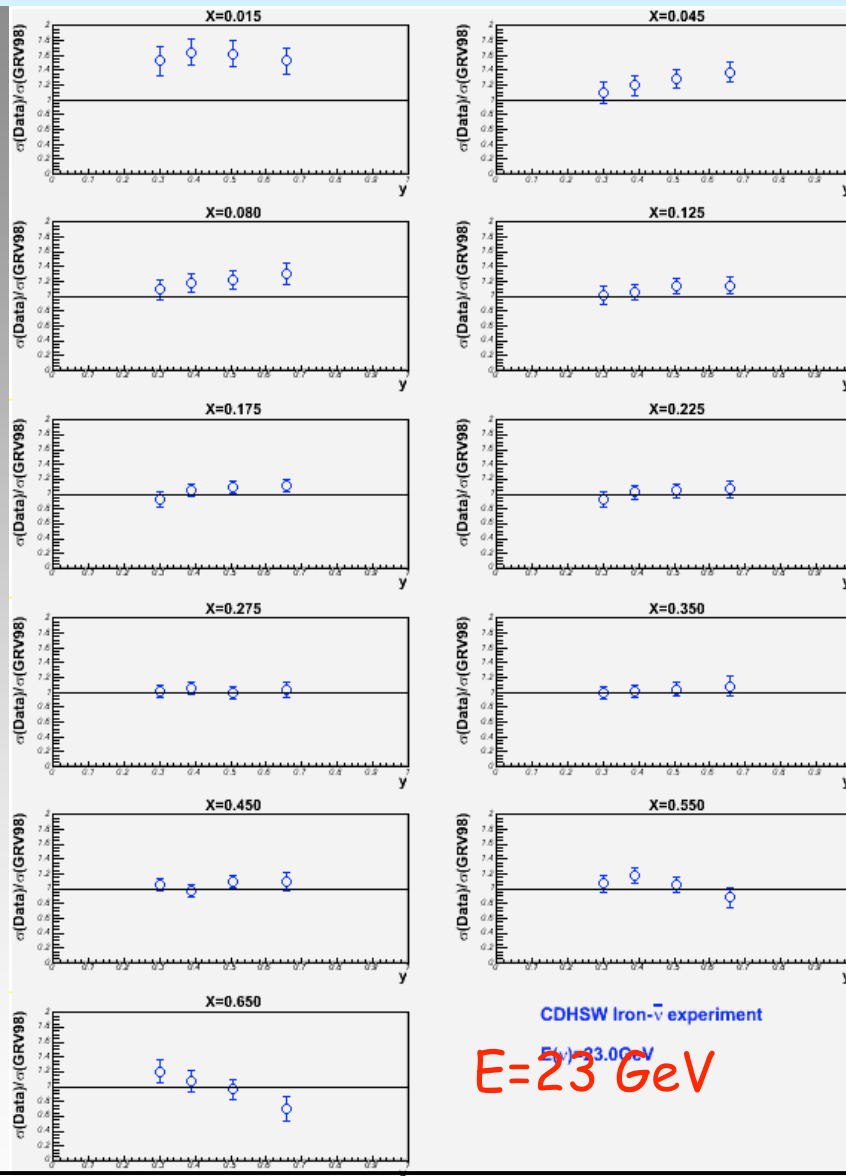
Model underestimates neutrino data at lowest x bin. At high energy, some may be from missing radiative corrections and c-bar contribution

Comparison with CDHSW neutrino data (Fe)



