Quarks for Dummies TM *

Modeling (e/ μ /ν)-N Cross Sections from Low to High Energies: from DIS to Resonance, to Quasielastic Scattering

Modified LO PDFs, ξ_w scaling, Quarks and Duality

Fermilab Wine/Cheese Talk

August 16, 2002 (updated Aug. 30,2002)

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*TM- (present company excepted)

Some of this QCD/PDF work has been published in

1. Studies in QCD NLO+TM+ renormalon HT - Yang, Bodek Phys. Rev. Lett 82, 2467 *(1999)*
2. Studies in QCD NNLO+TM+ renormalon HT - Yang, Bodek: Eur. Phys. J. C13, 241 *(2000)*

Scaling variable PDF Studies (X_w, ξ_w, ξ'_w)

- 0th ORDER PDF (QPM + X_w scaling) studies - A. Bodek, et al PRD 20, 1471 *(1979)* + earlier papers in the 1970's.
- LO + Modified PDFs (X_w scaling) studies -

  Bodek, Yang: hep-ex/0203009 *(2002)* to appear in proc of NuInt01-KEK (Nuclear Physics B) +DPF02
- LO + Modified PDFs (ξ_w scaling) studies -

  Presented at NuFact 02-London (*July 2002*) - being written.

  - covered in *THIS TALK*

Neutrino cross sections at low energy

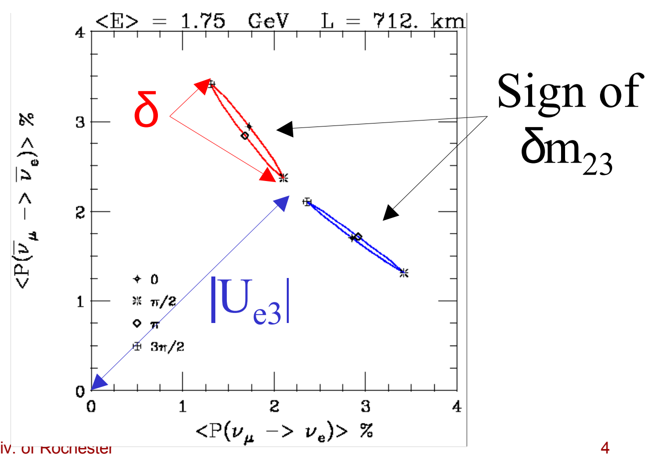
- Neutrino *oscillation experiments* (K2K, MINOS, CNGS, MiniBooNE, and future experiments with Superbeams at JHF,NUMI, CERN) are in the few GeV region
- Important to correctly model neutrino-nucleon and neutrino-nucleus reactions at 0.5 to 4 GeV (essential for precise next generation neutrino oscillation experiments with super neutrino beams) as well as at the 15-30 GeV (for future ν factories) - NuInt, Nufac
- The very high energy region in neutrino-nucleon scatterings (50-300 GeV) is well understood at the few percent level in terms QCD and Parton Distributions Functions (PDFs) within the framework of the quark-parton model (data from a series of e/μ/ν DIS experiments)
- However, neutrino differential cross sections and final states in the few GeV region are poorly understood. (especially, resonance and low Q^2 DIS contributions). *In contrast, there is enormous amount of e-N data from SLAC and Jlab in this region.*

- *Intellectually - Understanding Low Energy neutrino and electron scattering Processes is also a very way to understand quarks and QCD*. - common ground between the QCD community and the weak interaction community, and between medium and HEP physicists.

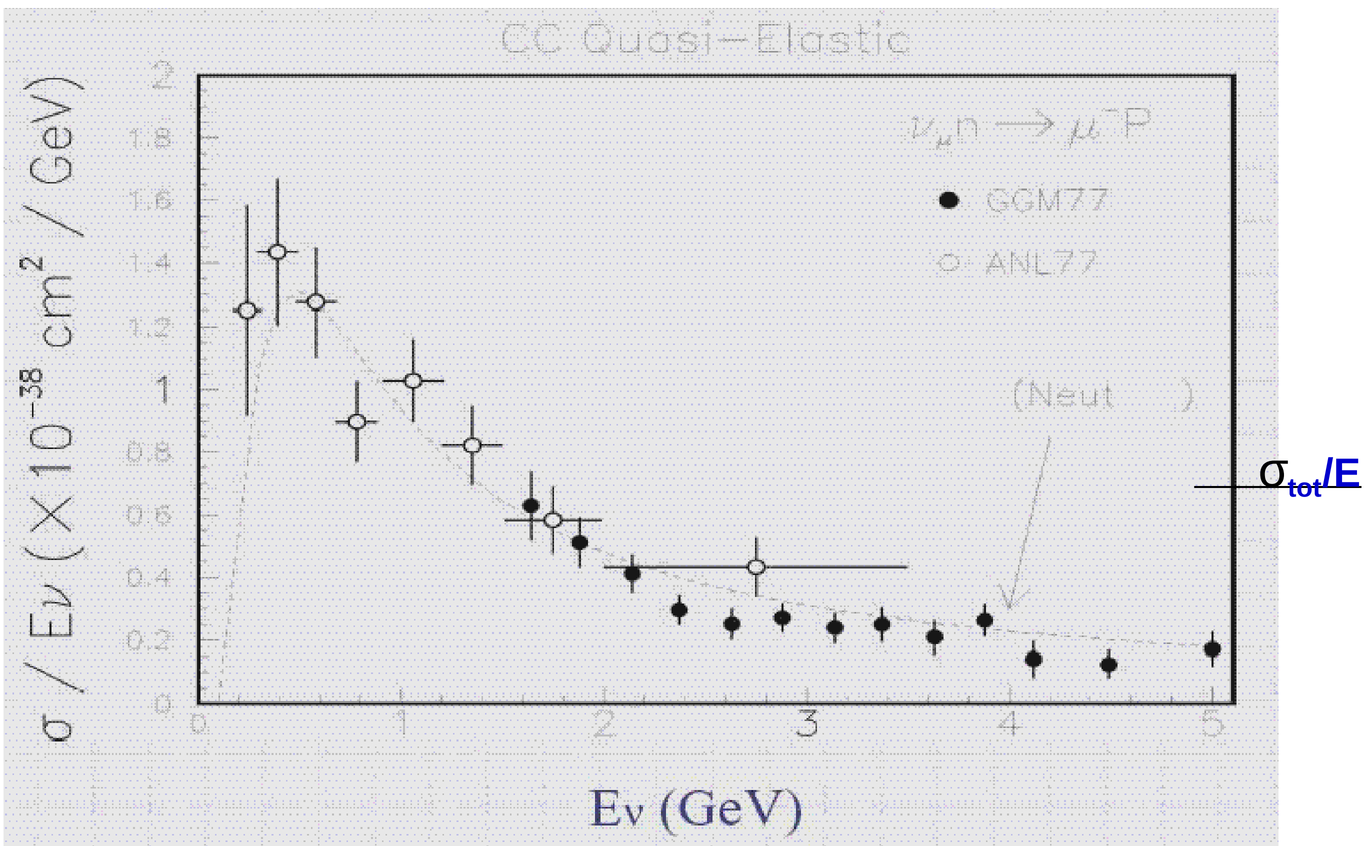
The Importance of Precision Measurement of the Oscillation Probability $P(\nu_\mu \to \nu_e)$ with Neutrino Superbeams

- Conventional "superbeams" (e.g. NUMI) will be our only windows into this suppressed transition
  - Analogous to $|V_{ub}|$ in quark sector
  - (Next steps: μ sources or "β beams" too far away)

