Name: *Partners' Names:*
Laboratory Section:
Laboratory Section Date: *Grade:*

Last Revised on May 27, 2008

EXPERIMENT 1
Statistical Errors & Error Propagation

0   Pre-Laboratory Work [2pts]

1. A group of ten children is trying to guess the number of jelly beans in a jar at a county fair. They decide to all make a guess and then enter the average of their guesses into the contest. Their guesses are 846, 994, 1000, 1112, 1212, 1250, 1299, 1534, 1600, and 1943. The actual number of jelly beans is 1286.

   a. What is the standard deviation of the children's guesses?

   b. Is their average within one standard deviation of the actual number? If not, is it within two?

   c. Is their average consistent with the actual number? Why or why not?

2. Throughout this experiment you will be asked to acquire data in many trials. Provide a qualitative answer as to why you are asked to do this.

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1. Purpose

The purpose of this experiment is to understand statistical distributions and error propagation that can occur in their calculations. In addition, students will become familiar with the use and capabilities of a standard scientific calculator, as well as a digital Geiger counter.

2. Introduction

Probability Distributions
Probability distributions are widely used in experiments that involve counting. Sampling errors that occur in counting experiments are called statistical errors. Statistical errors are one kind of error in a class of errors known as *random errors*. You will find that what you learn in this laboratory is relevant not only in the natural and social sciences, but also in everyday life.

You are probably familiar with polls conducted before a presidential election. If the sample of people who are polled was carefully chosen to represent the general population, then the error in the prediction would depend on the number of people in the sample. The larger the number of people in the sample, the smaller the error will be. If the sample were not properly chosen, it would result in a bias (i.e. a *systematic error*).

Propagation of Error
In any calculation that occurs in science, there is always some inherent uncertainty as to the accuracy of the original measurement. Often times, this uncertainty stems from limitations in the measurement apparatus of the experiment (e.g. a ruler reading to only 1/10 of a centimeter). This uncertainty in measurement will propagate throughout the remaining calculations of that problem and will affect any further calculations and results, as we shall see in this experiment.

In an experiment involving simple motion, we can see that if an object is dropped it will follow Newton's equations of motion (a free falling object) and should be defined by the equation:

$$x = \frac{1}{2}gt^2 \qquad\qquad \textbf{Equation 1}$$

where g is the gravitational constant and $g = 9.8039 \, m/s^2$ on the earth's surface. To solve for $\dfrac{\partial}{\partial x}$ we must first isolate t on one side of the equation and then take the partial derivative with respect to x.

$$t^2 = \frac{2x}{g} \qquad\qquad \textbf{Equation 2}$$

$$t = \sqrt{\frac{2x}{g}} \qquad\qquad \textbf{Equation 3}$$

$$\frac{\partial t}{\partial x} = \frac{1}{2}\sqrt{\frac{2}{g}}x^{-1/2} = \sqrt{\frac{1}{2gx}} \qquad\qquad \textbf{Equation 4}$$

(See Appendix B and the end of this instructional section for more information).

Statistical Tools

   Quite possibly the most used statistical measurement is the mean, or average. This is simply the average value of all data points within a specific set. To calculate this we sum across all points and divide by the number of points:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{N} \qquad\qquad \textbf{Equation 5}$$

   Within any scientific field, the idea of standard deviation is one of the more commonly used statistical tools in data analysis and representation. Calculating the standard deviation tells us the spread of the values of the data. In other words, it is used as a measure of the average difference between the values of the data in the set.

   The standard deviation will always be a positive number and will always be in the same units as the data used in the calculation. For example, when measuring distances in meters, the standard deviation will have units of meters as well. We define the standard deviation as:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \qquad\qquad \textbf{Equation 6}$$

   We can also calculate the root mean square (RMS) of a specific variable or data set. If we were to take the average of particular data set that had negatives, they would cancel out their positive counterparts. (Think of one period of a sinusoidal wave; there are an equal number of points above and below the axis resulting in an average value of 0 for the function.) The RMS, however, helps us to determine the overall magnitude of a set of data points:

$$x_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}x_i^2} = \sqrt{\frac{x_1^2 + x_2^2 + \ldots + x_N^2}{N}} \qquad\qquad \textbf{Equation 7}$$

