Modeling Neutrino and Electron Scattering Cross Sections in the Few GeV Region with Effective *LO PDFs*

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We use new scaling variables x_w and ξ_w, and add low Q^2 modifications to GRV94 and GRV98 leading order parton distribution functions such that they can be used to model electron, muon and neutrino inelastic scattering cross sections (and also photoproduction) at both very low and high energies (Presented by Arie Bodek at NuInt02, the Second International Workshop on Neutrino-Nucleus Interactions in the Few GeV Region, Dec. 2002, Irvine, CA, USA [1])

1. Origin of Higher Twist Terms

The quark distributions in the proton and neutron are parametrized as Parton Distribution Functions (PDFs) obtained from global fits to various sets of data at very high energies. These fits are done within the theory of Quantum Chromodynamics (QCD) in either leading order (LO) or next to leading order (NLO). The most important data come from deep-inelastic e/μ scattering experiments on hydrogen and deuterium, and ν_μ and $\overline{\nu}_\mu$ experiments on nuclear targets. In previous publications [2,3,4] we have compared the predictions of the NLO MRSR2 PDFs to deep-inelastic e/μ scattering data [5] on hydrogen and deuterium from SLAC, BCDMS and NMC. In order to get agreement with the lower energy SLAC data for F_2 and R down to Q^2=1 GeV^2, and at the highest values of x ($x = 0.9$), we found that the following modifications to the NLO MRSR2 PDFs must be included.

1. The relative normalizations between the various data sets and the BCDMS systematic error shift must be included [2,3].

2. Deuteron binding corrections need to be applied and the ratio of d/u at high x must be increased as discussed in ref. [2].

3. Kinematic higher-twist originating from target mass effects [6] are very large and must be included.

4. Dynamical higher-twist corrections are smaller but also need to be included [2,3].

5. In addition, our analysis including QCD Next to NLO (NNLO) terms shows [3] that most of the dynamical higher-twist corrections needed to fit the data within a NLO QCD analysis originate from the *missing NNLO higher order terms*.

Our analysis shows that the NLO MRSR2 PDFs with target mass and NNLO higher order terms describe electron and muon scattering F_2 and R data with a very small contribution from higher twists. Studies by other authors [7] also show that in NNLO analyses the dynamic higher twist corrections are very small. If (for $Q^2 > 1$ GeV^2) most of the higher-twist terms needed to obtain agreement with the low energy data actually originate from target mass effects and missing NNLO terms (i.e. not from interactions with spectator quarks) then these terms should be the same in ν_μ and e/μ scattering. Therefore, low energy ν_μ data should be described by the PDFs which are fit to high energy data and are modified to include target mass and higher-twist corrections that fit low energy e/μ scattering data. However, for $Q^2 <$ 1 GeV^2 additional non-perturbative effects from