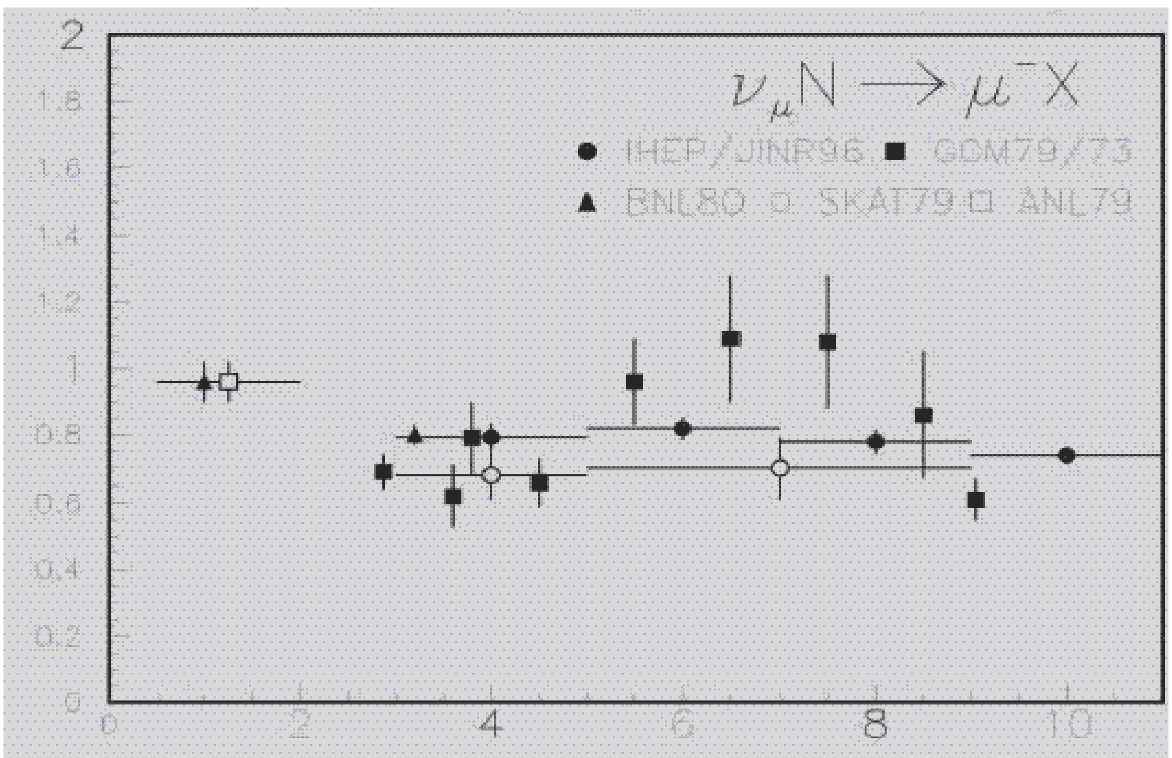
- Studying $P(\nu_\mu \to \nu_e)$ in neutrinos and anti-neutrinos gives us magnitude and phase information on $|U_{e3}|$



Examples of Current Low Energy Neutrino Data: Quasi-elastic cross section

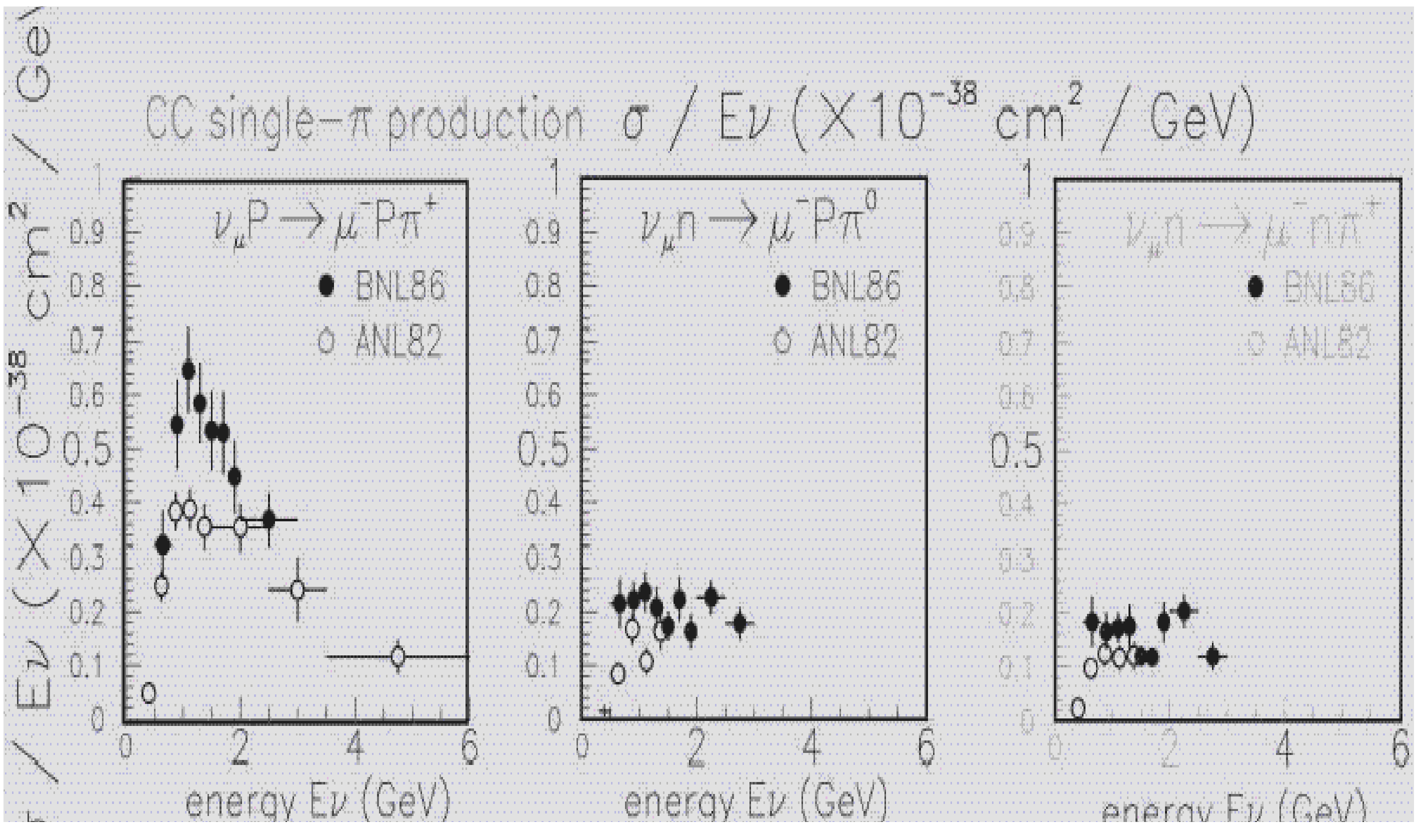


Examples of Low Energy Neutrino Data: Total (inelastic and quasielastic) cross section



E GeV

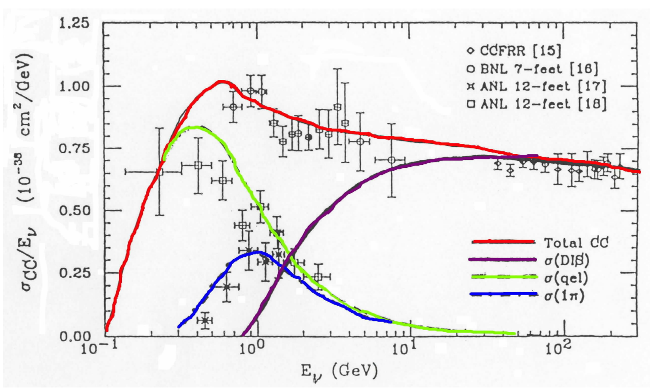
Examples of Current Low Energy Neutrino Data: Single charged and neutral pion production