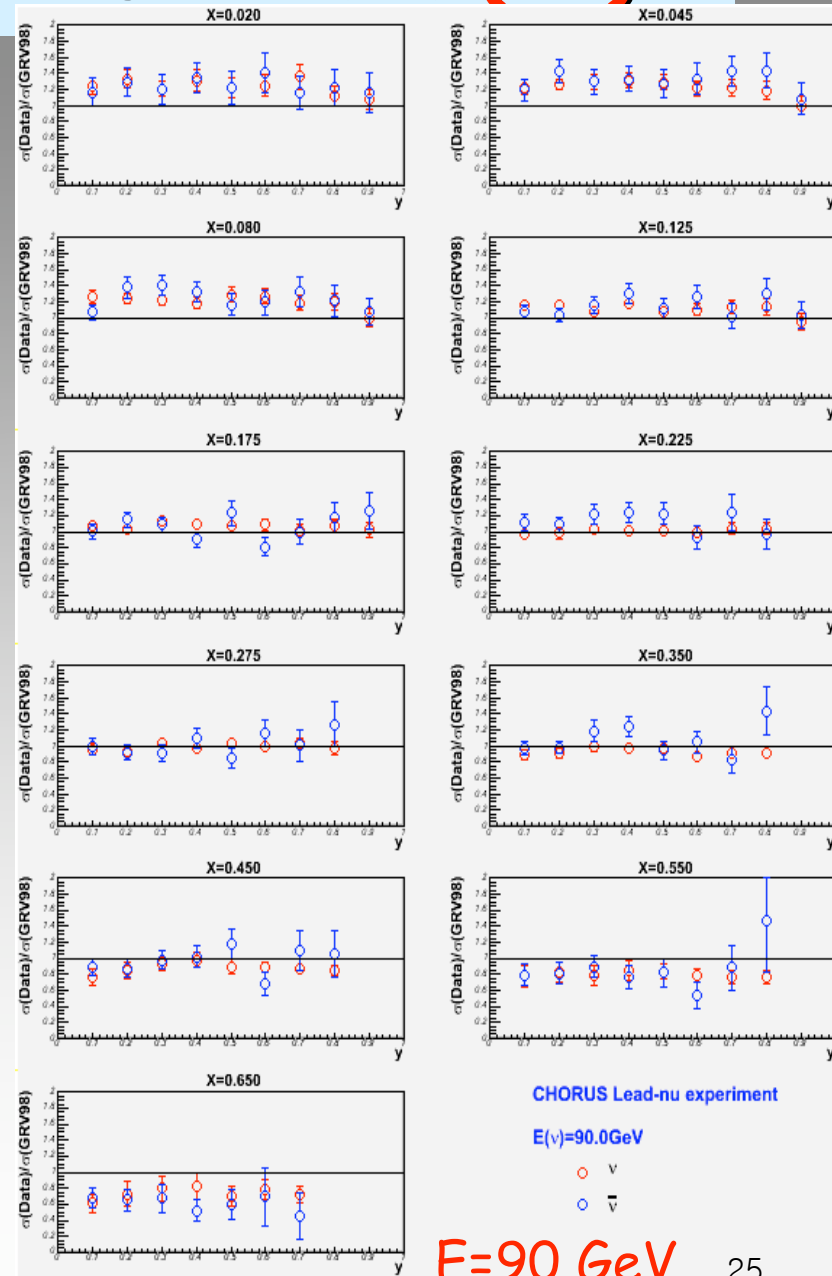
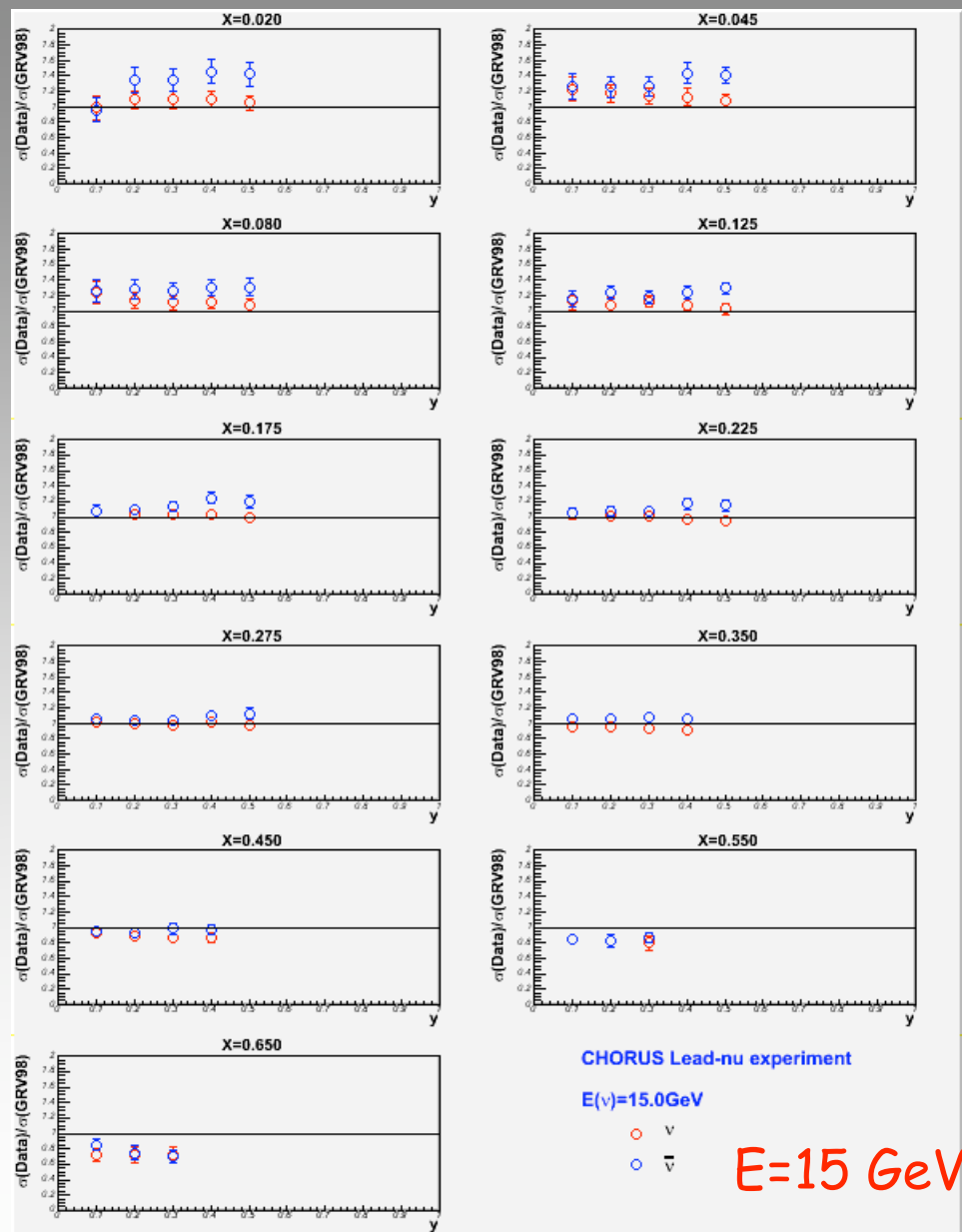
Model underestimates neutrino data at lowest x bin. At high energy, some may be from missing radiative corrections and c-bar contribution