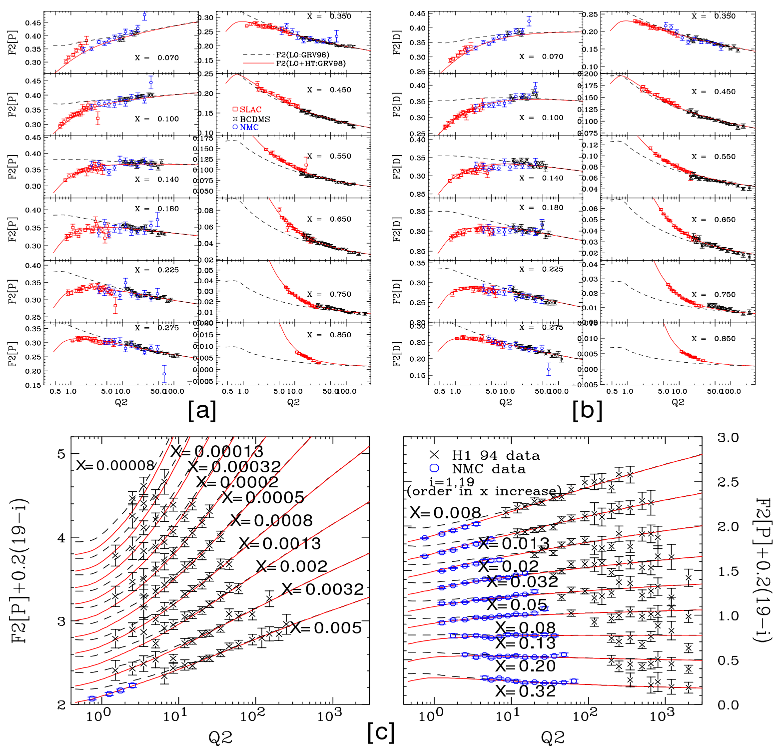

Figure 1. Electron and muon F_2 data (SLAC, BCDMS, NMC, H1 94) used in our GRV98 ξ_w fit compared to the predictions of the unmodified GRV98 PDFs (LO, dashed line) and the modified GRV98 PDFs fits (LO+HT, solid line); [a] for F_2 proton, [b] for F_2 deuteron, and [c] for the H1 and NMC proton data at low x.

spectator quarks must also be included [8].

2. Previous Results with GRV94 PDFs and x_w

In a previous communication [8] we used a modified scaling variable x_w and fit for modifications to the GRV94 leading order PDFs such that the PDFs describe both high energy and low energy e/μ data. In order to describe low energy data down to the photoproduction limit ($Q^2 = 0$), and account for both target mass and higher twist effects, the following modifications of the GRV94 LO PDFs are need:

1. We increased the d/u ratio at high x as described in our previous analysis [2].

2. Instead of the scaling variable x we used the scaling variable $x_w = (Q^2 + B)/(2M\nu + A)$ (or $=x(Q^2 + B)/(Q^2 + Ax)$). This modification was used in early fits to SLAC data [10]. The parameter A provides for an approximate way to include *both* target mass and higher twist effects at high x, and the parameter B allows the fit to be used all the way down to the photoproduction limit (Q^2=0).

3. In addition as was done in earlier non-QCD based fits [11] to low energy data, we multiplied all PDFs by a factor K=Q^2 / (Q^2

+C). This was done in order for the fits to describe low Q^2 data in the photoproduction limit, where F_2 is related to the photoproduction cross section according to

$$\sigma(\gamma p) = \frac{4\pi^2\alpha_{\rm EM}}{Q^2}F_2 = \frac{0.112mb\ GeV^2}{Q^2}F_2$$

4. Finally, we froze the evolution of the GRV94 PDFs at a value of $Q^2 = 0.24$ (for $Q^2 < 0.24$), because GRV94 PDFs are only valid down to $Q^2 = 0.23$ GeV^2.

In our analyses, the measured structure functions were corrected for the BCDMS systematic error shift and for the relative normalizations between the SLAC, BCDMS and NMC data [2,3]. The deuterium data were corrected for nuclear binding effects [2,3]. A simultaneous fit to both proton and deuteron SLAC, NMC and BCDMS data (with $x > 0.07$ only) yields A=1.735, B=0.624 and C=0.188 (GeV^2) with GRV94 LO PDFs ($\chi^2 = 1351/958$ DOF). Note that for x_w the parameter A accounts for *both* target mass and higher twist effects.

3. New Analysis with ξ_w, G_D and GRV98 PDFs

In this publication we update our previous studies, [9] which were done with a new improved scaling variable ξ_w, and fit for modifications to the more modern GRV98 LO PDFs such that the PDFs describe both high energy and low energy electron/muon data. We now also include NMC and H1 94 data at lower x. Here we freeze the evolution of the GRV98 PDFs at a value of $Q^2 = 0.8$ (for $Q^2 < 0.8$), because GRV98 PDFs are only valid down to $Q^2 = 0.8$ GeV^2. In addition, we use different photoproduction limit multiplicative factors for valence and sea. Our proposed new scaling variable is based on the following derivation. Using energy momentum conservation, it can be shown that the factional momentum $\xi = (p_z + p_0)/(P_z + P_0)$ carried by a quark of 4-mometum p in a proton target of mass M and 4-momentum P is given by $\xi = xQ^{'2}/[0.5Q^2(1 + [1 + (2Mx)^2/Q^2]^{1/2})]$, where $2Q^{'2} = [Q^2 + {M_f}^2 - {M_i}^2] + [(Q^2 + {M_f}^2 - {M_i}^2)^2 + 4Q^2({M_i}^2 + P_T^2)]^{1/2}$.