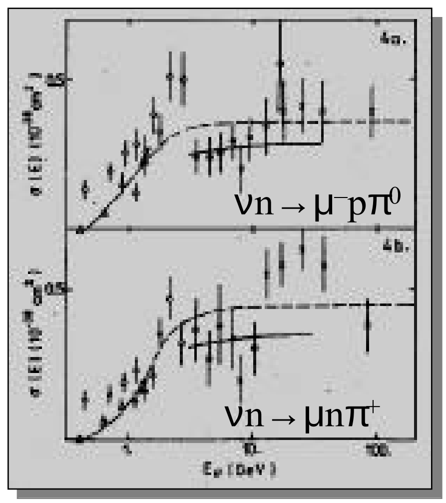


Old bubble chamber language

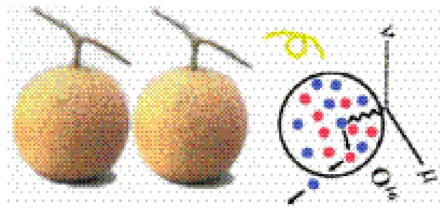
Status of Cross-Sections

- Not well-known, especially in region of NUMI 0.7^0 off-axis proposal (~2 GeV)





Note: 2nd conf.
NuInt02 to
Be held at
UC Irvine
Dec 12-15,2002
Needed even for the
Low statistics at K2K
Bring people of
All languages
And nuclear and
Particle physicists
Together.



NuInt01 : The First International Workshop on Neutrino-Nucleus Interactions in the Few GeV Region

December 13-16, 2001, KEK, Tsukuba, Japan

List of participants(PDF)

What do we want to know about low energy neutrino reactions and why- 1

Reasons

- <u>Intellectual Reasons:</u>
- **Understand how QCD works in both neutrino and electron scattering at low energies - different spectator quark effects. (*There are fascinating issues here as we will show*)**
- **How is fragmentation into final state hadrons affected by nuclear effects in electron versus neutrino reactions.**
- **Of interest to : Nuclear Physics/Medium Energy, QCD/ Jlab communities**

- <u>**Practical Reasons**</u>**:**

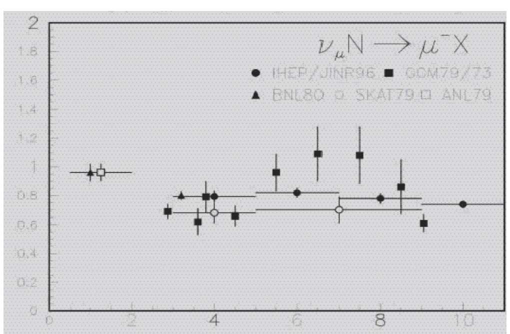
- **Determining the neutrino sector mass and mixing matrix precisely**
  - **requires knowledge of both Neutral Current (NC) and Charged Current(CC) differential Cross Sections and Final States**
  - **These are needed for the NUCLEAR TARGET from which the Neutrino Detector is constructed (e.g Water, Carbon, Iron).**

- **Particle Physics/ HEP/ FNAL /KEK/ Neutrino communities**

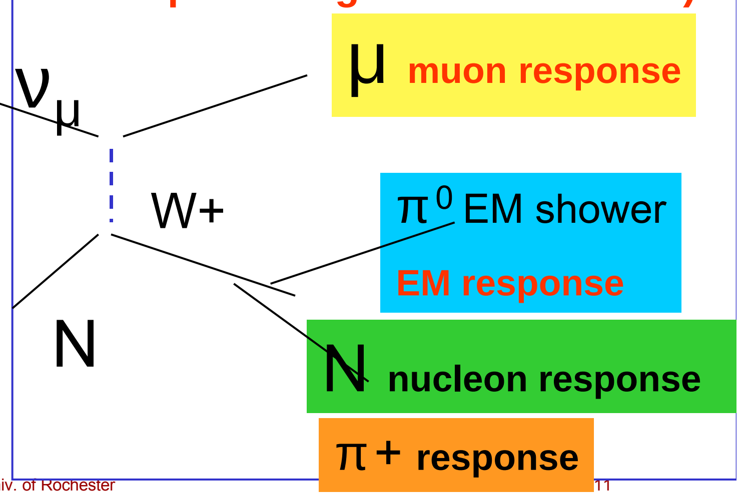
Neutrino Processes of Interest- 2

Charged - Current: both differential cross sections and final states

- **<u>Neutrino mass ΔM^2: -></u>** **<u>Charged Current Cross Sections and Final States are needed</u>:**
- The level of neutrino charged current cross sections versus energy provide the baseline against which one measures ΔM^2 at the oscillation maximum.



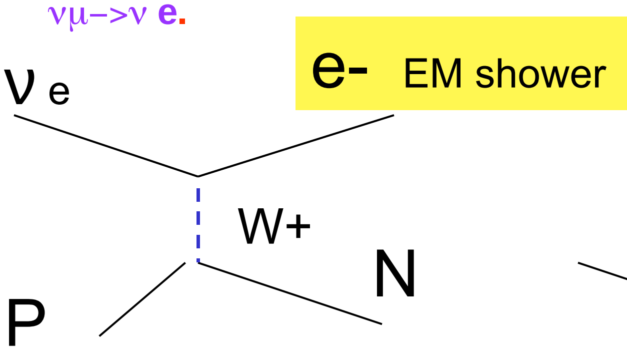
- **Measurement of the neutrino energy in a detector depends on the composition of the final states (different response to charged and neutral pions, muons and final state protons (e.g. Cerenkov threshold, non compensating calorimeters etc).**

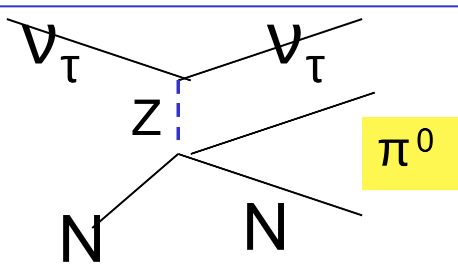


Neutrino Processes of Interest- 3

Neutral - Current both differential cross sections and final states

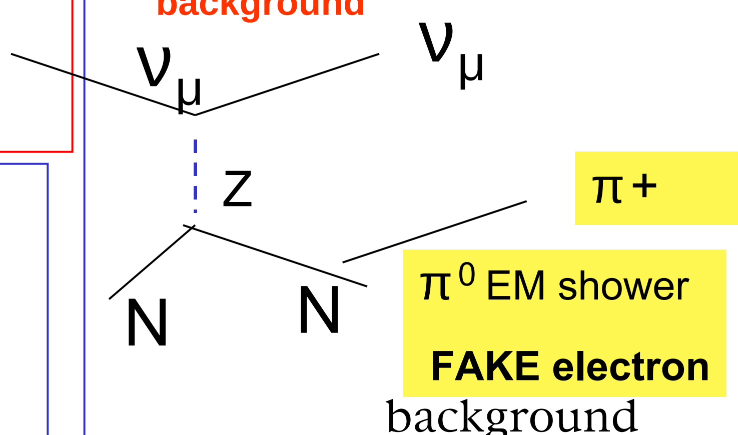
- **SIGNAL $\nu_\mu \to \nu_e$ transition**
- ~ **0.1% oscillations probability of $\nu_\mu \to \nu_e$.**