Comparison with CDHSW anti-neutrino data (Fe)



Model underestimates antineutrino data at lowest x bin -also lowest Q^2 . At high energy, some may be from missing radiative corrections and c-bar contribution

Comparison with CHORUS data (Pb)



How is the model be used

- **Duality is not expected to work for quasielastic or the delta**
This is because these cross section have definite isospin final states. Therefore PDFs will not give the correct ratio of neutrino vs antineutrino and proton versus neutron scattering for quasielastic and delta production.
- **Duality should work in the region of higher resonances since these regions include several resonances with different isospins..**
- People use the 2004 Bodek-Yang model above $W=1.8$
- They used other models for quasielastic, the delta, and the 1520 resonance region and matched them to Bodek-Yang in the $W=1.8$ region .
- 2009 -> make it work in the delta region

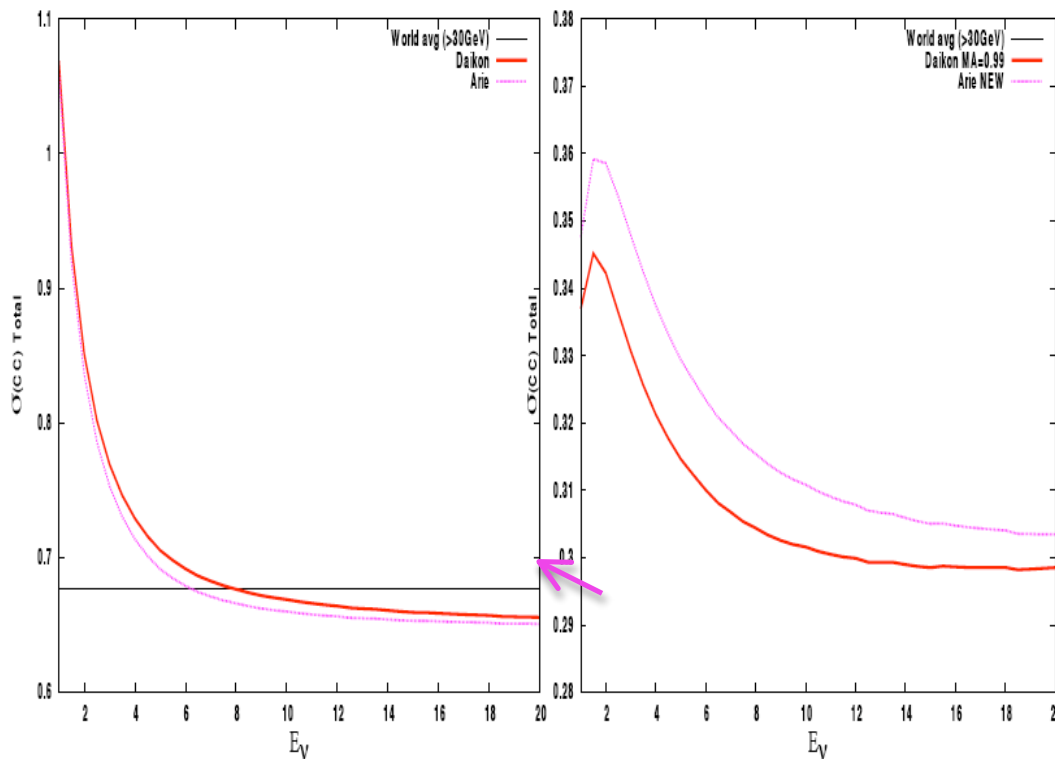
Total cross sections
Bodek-Yang 2004 used above $W=1.8$ AND
matched to resonance and quaselastic models.

Find that predicted total neutrino and antineutrino cross sections
are lower than high energy measurements (5%). The antineutrino
to neutrino ratio is also low.

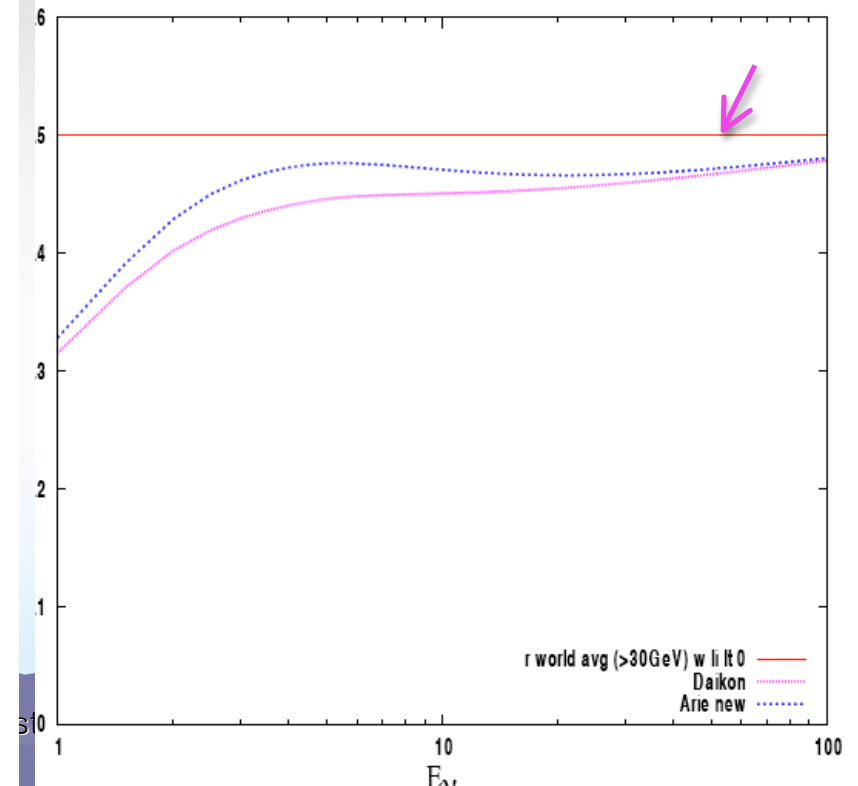
Some may come from the need to apply radiative corrections and include the c - \bar{c} sea at
very high energy (no c - \bar{c} sea in GRV98). Some may be differences in nuclear effects
between electrons and neutrinos- But is this all?