We can also calculate the RMS if we know the standard deviation:

$$x_{rms}^2 = \bar{x}^2 + \sigma^2 \qquad\qquad \textbf{Equation 8}$$

   We can relate the true standard deviation, σ, to the mean standard deviation, σ_{Mean}:

$$\sigma_{Mean} = \sigma\frac{1}{\sqrt{N}} \qquad\qquad \textbf{Equation 9}$$

3. Laboratory Work

3.1 Gravitational Acceleration of Ruler

Introduction

This experiment consists of dropping a ruler and determining which lab partner has the faster reaction time based upon the distance that he or she allows the ruler to pass before they catch the ruler with their hands.

The following items are to be given to each student:

        One (1) ½ meter stick

3.1.1 Setting up the Data Sample

Procedure

1. One lab partner will hold the ½ meter stick at the end of the meter stick (the 50 cm end) and hold it up as high as they are comfortably able to.

2. The second lab partner will be catching the ½ meter stick as it falls. To set up for this, place your thumb and pointer finger about 1 ½ inches apart and place your hand at the 0 cm mark of the ½ meter stick, so that the tops of your fingers are roughly equal with the zero mark on the ½ meter stick.

3. Without warning, the lab partner holding the ½ meter stick should release it and the other lab partner should react as quickly as possible and catch it as quickly as they are able by closing their fingers and not moving their arms.

4. Note the distance that the ruler has traveled according to the tops of your fingers and round up to the nearest whole centimeter. Record your result in *Table 4.1.1*. Repeat this procedure 24 more times, for a total of 25 trials.

5. Lab partners should switch roles and repeat steps 1-4 so that there are two data sets, each with 25 trials.

3.2 Radioactive Decay of β-Source Sample

Introduction

For this portion of the experiment, we will be using a radioactive Sr^+_{90} button source that will emit a very small amount (0.10 μCi) of β rays. Don't worry, this sample is very weak and it is encapsulated within a steel box for your safety.

The following items are to be given to each student:

        One sealed unit containing a Sr^+_{90} button source
        One digital Geiger counter with external Geiger tube wand

Procedure
1. Turn on the Geiger counter (it is up to you whether you want the audio on or off).
2. Set the Geiger counter to read in mR/hr (milli-Roentgens per hour) and to take one second readings. A one-second reading with empty space between the source and the Geiger counter wand should give about 100 mR/hr. Talk to your TA if your reading is off.
3. Take one reading with nothing inserted between the source and the Geiger counter wand, one reading with the .12 cm–thick aluminum plate inserted, and one reading with the .60 cm–thick aluminum plate inserted. Record your data in *Table 4.2.1*.
4. Take ten readings with nothing inserted between the source and the wand, and record your data in *Table 4.2.2*.
5. Take ten readings with the .12 cm–thick aluminum plate, and ten readings with the .60 cm–thick aluminum plate. Record this data in *Table 4.2.2* as well.

Excerpt from Appendix B Error Analysis
This excerpt is from *Appendix B. Error Analysis*, which can be found on the Physics Laboratory Website (www.pas.rochester.edu/~physlabs).

Propagation of Error
Frequently, the result of an experiment will not be measured directly. Rather, it will be calculated from several measured physical quantities (each of which has a mean value and an error). What is the resulting error in the final result of such an experiment? For instance, what is the error in $Z = A + B$ where A and B are two measured quantities with errors ΔA and ΔB respectively?

A first thought might be that the error in Z would be just the sum of the errors in A and B. After all,

$$(A + \Delta A) + (B + \Delta B) = (A + B) + (\Delta A + \Delta B) \qquad \textbf{Equation 1.10}$$

and

$$(A - \Delta A) + (B - \Delta B) = (A + B) - (\Delta A + \Delta B). \qquad \textbf{Equation 1.11}$$

But this assumes that, when combined, the errors in A and B have the same sign and maximum magnitude; that is they always combine in the worst possible way. This could only happen if the errors in the two variables were perfectly correlated, (i.e. if the two variables were not really independent).