SIGNAL ν_τ

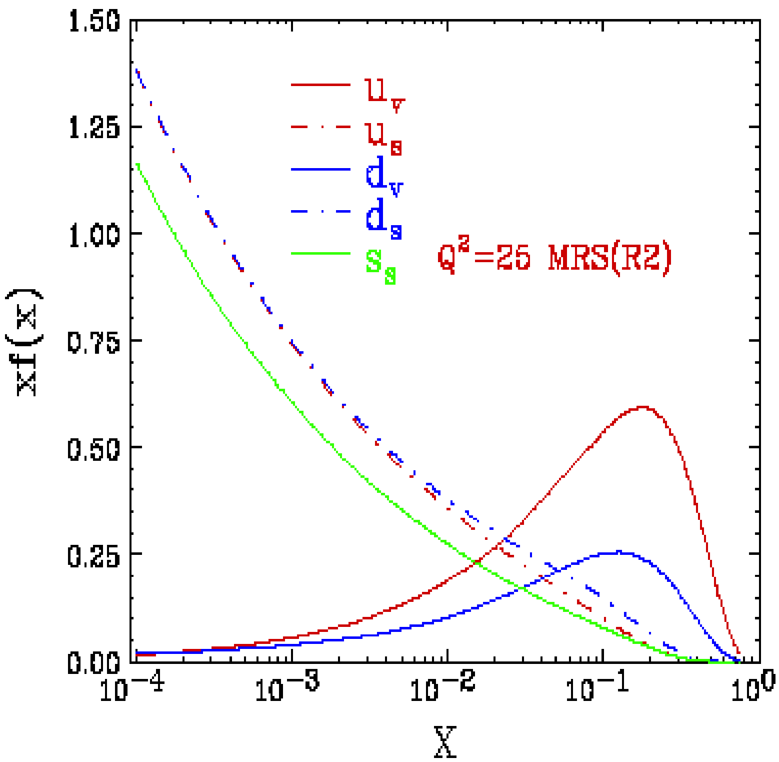
- **Backgrounds: <u>Neutral Current Cross Sections and Final State Composition are needed</u>:**
- **Electrons from Misidentified π^0 in NC events without a muon from higher energy neutrinos are a background**



How are PDFs Extracted from global fits to High Q2 Deep Inelastic e/μ/ν Data

Note: additional information on Antiquarks from Drell-Yan and on Gluons from p-pbar jets also used.

MRSR2 PDFs



$$u_V + d_V \xrightarrow{from} F_2^{\nu} \approx x(u+\bar{u}) + x(d+\bar{d})$$

$$xF_3^{\nu} \approx x(u-\bar{u}) + x(d-\bar{d})$$

$$u+\bar{u} \xrightarrow{from} {}^{\mu}F_2^{p} \approx \frac{4}{9}x(u+\bar{u}) + \frac{1}{9}x(d+\bar{d})$$

$$d+\bar{d} \xrightarrow{from} {}^{\mu}F_2^{n} \approx \frac{1}{9}x(u+\bar{u}) + \frac{4}{9}x(d+\bar{d})$$

$$\left.\begin{array}{l}\textit{nuclear effects}\\ \textit{typically ignored}\end{array}\right\} \; {}^{\mu}F_2^{n} = 2\frac{{}^{\mu}F_2^{d}}{{}^{\mu}F_2^{p}} - 1$$

$$d/u \xrightarrow{from} p\bar{p}W^{\;Asymmetry} \approx \frac{d/u(x_1) - d/u(x_2)}{d/u(x_1) + d/u(x_2)}$$

At high x, deuteron binding effects introduce an uncertainty in the d distribution extracted from F2d data (but not from the W asymmetry data).

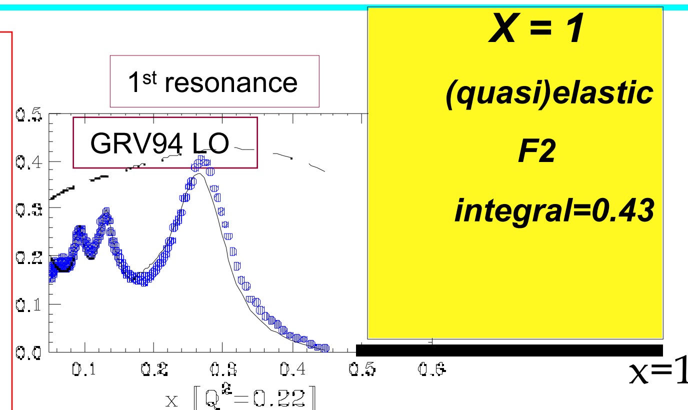
Neutrino cross sections at low energy

Neutrino interactions --

- **Quasi-Elastic / Elastic (W=Mp)**
  ν_μ + n --> μ^- + p (x =1, W=Mp)
  well measured and described by form factors (but need to account for Fermi Motion/binding effects in nucleus) e.g. Bodek and Ritchie (Phys. Rev. D23, 1070 (1981)

- **Resonance (low Q^2, W< 2)**
  ν_μ + p --> μ^- + p + π Poorly measured and only 1st resonance described by Rein and Seghal

- **Deep Inelastic**
  ν_μ + p --> μ^- + X (high Q^2, W> 2)
  well measured by high energy experiments and well described by quark-parton model (pQCD with NLO PDFs), but doesn't work well at low Q^2 region.



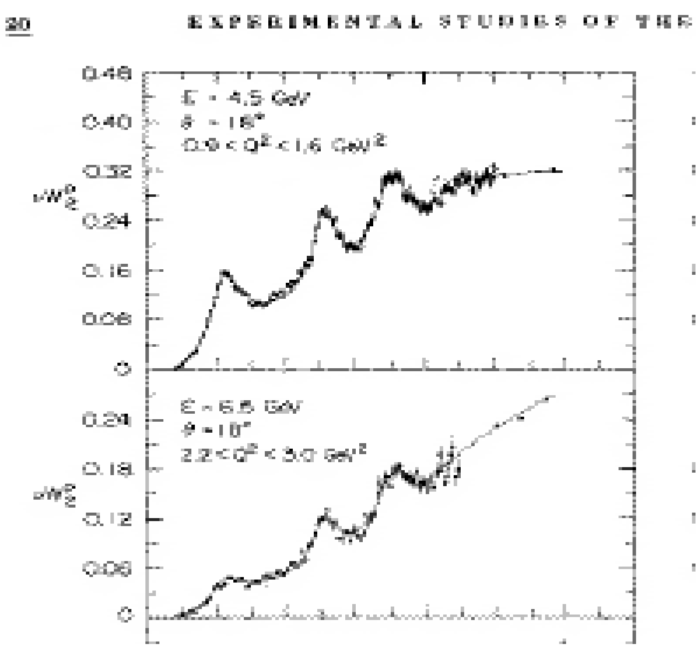
(e.g. SLAC data at Q^2=0.22)

- **Issues at few GeV :**
- **Resonance production and low Q^2 DIS contribution meet.**
- **The challenge is to describe ALL THREE processes at ALL neutrino (or electron) energies**
- **HOW CAN THIS BE DONE? - Subject of this TALK**

MIT SLAC DATA 1972 e.g. E0 = 4.5 and 6.5 GeV

e-P scattering A. Bodek PhD thesis 1972

[PRD 20, 1471(1979)] Proton Data

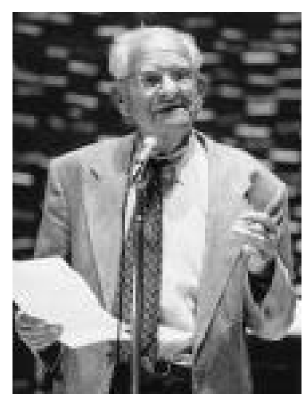


‘ The electron scattering data in the Resonance Region is the “Frank Hertz Experiment” of the Proton. The Deep Inelastic Region is the “Rutherford Experiment” of the proton’

V. Weisskopf * (former faculty member at Rochester and at MIT when he showed these data at an MIT Colloquium in 1971 (* died April 2002 at age 93)

What do The Frank Hertz” and “Rutherford Experiment” of the proton’ have in common?