Here M_i is the initial quark mass with average initial transverse momentum P_T and M_f is the mass of the quark in the final state. The above expression for ξ was previously derived [6] for the case of $P_T = 0$. Assuming $M_i = 0$ we use instead: $\xi_w = x(Q^2 + B + {M_f}^2)/(0.5Q^2(1 + [1 + (2Mx)^2/Q^2]^{1/2}) + Ax)$

Here M_f=0, except for charm-production processes in neutrino scattering for which M_f=1.5 GeV. For ξ_w the parameter A is expected to be much smaller than for x_w since now it only accounts for the higher order (dynamic higher twist) QCD terms in the form of an *enhanced* target mass term (the effects of the proton target mass are already taken into account using the exact form in the denominator of ξ_w). The parameter B accounts for the initial state quark transverse momentum and final state quark $effective$ $\Delta {M_f}^2$ (originating from multi-gluon emission by quarks).

Using closure considerations [12] (*e.g.*the Gottfried sum rule) it can be shown that, at low Q^2, the scaling prediction for the *valence* quark part of F_2 should be multiplied by the factor K=[1-$G_D^2(Q^2)$][1+M(Q^2)] where $G_D = 1/(1{+}Q^2/0.71)^2$ is the proton elastic form factor, and M(Q^2) is related to the magnetic elastic form factors of the proton and neutron. At low Q^2, [1-$G_D^2(Q^2)$] is approximately $Q^2/(Q^2$ +C) with $C = 0.71/4 = 0.178$ (versus our fit value C=0.18 with GRV94). In order to satisfy the Adler Sum rule [13] we add the function M(Q^2) to account for terms from the magnetic and axial elastic form factors of the nucleon). Therefore, we try a more general form $K_{valence}$=[1-$G_D^2(Q^2)$][Q^2+C_{2v}]/[Q^2 +C_{1v}], and K_{sea}=$Q^2/(Q^2$+C_{sea}). Using this form with the GRV98 PDFs (and now also including the very low x NMC and H1 94 data in the fit) we find A=0.419, B=0.223, and C_{1v}=0.544, C_{2v}=0.431, and C_{sea}=0.380 (all in GeV^2, $\chi^2 = 1235/1200$ DOF). As expected, A and B are now smaller with respect to our previous fits with GRV94 and x_w. With these modifications, the GRV98 PDFs must also be multiplied by N=1.011 to *normalize* to the SLAC F_{2p} data. The fit (Fig-