If the variables are independent then sometimes the error in one variable will happen to cancel out some of the error in the other and so, on the average, the error in Z will be less than the sum of the errors in its parts. A reasonable way to try to take this into account is to treat the perturbations in Z produced by perturbations in its parts as if they were "perpendicular" and added according to the Pythagorean theorem

$$\Delta Z = \sqrt{(\Delta A)^2 + (\Delta B)^2}. \qquad \textbf{Equation 1.12}$$

For example, if $A = (100 \pm 3)$ and $B = (6 \pm 4)$ then $Z = (106 \pm 5)$ since $5 = \sqrt{3^2 + 4^2}$.

This idea can be used to derive a general rule. Suppose there are two measurements, A and B, and the final result is $Z = F(A, B)$ for some function F. If A is perturbed by ΔA, then Z will be perturbed by $\left(\dfrac{\partial F}{\partial A}\right)\Delta A$, where (the partial derivative) $\dfrac{\partial F}{\partial A}$ is the derivative of F with

respect to A with B held constant. Similarly, the perturbation in Z due to a perturbation in B is $\left(\frac{\partial F}{\partial B}\right)\Delta B$. Combining these by the Pythagorean Theorem yields

$$\Delta Z = \sqrt{\left(\frac{\partial F}{\partial A}\right)^2 (\Delta A)^2 + \left(\frac{\partial F}{\partial B}\right)^2 (\Delta B)^2} \ . \qquad \textbf{Equation 1.13}$$

In the example of $Z = A + B$ considered above,

$$\frac{\partial F}{\partial A} = 1 \ \text{ and } \ \frac{\partial F}{\partial B} = 1 \ ,$$

so this gives the same result as before. Similarly, if $Z = A - B$, then

$$\frac{\partial F}{\partial A} = 1 \ \text{ and } \ \frac{\partial F}{\partial B} = -1 \ ,$$

which also gives the same result.
However, if $Z = AB$, then,

$$\frac{\partial F}{\partial A} = B \ \text{ and } \ \frac{\partial F}{\partial B} = A \ ,$$

so

$$\Delta Z = \sqrt{B^2 (\Delta A)^2 + A^2 (\Delta B)^2} \ . \qquad \textbf{Equation 1.14}$$

Thus,

$$\frac{\Delta Z}{Z} = \frac{\Delta Z}{AB} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2} \qquad \textbf{Equation 1.15}$$

or the fractional error in Z is the square root of the sum of the squares of the fractional errors in its parts.

It should be noted that since the above applies *only* when the two measured quantities are *independent* of each other, it does not apply when, for example, one physical quantity is measured and what is required is its square. If $Z = A^2$, then the perturbation in Z due to a perturbation in A is

$$Z = \frac{\partial F}{\partial A} \Delta A = 2A\Delta A \ . \qquad \textbf{Equation 1.16}$$

Thus, in this case,

$$(A + \Delta A)^2 = A^2 \pm 2A\Delta A = A^2 \left(1 \pm 2\frac{\Delta A}{A}\right) \qquad \textbf{Equation 1.17}$$

and not $A^2 \left(1 \pm \sqrt{2}\frac{\Delta A}{A}\right)$ as would be obtained by misapplying the rule for independent variables. For example, $(10 \pm 1)^2 = 100 \pm 20$ and not 100 ± 14.

If a variable Z depends on one or two variables A or/and B, which have independent errors ΔA and ΔB, then the rules for calculating the error in Z is tabulated in *Table 1.4* for a variety of simple relationships. These rules may be compounded for more complicated situations.

Combination of Errors

	Relation between Z and (A, B)	Relation between errors ΔZ and $(\Delta A, \Delta B)$
1	$Z = A + B$ $Z = A - B$	$(\Delta Z)^2 = (\Delta A)^2 + (\Delta B)^2$
2	$Z = AB$ $Z = \dfrac{A}{B}$	$\left(\dfrac{\Delta Z}{Z}\right)^2 = \left(\dfrac{\Delta A}{A}\right)^2 + \left(\dfrac{\Delta B}{B}\right)^2$
3	$Z = A^n$	$\dfrac{\Delta Z}{Z} = n\dfrac{\Delta A}{A}$
4	$Z = \ln A$	$\Delta Z = \dfrac{\Delta A}{A}$
5	$Z = e^A$	$\dfrac{\Delta Z}{Z} = \Delta A$
6	$Z = \dfrac{A + B}{2}$	$\Delta Z = \dfrac{1}{2}\sqrt{(\Delta A)^2 + (\Delta B)^2}$