A: Quarks! And QCD



Building up a model for all Q^2.

- **Can we build up a model to describe all Q^2 region from high down to very low energies ? [resonance, DIS, even photo production]**
- **Advantage if we describe it in terms of the quark-parton model.**
- **then it is straightforward to convert charged-lepton scattering cross sections into neutrino cross section. (just matter of different couplings)**

o Final state hadrons implemented in terms of fragmentation functions.

o Nuclear dependence of PDFs and fragmentation functions can be included.

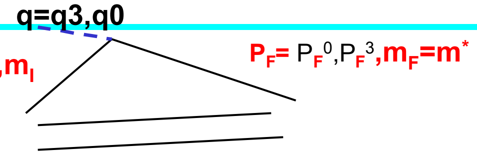
Challenges

- Understanding of high x PDFs at very low Q^2?
- There is a of wealth SLAC, JLAB data, but it requires understanding of non-perturbative QCD effects.
- Need better understanding of resonance scattering in terms of the quark-parton model? (duality works, many studies by JLAB)
- Need to satisfy photoproduction limits at Q^2=0.
- At high Q^2 should agree with QCD PDFs and sum rules
- At ALL Q2 should agree with Current Algebra sum rules.
- Should have theoretical basis
- If one knows where the road begins (high Q2 PDFs) and ends (Q2=0 photo-production), it is easier to build it.

Initial quark mass m_I and final mass ,m_F=m^* bound in a proton of mass M -- Summary: INCLUDE quark initial Pt) Get ξ scaling (not $x=Q^2/2M\nu$)

ξ *Is the correct variable which is Invariant in any frame : q3 and P in opposite directions.*

ξ



$PI, P0$ $\quad$ $q3, q0$

$$\xi = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \qquad \text{quark} \longrightarrow \quad \longleftarrow \text{photon}$$

$$(q + P_I)^2 = P_F^2 \quad \rightarrow q^2 + 2P_I\, q + P_I^2 = m_F^2$$

$$\xi = \frac{Q^2 + m_F^2}{M\nu[1 + \sqrt{(1 + Q^2/\nu^2)}]} \qquad for\ m_I^2, Pt = 0$$

Special cases:
Numerator m_F^2: Slow Rescaling ξ as in charm production

Denominator: Target mass effect, e.g. Nachtman Variable ξ, Light Cone Variable ξ, Georgi Politzer Target Mass ξ

Most General Case:

$$\xi'_w = [Q'^2 + B] / [M\nu (1+(1+Q^2/\nu^2))^{1/2} + A]$$

where $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + [(Q^2 + m_F^2 - m_I^2)^2 + 4Q^2 (m_I^2 + P^2t)]^{1/2}$

For the case of Pt2=0 see R. Barbieri et al Phys. Lett. 64B, 1717 (1976) and Nucl. Phys. B117, 50 (1976)

Add B and A to account for effects of additional Δ m^2 from NLO and NNLO (up to infinite order) QCD effects.

Initial quark mass m_I and final mass ,m_F=m * bound in a proton of mass M -- Page 1 INCLUDE quark initial Pt) Get ξ scaling (not x=Q²/2Mν) DETAILS

ξ *Is the correct variable which is Invariant in any frame : q3 and P in opposite directions.*

$$PI,P0 \qquad q3,q0$$

$$\xi = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \quad \xrightarrow{quark} \quad \xleftarrow{photon}$$

$$In - LAB - Frame : \rightarrow \quad P_P^0 = M, P_P^3 = 0$$

$$\xi = \frac{P^0_{I-LAB} + P^3_{I-LAB}}{M} \rightarrow P^0_{I-LAB} + P^3_{I-LAB} = \xi M$$

$$\xi = \frac{(P_I^0 + P_I^3)(P_I^0 - P_I^3)}{M(P_I^0 - P_I^3)} = \frac{(P_I^0)^2 - (P_I^3)^2}{M(P_I^0 - P_I^3)}$$

$$\xi M(P_I^0 - P_I^3) = (m_I^2 + Pt^2)$$

$$\rightarrow P_I^0 - P_I^3 = (m_I^2 + Pt^2)/(\xi M)$$

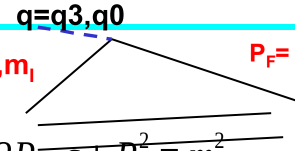
$$(1): \quad P_I^0 - P_I^3 = (m_I^2 + Pt^2)/(\xi M)$$

$$(2): \quad P_I^0 + P_I^3 = \xi M$$

$$2P_I^0 = \xi M + (m_I^2 + Pt^2)/(\xi M) \xrightarrow{m, Pt=0} \xi M$$

$$2P_I^3 = \xi M - (m_I^2 + Pt^2)/(\xi M) \xrightarrow{m, Pt=0} \xi M$$

ξ



$$(q + P_I)^2 = P_F^2 \quad \rightarrow q^2 + 2P_I q + P_I^2 = m_F^2$$

$$2(P_I^0 q^0 + P_I^3 q^3) = Q^2 + m_F^2 - m_I^2 \qquad Q^2 = -q^2 = (q^3)^2 - (q^0)^2$$

$$In - LAB - Frame : \rightarrow \qquad Q^2 = -q^2 = (q^3)^2 - \nu^2$$

$$[\xi M + (m_I^2 + Pt^2)/(\xi M)]\nu + [\xi M - (m_I^2 + Pt^2)/(\xi M)]q^3$$

$$= Q^2 + m_F^2 - m_I^2 \quad : General$$

$$Set: m_I^2, Pt = 0 \qquad (for\ now)$$

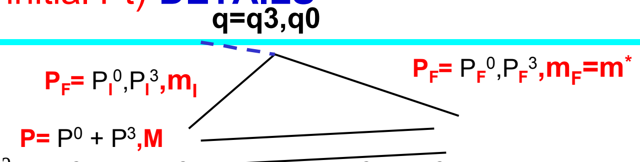
$$\xi M\nu + \xi M q^3 = Q^2 + m_F^2$$

$$\xi = \frac{Q^2 + m_F^2}{M(\nu + q^3)} = \frac{Q^2 + m_F^2}{M\nu(1 + q^3/\nu)} \quad for\ m_I^2, Pt = 0$$

$$\xi = \frac{Q^2 + m_F^2}{M\nu[1 + \sqrt{(1 + Q^2/\nu^2)}]} \qquad for\ m_I^2, Pt = 0$$

Special cases : Denom − TM term, Num − Slow rescaling

initial quark mass m_I and final mass $m_F=m^*$ bound in a proton of mass M -- Page 2 INCLUDE quark initial Pt) DETAILS