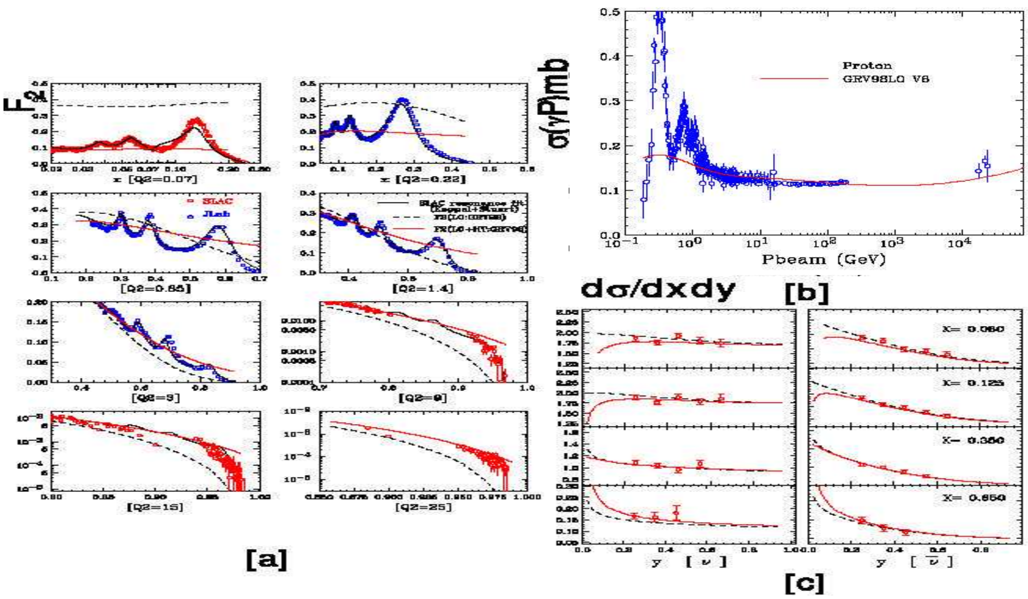

Figure 2. Comparisons to proton and iron data not included in our GRV98 ξ_w fit. (a) Comparison of SLAC and JLab (electron) F_{2p} data in the resonance region (or fits to these data) and the predictions of the GRV98 PDFs with (LO+HT, solid) and without (LO, dashed) our modifications. (b) Comparison of photoproduction data on protons to predictions using our modified GRV98 PDFs. (c) Comparison of representative CCFR ν_μ and $\overline{\nu}_\mu$ charged-current differential cross sections [4,14] on iron at 55 GeV and the predictions of the GRV98 PDFs with (LO+HT, solid) and without (LO, dashed) our modifications.

ure 1) yields the following normalizations relative to the SLAC F_{2p} data ($SLAC_D$=0.986, $BCDMS_P$=0.964, $BCDMS_D$=0.984, NMC_P=1.00, NMC_D=0.993, $H1_P$=0.977, and BCDMS systematic error shift of 1.7).(Note, since the GRV98 PDFs do not include the charm sea, for $Q^2 > 0.8$ GeV^2 we also include charm production using the photon-gluon fusion model in order to fit the very high ν HERA data. This is not needed for any of the low energy comparisons but is only needed to describe the highest ν HERA electro and photoproduction data).

Comparisons of *predictions* using these modified GRV98 PDFs to other data which were *not included* in the fit is shown in Figures 2 and 3. From duality [15] considerations, with the ξ_w scaling variable, the modified GRV98 PDFs should also provide a reasonable description of the average value of F_2 in the resonance region. Figures 2(a) and 3(a) show a comparison between resonance data (from SLAC and Jefferson Lab, or parametrizations of these data [16]) on protons and deuterons versus the predictions with the standard GRV98 PDFs (LO) and with our modified GRV98 PDFs (LO+HT). The modified GRVB98 PDFs are in good agreement with SLAC and JLab resonance data down to $Q^2 = 0.07$ (although resonance data were not included in our fits). There is also very good agreement of the *predictions* of our modified GRV98 in the $Q^2 = 0$ limit with photoproduction data on protons and deuterons as shown in Figure 2(b) and 3(b). In