Table 3.3.1

Calculator Use

Provided in the lab is a TI-30XIIS scientific calculator that will simplify the calculations for standard deviation. To define statistical data points:

1. Press ┊ 2nd [STAT]. Select 1-VAR and press ENTER. The **STAT** indicator on the bottom of the screen will display.
2. Press DATA.
3. Enter a value for X1.
4. Press ⊙ and enter the frequency of that value (e.g. You received a measurement of 0.14 m 5 times, 5 would be the frequency of your measurement.
5. Repeat steps 3 and 4 until all data points are entered. You must press ENTER or ⊙ to save the last data point or FRQ value entered. If you add or delete data points, the TI-30X IIS automatically reorders the list.
6. When all points and frequencies are entered press STATVAR to display the menu of variables and their current values. For the purposes of our lab you will want to look at the total number of entries, n, mean value, $\bar{x}$, and the standard deviation, S_x.

7. When finished:
   Press ┊ 2nd [STAT] and select CLRDATA to clear all data points without exiting STAT mode, or
   Press ┊ 2nd [EXIT STAT] ENTER to clear all data points, variables, and FRQ values, and to exit **STAT** mode (**STAT** indicator turns off).

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EXPERIMENT 1
Statistical Errors & Error Propagation

4. Post-Laboratory Work [18 pts]

4.1 Gravitational Acceleration of Ruler [12 pts]

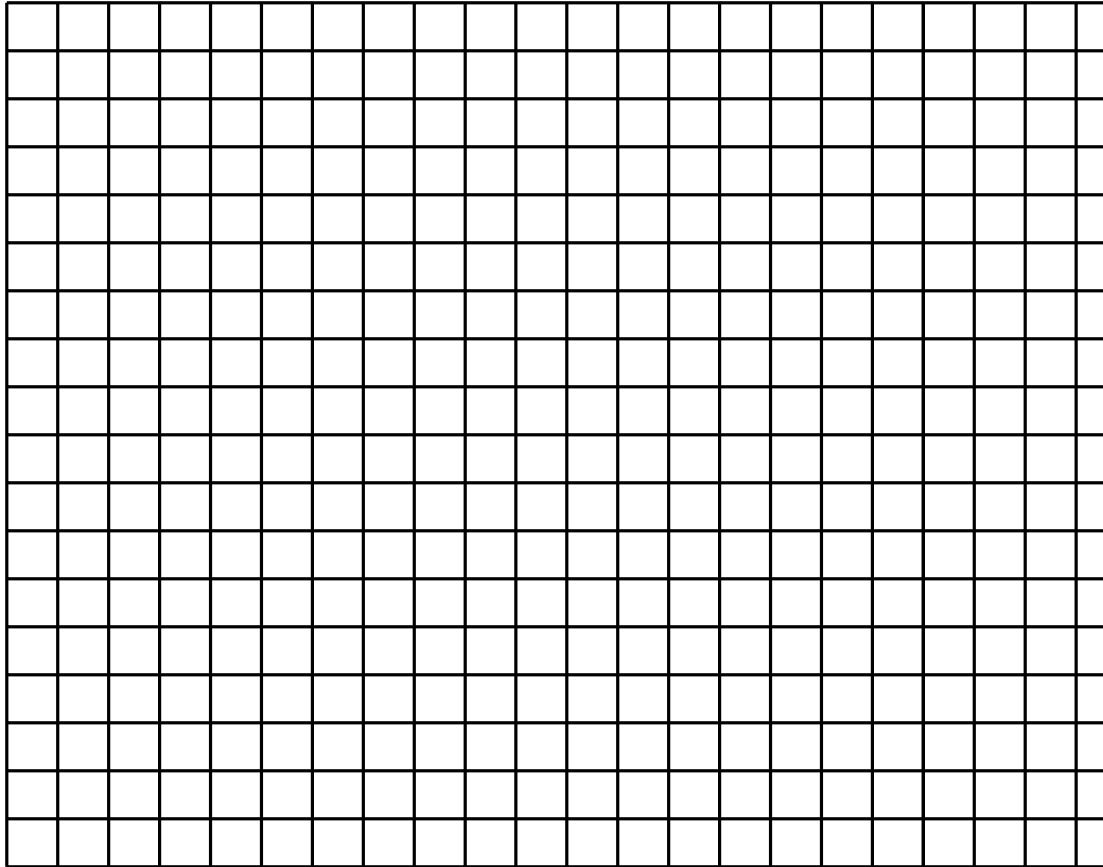| Trial	X_{P1}	T_{P1}	X_{P2}	T_{P2}	Trial	X_{P1}	T_{P1}	X_{P2}	T_{P2}
1					14				
2					15				
3					16				
4					17				
5					18				
6					19				
7					20				
8					21				
9					22				
10					23				
11					24				
12					25				
13									