ξ *For the case of non zero* m_I, P_t

(note P and q3 are opposite)

$$\xi = \frac{P_I^0 + P_I^3}{P_P^0 + P_P^3} \qquad \begin{matrix} PI,P0 & q3,q0 \\ \text{quark} \rightarrow & \leftarrow \text{photon} \end{matrix}$$

$$In-LAB-Frame:\rightarrow \quad P_P^0 = M, P_P^3 = 0$$

$$(1) : 2P_I^0 = \xi M + (m_I^2 + Pt^2)/(\xi M) \quad \rightarrow\rightarrow\rightarrow\rightarrow$$

$$(1) : 2P_I^3 = \xi M - (m_I^2 + Pt^2)/(\xi M) \quad \rightarrow\rightarrow\rightarrow\rightarrow$$

$$(q+P_I)^2 = P_F^2 \quad \rightarrow q^2 + 2P_I\, q + P_I^2 = m_F^2$$

$$Q^2 = -q^2 = (q^3)^2 - \nu^2$$

$$[\xi M + (m_I^2 + Pt^2)/(\xi M)]\nu + [\xi M - (m_I^2 + Pt^2)/(\xi M)]q^3$$

$$= Q^2 + m_F^2 - m_I^2$$

Keep all terms here and : multiply by ξ **M** and group terms in ξ qnd ξ **2**

$\xi^2\ M^2(\nu+q3) - \xi\ M[Q^2 + m_F^2 - m_I^2] + [m_I^2 + Pt^2(\nu - q3)^2] = 0$ General Equation

a b c

=> solution **of quadratic equation** $\xi = [-b + (b^2 - 4ac)^{1/2}] / 2a$

use $(\nu^2 - q3^2) = q^2 = -Q^2$ and $(\nu + q3) = \nu + \nu[1 + Q^2/\nu^2]^{1/2} = \nu + \nu[1 + 4M^2 x^2/Q^2]^{1/2}$

$\xi'_w = [Q'^2 + B] / [M\nu(1+(1+Q^2/\nu^2))^{1/2} + A]$

where $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + [(Q^2 + m_F^2 - m_I^2)^2 + 4Q^2(m_I^2 + P^2t)]^{1/2}$

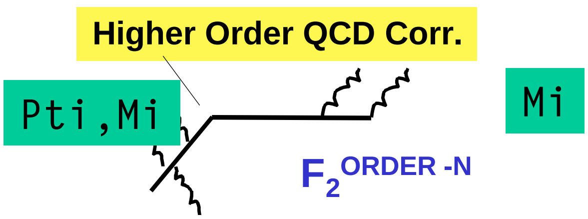
Add B and A to account for effects of additional Δm^2 from NLO and NNLO effects.

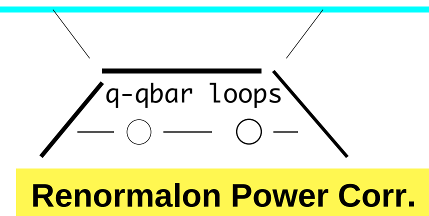
or $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + [Q^4 + 2Q^2(m_F^2 + m_I^2 + 2P^2t) + (m_F^2 - m_I^2)^2]^{1/2}$

$\xi_w = [Q'^2 + B] / [M\nu(1 + [1 + 4M^2 x^2/Q^2]^{1/2}) + A]$ (equivalent form)

$\xi_w = x[2Q'^2 + 2B] / [Q^2 + (Q^4 + 4x^2 M^2 Q^2)^{1/2} + 2Ax]$ (equivalent form)

QCD is an asymptotic series, not a converging series- at any order, there are power corrections





$$F_2^{ORDER\ -N} = F_2^{QCD-0}\{1 + C_1(x,Q)\,\alpha_S + C_2(x,Q)\,\alpha_S^2 + \ldots\ldots\ C_N(x,Q)\,\alpha_S^{\ N}\}$$

$\Delta F_2 = F_2^{ALL\ ORDERS} - F_2^{ORDER\ N}$ --> (The series is Truncated)

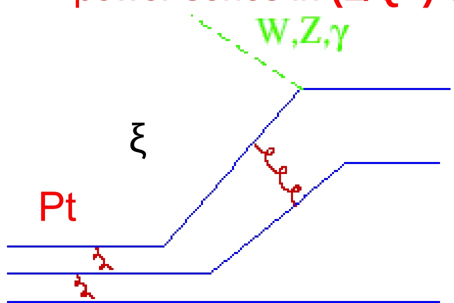
$$\Delta F_2 = \text{Power Corrections} = (1/Q^2)\ a_{2,N}\,D_2(x,Q^2) + (1/Q^4)\ a_{4,N}\,D_4(x,Q^2)$$

In pQCD the $(1/Q^2)$ terms from the interacting quark are the missing higher order terms. Hence, $a_{2,N}$ and $a_{4,N}$ should become smaller with N. The only other HT terms are from the final state interaction with the spectator quarks, which should only affect the low W region. Our studies have shown that to a good approximation, if one includes the known target mass (TM) effects, the spectator quarks do not affect the average level of the low W cross section as predicted by pQCD if the power corrections from the interacting quark are included.

Preview: Will model multi-gluon emission with Mi,Pti, Mf

What are Higher Twist Effects- page 1

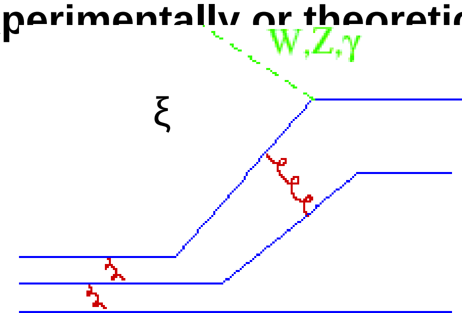
- **Higher Twist Effects** are terms in the structure functions that behave like a power series in **$(1/Q^2)$ or $[Q^2/(Q^4+A)]$,… $(1/Q^4)$** etc.…



(a)**Higher Twist: Interaction between Interacting and Spectator quarks via gluon exchange at Low Q2-at low W**
(b) **Interacting quark TM binding, initial Pt and Missing Higher Order QCD terms DIS region. ->$(1/Q^2)$ or $[Q^2/(Q^4+A)]$,… $(1/Q^4)$.**

•**While pQCD** predicts terms in α_s^{2} **$(\sim 1/[\ln(Q^2/\Lambda^2)])$…** α_s^{4} etc…

•(i.e. LO, NLO, NNLO etc.) **In the few GeV region, the terms of the two power series cannot be distinguished, experimentally or theoretically**



high Q2 impulse
ion, the interacting
spectator quarks
ved and do not
ct each other.

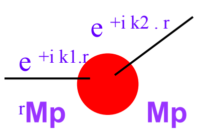
In NNLO p-QCD additional gluons emission: terms like α_s^{2} **$(\sim 1/[\ln(Q^2/\Lambda^2)])$…** α_s^{4}
Spectator quarks are not Involved.