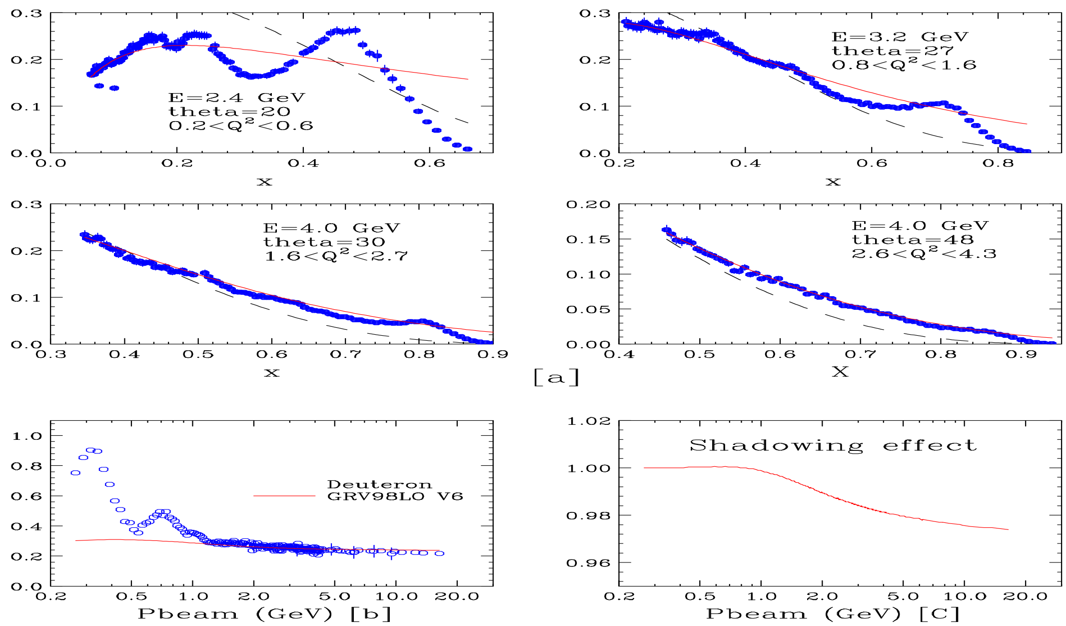

Figure 3. Comparisons to data on deutrerium which were not included in our GRV98 ξ_w fit. (a) Comparison of SLAC and JLab (electron) F_{2d} data in the resonance region and the predictions of the GRV98 PDFs with (LO+HT, solid) and without (LO, dashed) our modifications. (b) Comparison of photoproduction data on deuterium to predictions using our modified GRV98 PDFs (including shadowing corrections). (c) The shadowing corrections that were applied to the PDFs for predicting the photoproduction cross section on deuterium.

predicting the photoproduction cross sections on deuterium, we have applied shadowing corrections [20] as shown in Figure 3(c). We also compare the *predictions* with our modified GRV98 PDFs (LO+HT) to a few representative high energy CCFR ν_μ and $\overline{\nu}_\mu$ charged-current differential cross sections [4,14] on iron (neutrino data were not included in our fit). In this comparison we use the PDFs to obtain F_2 and xF_3 and correct for nuclear effects in iron [8]. The structure function $2xF_1$ is obtained by using the R_{world} fit from reference [5]. There is very good agreement of our *predictions* with these neutrino data on iron.

In order to have a full description of all charged current ν_μ and $\overline{\nu}_\mu$ processes, the contribution from quasielastic scattering [17] must be added separately at $x = 1$. The best prescription is to use our model in the region above the first resonance (above W=1.35 GeV) and add the contributions from quasielastic and first resonance [18] (W=1.23 GeV) separately. This is because the $W = M$ and W=1.23 GeV regions are dominated by one and two isospin states, and the amplitudes for neutrino versus electron scattering are related via Clebsch-Gordon rules [18] instead of quark charges (also the V and A couplings are not equal at low W and Q^2). In the region of higher mass resonances (e.g. W=1.7 GeV) there is a significant contribution from the deep-inelastic continuum which is not well modeled by the existing fits [18] to neutrino resonance data (and using our modified PDFs should be better). For nuclear targets, nuclear corrections [8] must also be applied. Recent results from Jlab indicate that the Fe/D ratio in the resonance region is the same as the Fe/D ratio from DIS data for the same value of ξ (or ξ_w). The effects of terms proportional to the muon mass and F_4 and F_5 structure functions in neutrino scattering are small and are discussed in Ref. [17,19]. In the future, we plan to investigate the effects of including the initial state quark P_T in ξ_w, and institute further improvements such as allowing for different higher twist parameters for u, d, s, c, b quarks in the sea, and the small difference (expected in the Adler sum rule) in the K factors for axial and vector terms in neutrino scattering. In addition, we can multiply the PDFs by a modulating function [10,12] A(W,Q^2) to improve modeling in the resonance region (for hydrogen) by including (instead of *predicting*) the resonance data [16] in the fit. We can also include resonance data on deuterium [16] and heavier nuclear targets in the fit, and low energy neutrino data. Note that because of the effects of experimental resolution and Fermi motion [21] (for nuclear targets), a description of the average cross section in the resonance region is sufficient for most neutrino experiments.