Table 4.1.1

1. With the data from each lab partner established for the columns of X_{P1} and X_{P2}, calculate the time for each value, T_{P1} and T_{P2}, using Equation 3. (1 pt)

2. Using the data set from above (2 pts):

What is your average position? $\overline{X}_{P1} =$_____
What is your partner's average position? $\overline{X}_{P2} =$_____

Using the supplied scientific calculator and Equation 6:

What is the σ of your data? $\sigma_{X_{P1}} =$_____
What is the σ of your partner's data? $\sigma_{X_{P2}} =$_____

3. Using the data from above, plot a histogram of frequency vs. distance (y vs. x) for your data. Also, draw a best fit curve around your data. Do not forget to label your axis and provide a title to your plot. (2 pts)

4. After calculating T$_{P1}$ and T$_{P2}$, using Equation 3 (2 pts):

What is your average time? $\overline{T}_{P1} =$ _____

What is your partner's average time? $\overline{T}_{P2} =$ _____

What is the σ of your calculated times? $\sigma_{T_{P1}} =$ _____

What is the σ of your partner's calculated times? $\sigma_{T_{P2}} =$ _____

5. Using your average position, $\overline{X}_{P1}$ and $\overline{X}_{P2}$, calculate the propagation of error to time with Equation 4. (1 pt)

6. We can relate the standard deviation of position and the standard deviation of time with a calculated quantity. If we know that the ratio of the two is satisfied by a constant, do you have any intuition as to what this constant could be? (Hint: You have calculated this value by now). (2 pts)

7. If you had been instructed to only drop the ruler one time for each partner would this have provided a good sample for the experiment? Why or why not? How can we carry this idea into experiments that we will do throughout the semester? (2 pts)

4.2 Radioactive Decay of β-Source Sample [6 pts]

empty space	.12 cm sheet	.60 cm sheet

Table 4.2.1

8. The error in each of these readings can be approximated as $\sqrt{N}$, where N is the reading. (So if you had a reading of 100 mR/hr, the error can be taken as $\sqrt{100}$ mR/hr = 10 mR/hr.) Record the errors for each of the readings in *Table 4.2.1* below, writing them as $N \pm \sqrt{N}$. (1 pt)

empty space:

.12 cm sheet:

.60 cm sheet:

9. Based on the errors you just computed, are your readings distinguishable from each other? (That is, if you were given one of the readings above without being told which setup it came from, could you say whether that reading was taken with empty space, the .12 cm sheet, or the .60 cm sheet?) Why or why not? (1 pt)

trial #	empty space	.12 cm sheet	.60 cm sheet
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Table 4.2.2

10. Calculate the average of each of the three setups, and the error (approximated as $\sqrt{N}$) in each of these averages. Record your data below, again in the form $N \pm \sqrt{N}$. (1 pt)

empty space:

.12 cm sheet:

.60 cm sheet:

11. Based on the errors you computed, are these averages distinguishable from each other? Why or why not? Compare your answer with your answer to question 9 and explain any differences. (1 pt)

12. Sketch a picture of what the inside of the black box looks like – where is the radioactive source? Where is the Geiger counter wand? Where does the aluminum sheet that you insert go? (Think about why inserting this sheet causes the reading of mR/hr to decrease.) (1 pt)

13. If you were to take more readings with the .60 cm-thick sheet in place, how would you expect the standard deviation of this larger data set to compare to the standard deviation of your 10-point data set? There is no need to take more data – answer qualitatively. (1 pt)