What are Higher Twist Effects - Page 2-details

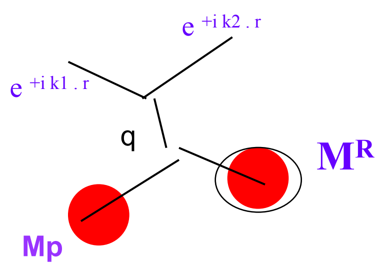
- **Nature has "evolved" the high Q^2 PDF from the low Q^2 PDF,** therefore, **the high Q^2 PDF** include the information about the higher twists .
- High Q^2 manifestations of higher twist/non perturbative effects include: difference between u and d, the difference between d-bar, u-bar and s-bar etc. High Q^2 PDFs "remember" the higher twists, which originate from the non-perturbative QCD terms.
- **Evolving back the high Q^2 PDFs *to low Q^2* (e.g. NLO-QCD) and comparing to low Q^2 data is one way to check for the effects of higher order terms.**
- What do these higher twists come from?
  - **Kinematic higher twist** – initial state target mass binding (Mp, ξ_{TM}) initial state and final state quark masses (e.g. charm production)- ξ_{TM} important at high x
  - **Dynamic higher twist** – correlations between quarks in initial or final state.==> Examples : Initial or final state multiquark correlations: diquarks, elastic scattering, excitation of quarks to higher bound states e.g. resonance production, exchange of many gluons: important at low W
  - **Non-perturbative effects** to satisfy gauge invariance and connection to photo-production [e.g. $F_2(\nu, Q^2=0)$ = **Q^2 / [Q^2 +C]=** 0]. important at very low Q2.
  - **Higher Order QCD effects -** to e.g. NNLO+ multi-gluon emission"looks like" Power higher twist corrections since a LO or NLO calculation do not take these into account, also quark intrinsic P_T (terms like P_T^2/Q^2). Important at all x (look like **Dynamic Higher Twist**)

Old Picture of fixed W scattering - form factors (the Frank Hertz Picture)

- **OLD Picture fixed W: Elastic Scattering, Resonance Production. Electric and Magnetic Form Factors (G_E and G_M) versus Q^2 measure size of object (the electric charge and magnetization distributions).**



- **Elastic scattering W = M^p = M**, single final state nucleon: Form factor measures size of nucleon.Matrix element squared $| \langle p_f | V(r) | p_i \rangle |^2$ between initial and final state lepton plane waves. Which becomes:
- $| \langle e^{-i\,k2 . r} | V(r) | e^{+i\,k1 . r} \rangle |^2$
- q = k1 - k2 = momentum transfer
- $G_E(q) = \int \{ e^{\,i\,q\,.\,r} \rho(r)\, d^3r \}$ = Electric form factor is the Fourier transform of the charge distribution. Similarly for the magnetization distribution for G_M Form factors are relates to structure function by:
- $2xF_1(x, Q^2)_{elastic} = x^2 G_M{}^2{}_{elastic}(Q^2)\, \delta(x-1)$



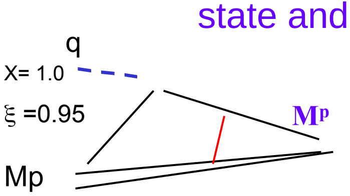
- **Resonance Production, W=M^R,** Measure transition form factor between a quark in the ground state and a quark in the first excited state. For the Delta 1.238 GeV first resonance, we have a Breit-Wigner instead of δ (x-1).
- $2xF_1(x, Q^2)_{resonance} \sim x^2 G_M{}^2{}_{Res.\ transition}(Q^2)\, BW(W-1.238)$

Duality: Parton Model Pictures of Elastic and Resonance Production at Low W (High Q2)

Elastic Scattering, Resonance Production: Scatter from one quark with the correct parton momentum ξ, and the two spectator are just right such that a final state interaction $\boldsymbol{A_w}$ **(w, Q²)** makes up a proton, or a resonance.

Elastic scattering W = M^p = M, single nucleon in final state.

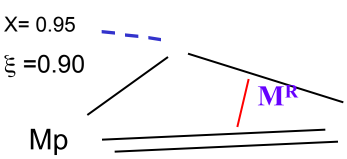
The scattering is from a quark with a very high value of ξ, is such that one cannot produce a single pion in the final state and the final state interaction makes a proton.



$\boldsymbol{A_w}$ **(w, Q²) =** δ (**x-1) and the level is the {integral over** ξ, from pion threshold to ξ =1 } : **local duality**

(This is a check of local duality in the extreme, better to use measured Ge,Gm, Ga, Gv)

Note: in Neutrinos (axial form factor within 20% of vector form factor)



Resonance Production, W=M^R, e.g. delta 1.238 resonance. The scattering is from a quark with a high value of ξ, is such that that the final state interaction makes a low mass resonance. $\boldsymbol{A_w}$ **(w, Q²) includes** Breit-Wigners. *Local duality*

Also a check of local duality for electrons and neutrinos

With the correct scaling variable, and if we account for low W and low Q2 higher twist effects, the prediction using QCD PDFs q (ξ, Q^2) should give an average of F2 in the elastic scattering and in the resonance region. (including both resonance and continuum contributions). If we **modulate** the PDFs with a final state interaction **resonance *A* (w, Q²)** we could also reproduce the various Breit-Wigners + continuum.

Photo-production Limit $Q^2=0$
Non-Perturbative - QCD evolution freezes

- Photo-production Limit: Transverse Virtual and Real Photo-production cross sections must be equal at $Q^2=0$. Non-perturbative effect.
- There are no longitudinally polarized photons at $Q^2=0$
-
  - $\sigma_L(\nu, Q^2) = 0$ limit as $Q^2 \to 0$
  - Implies $R(\nu, Q^2) = \sigma_L/\sigma_T \sim Q^2 / [Q^2 + \text{const}] \to 0$ limit as $Q^2 \to 0$
  -
  - $\sigma(\gamma\text{-proton}, \nu) = \sigma_T(\nu, Q^2)$ limit as $Q^2 \to 0$
  - implies $\sigma(\gamma\text{-proton}, \nu) = 0.112 \text{ mb } 2xF_1(\nu, Q^2) / (KQ^2)$ limit as $Q^2 \to 0$
  - $\sigma(\gamma\text{-proton}, \nu) = 0.112 \text{ mb } F_2(\nu, Q^2) D / (KQ^2)$ limit as $Q^2 \to 0$
  - or $F_2(\nu, Q^2) \sim Q^2 / [Q^2 + C] \to 0$ limit as $Q^2 \to 0$
- $K = [1 - Q^2/2M\nu]$ $D = (1 + Q^2/\nu^2)/(1+R)$
- If we want PDFs to work down to $Q^2=0$ where pQCD freezes
  - The PDFs must be multiplied by a factor $Q^2 / [Q^2 + C]$ (where C is a small number).
  - The scaling variable x does not work since $\sigma(\gamma\text{-proton}, \nu) = \sigma_T(\nu, Q^2)$
  - At $Q^2 = 0$ $F_2(\nu, Q^2) = F_2(x, Q^2)$ with $x = Q^2 / (2M\nu)$ reduces to one point x=0
  - However, a scaling variable $\xi_w = (Q^2 + B) / (2M\nu)$ works at $Q^2 = 0$
  - $F_2(\nu, Q^2) = F_2(\xi_c, Q^2) = F_2\ [B/(2M\nu), 0]$ limit as $Q^2 \to 0$

How do we "measure" higher twist (HT)