The current analysis assumes that the axial and vector structure functions are equal at all Q^2. However, at very low Q^2, the vector structure function must go to zero, while the axial-vector part is finite. We are currently (August 2003) in the process of including low energy data from Chorus (on Pb) in our fit, in order to constrain the low Q^2 axial-vector contribution. As for the vector case, the form of the fits is motivated by the Adler sum rule for the axial-vector contribution as follows: $K_{valence-ax}$= [1-$F_A^2(Q^2)$][Q^2+D_{2v-ax}]/[Q^2 +D_{1v-ax}], and K_{sea-ax}=(Q^2+D_{sea-ax})/(Q^2+B_{sea-ax}). Here [17] F_A = -1.267/(1+Q^2/1.00)^2.

4. Appendix

In leading order QCD (e.g. GRV98 LO PDFs), F_2 for the scattering of electrons and muons on proton (or neutron) targets is given by the sum of quark and anti-quark distributions (each weighted the square of the quark charges):

$$F_2(x) = \Sigma_i e_i^2 \left[x q_i(x) + x\overline{q}_i(x) \right] \quad (1)$$

$$2xF_1(x) = F_2(x)(1+4Mx^2/Q^2)/(1+R_w) \quad (2)$$

Here, $R_w(x,Q^2)$ is parameterized [5] by:

$$R_w = \frac{0.0635}{log(Q^2/0.04)}\theta(x,Q^2) + \frac{0.5747}{Q^2} - \frac{0.3534}{Q^4+0.09}, \quad (3)$$

where $\theta = 1. + \frac{12Q^2}{Q^2+1.0} \times \frac{0.125^2}{0.125^2+x^2}$.

The R_w function provides a good description of the world's data in the $Q^2 > 0.5$ and $x > 0.05$ region. Note that the R_w function breaks down below $Q^2 = 0.3$. Therefore, we freeze the function at $Q^2 = 0.35$ and introduce the following function for R in the $Q^2 < 0.35$ region. The new function provides a smooth transition from $Q^2 = 0.35$ down to $Q^2 = 0$ by forcing R to approach zero at $Q^2 = 0$ as expected in the photoproduction limit (while keeping a $1/Q^2$ behavior at large Q^2 and matching to R_w at $Q^2 = 0.35$).

$$R = 3.207 \times \frac{Q^2}{Q^4+1} \times R_w(x, Q^2 = 0.35). \quad (4)$$

In the comparison with CCFR charged-current differential cross section on iron, a nuclear correction for iron targets is applied. We use the following parameterized function, $f(x)$ (fit to experimental electron and muon scattering data for the ratio of iron to deuterium cross sections), to convert deuterium structure functions to (isoscalar) iron structure functions [8];

$$f(x) = 1.096 - 0.364x - 0.278e^{-21.94x} + 2.772x^{14.417} \quad (5)$$

For the ratio of deuterium cross sections to cross sections on free nucleons we use the following function obtained from a fit to SLAC data on the nuclear dependence of electron scattering cross sections [4].

$$f = (0.985 \pm 0.0013) \times (1 + 0.422x - 2.745x^2 + 7.570x^3 - 10.335x^4 + 5.422x^5). \quad (6)$$

This correction is only valid in the $0.05 < x < 0.75$ region. In neutrino scattering, we use the same nuclear correction factor for F_2, xF_3 and $2xF_1$.

The d/u correction for the GRV98 LO PDFs is obtained from the NMC data for F_2^D/F_2^P. Here, Eq. 6 is used to remove nuclear binding effects in the NMC deuterium F_2 data. The correction term, $\delta(d/u)(x)$ is obtained by keeping the total valence and sea quarks the same.

$$\delta(d/u) \quad = \quad -0.00817 + 0.0506x + 0.0798x^2, \quad (7)$$

where the corrected d/u ratio is $(d/u)' = (d/u) + \delta(d/u)$. Thus, the modified u and d valence distributions are given by

$$u'_v = \frac{u_v}{1 + \delta(d/u)\frac{u_v}{u_v+d_v}} \quad (8)$$

$$d'_v = \frac{d_v + u_v\delta(d/u)}{1 + \delta(d/u)\frac{u_v}{u_v+d_v}}. \quad (9)$$

The same formalism is applied to the modified u and d sea distributions. Accidently, the modified u and d sea distributions (based on NMC data) agree with the NUSEA data in the range of x between 0.1 and 0.4. Thus, we find that any futher correction on sea quarks is not necessary.

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