Total Cross Sections



$$r = \sigma^{\bar{\nu}} / \sigma^{\nu}$$



Note: GRV98 + B-Y 2004 is for free nucleons (H+D). Electron and muon data are corrected for radiative corrections. In addition, GRV98 has no charm sea.

Published neutrino differential cross sections:

- (1) Have no radiative corrections
- (2) Are on nuclear targets
- (3) Have contributions from XF3 and include both axial and vector contributions.
- (4) Some are at very high energy which include a contribution from the charm sea.

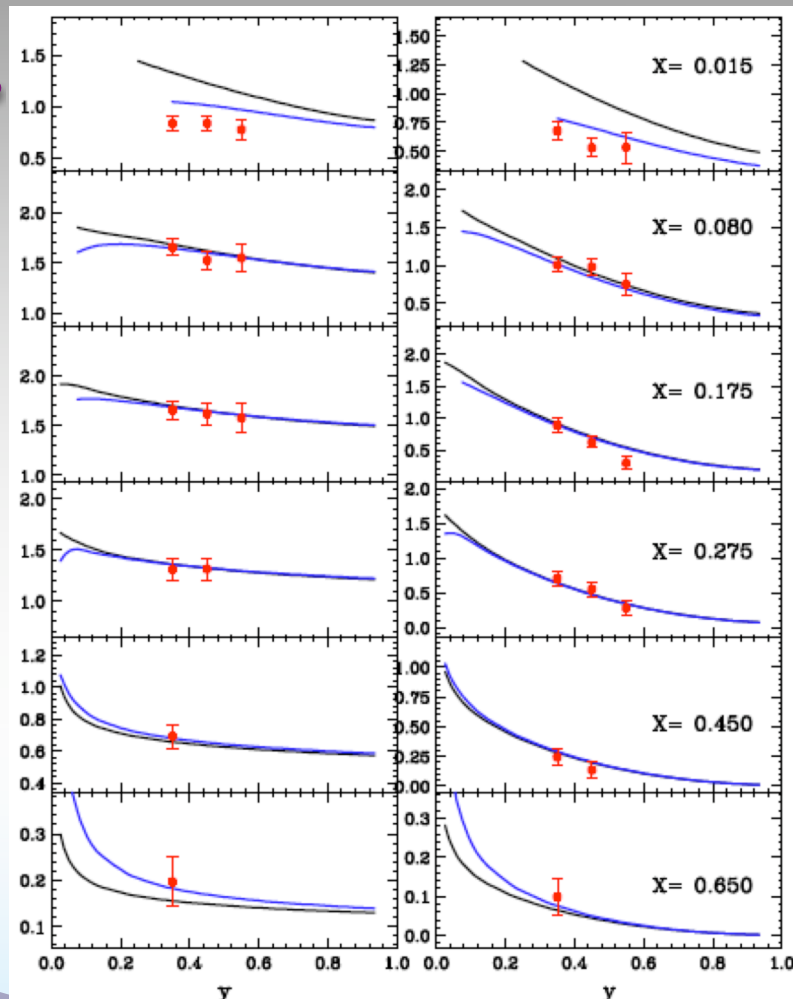
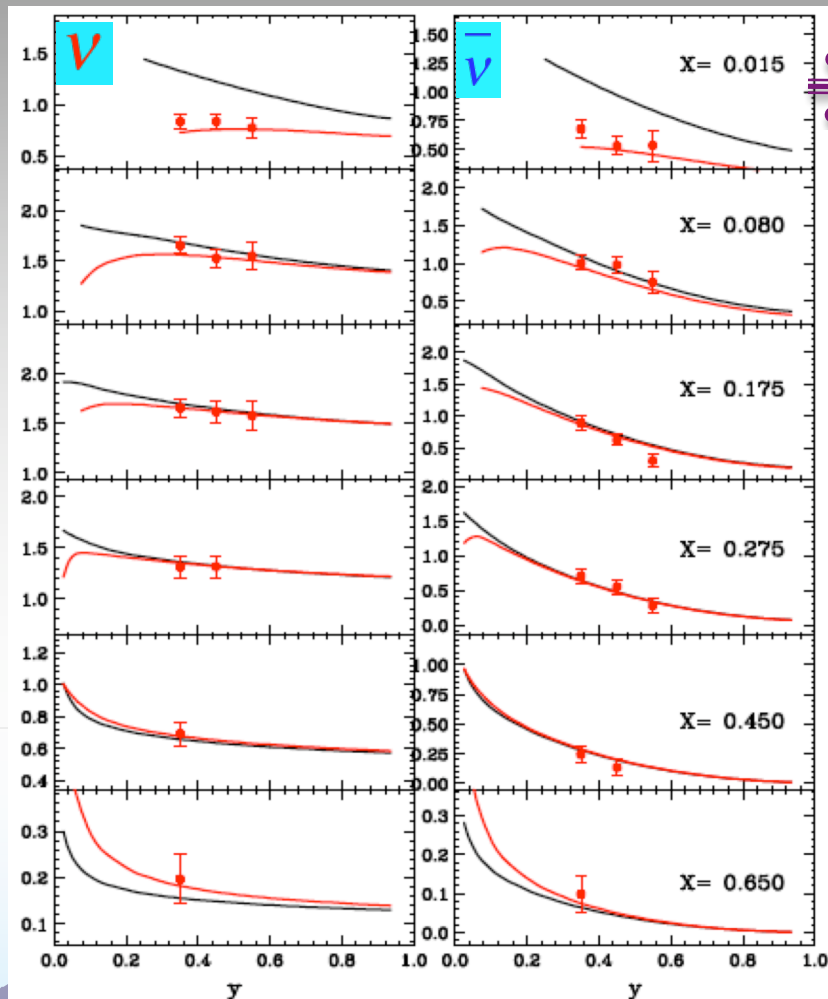
In order to compare to neutrino data:

- (1) We need to account for difference in the scaling violations in XF3 and F2 (2009 update 1)
- (2) We need to make duality work in the resonance region at very low Q^2 if we want to match to the resonance region, (2009 update 2)
- (3) We need to account for difference in axial and vector structure functions at low Q^2 (2009 update 3)
- (4) We need to apply an X dependent nuclear correction to convert from H-D to nuclei.
 - (1) However, nuclear effects may be different for muons and neutrinos, different for axial versus vector, different for F2, XF3 (will be studied in MINERva)
 - (2) We should add radiation to GRV98 + B-Y 2004 (or radiatively correct the neutrino data) - not done
 - (3) We should add charm sea contribution at very large energy (not done)

CCFR diff. cross at $E_\nu = 35 \text{ GeV}$

$$K_{\text{axial}} = K_{\text{vector}} = Q^2 / (Q^2 + C)$$

$$K_{\text{axial}} = 1$$



Black line GRV98; red line with B-Y 2004

And $K_{\text{axial}} = K_{\text{vector}}$, blue $K_{\text{axial}} = 1$

Things left to do

- Use $K_{\text{axial}}(Q^2)=1$ for now, but it will be tuned further in the future.
- We can tune the axial vector K factor by including low Q^2 neutrino and antineutrino differential cross sections in the fit

However, the electron data has been radiatively corrected. A proper comparison to neutrino differential cross sections needs to include both radiative corrections and the c -bar contribution at high energies (which are not included in the GRV98 PDFs). And what about the nuclear effects?

We plan to tune $K_{\text{axial}}(Q^2)$ to get better agreement with the neutrino and antineutrino measured total cross sections (Here we need to separately add the quasielastic, delta and c -bar contributions, (but no need to include the radiative corrections since these integrate away in the total cross section). We will have this comparison soon

- In the future more detailed information on the axial form factor would come from MINERvA: by combining JUPITER Jlab (e -N vector) with the MINERvA (neutrino-N vector+axial) data.
- There could be different nuclear effects (e vs ν), F_2 vs xF_3 , and for axial F_2 versus vector F_2 . This will also be studied in MINERvA