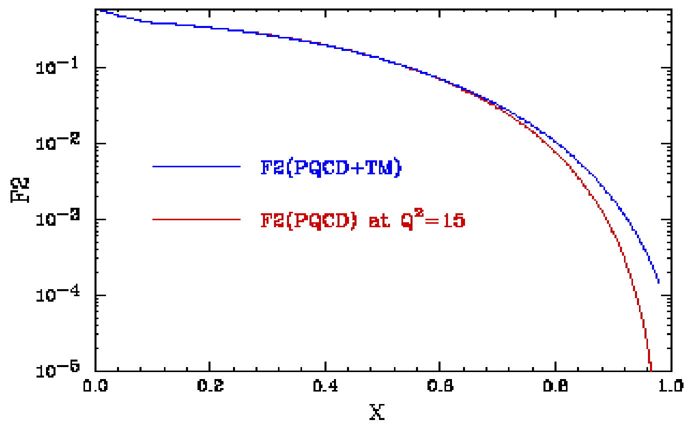
- Take a set of QCD PDF which were fit to high Q^2 ($e/\mu/\nu$) data (in Leading Order-LO, or NLO, or NNLO)
- Evolve to low Q2 (NNLO, NLO to Q^2=1 GeV^2) (LO to Q^2=0.24)
- Include the "known" kinematic higher twist from initial target mass (proton mass) and final heavy quark masses (e.g. charm production).
- Compare to low Q^2data in the DIS region (e.g. SLAC)
- The difference between data and QCD+target mass predictions is the extracted "effective" dynamic higher twists.
- Describe the extracted "effective" dynamic higher twist within a specific HT model (e.g. QCD renormalons, or a purely empirical model).
- Obviously - results will depend on the QCD order LO, NLO, NNLO (since in the 1 GeV region $1/Q^2$and $1/LnQ^2$ are similar). In lower orders, the "effective higher twist" will also account for missing QCD higher order terms. The question is the relative size of the terms.
    - o **Studies in NLO - Yang and Bodek: Phys. Rev. Lett 82, 2467 (1999) ;ibid 84, 3456 (2000)**
    - o **Studies in NNLO - Yang and Bodek: Eur. Phys. J. C13, 241 (2000)**
    - o **Studies in LO - Bodek and Yang: hep-ex/0203009 (2002)**
    - o **Studies in QPM 0th order - Bodek, el al PRD 20, 1471 (1979)**

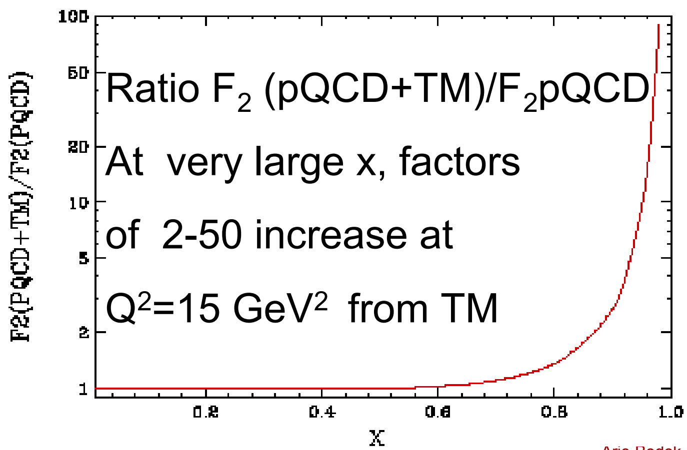
Lessons from Two 99,00 QCD studies

- Our NLO study comparing **NLO PDFs** to DIS SLAC, NMC, and BCDMS e/μ scattering data on H and D targets shows (for $Q^2 > 1$ GeV^2)
  [ref:Yang and Bodek: Phys. Rev. Lett 82, 2467 (1999)]
  - *Kinematic Higher Twist* (**target mass**) effects are **large** and important at large x, and must be included in the form of Georgi & Politzer ξ_{TM} scaling.
  - *Dynamic Higher Twist* effects are smaller, but need to be included. (A second NNLO study established their origin)
  - The ratio of d/u at high x must be increased if nuclear binding effects in the deuteron are taken into account.
  - *The Very high x (=0.9) region* - is described by **NLO QCD** (if target mass and renormalon higher twist effects are included) to better than 10%. SPECTATOR QUARKS modulate $A(W,Q^2)$ ONLY.
  - *Resonance region*: NLO pQCD + Target mass + Higher Twist describes average F_2 in the resonance region (duality works). ***Include*** A_w **(w, Q^2)** resonance modulating function from spectator quarks later.
- A similar NNLO study using **NNLO QCD** we find that the "empirically measured "effective" **Dynamic Higher Twist Effects** in the NLO study come from the **missing NNLO higher order QCD terms**. [ref: Yang and Bodek Eur. Phys. J**. C13**, 241 (2000)]

Kinematic Higher-Twist (GP target mass:TM)

Georgi and Politzer Phys. Rev. D14, 1829 (1976): **Well known**





ξ_{TM+c} = { 2x / [1 + k] } [1+ Mc^2/Q^2]

(last term only for heavy charm product)

k= $(1 + 4x^2 M^2/Q^2)^{1/2}$ (target mass part)

(Derivation of ξ_{TM} in Appendix)

For Q^2 large (valence) $F_2 = 2\xi F_1 = \xi F_3$

$F_2^{pQCD+TM}(x,Q^2) = F_2^{pQCD}(\xi, Q^2)\, x^2/[k^3\xi^2]$

$+J_{1*}\,(6M^2x^3/[Q^2k^4]) + J_{2*}(12M^4x^4/[Q^4k^5])$

$2F_1^{pQCD+TM}(x,Q^2) = 2F_1^{pQCD}(\xi, Q^2)\, x/[k\xi]$

$+J_{1*}\,(2M^2x^2/[Q^2k^2]) + J_{2*}(4M^4x^4/[Q^4k^5])$

$F_3^{pQCD+TM}(x,Q^2) = F_3^{pQCD}(\xi, Q^2)\, x/[k^2\xi]$

$+J_{1F3*}\,(4M^2x^2/[Q^2k^3])$

For charm production replace x above

With -> x [1+ Mc^2/Q^2]

$$J_1 = \int_\xi^1 du\, F_2^{pQCD}(u,Q^2)/u^2$$

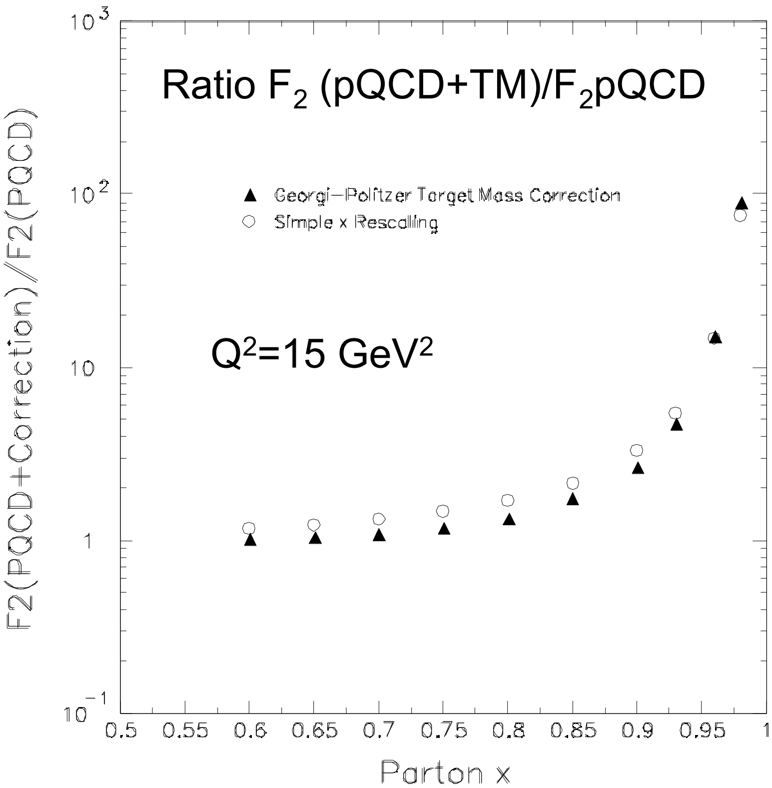
$$J_{1F3} = \int_\xi^1 du\, F_3^{pQCD}(u,Q^2)/u$$

$$J_2 = \int_\xi^1 du \int_u^1 dV\, F_2^{pQCD}(V,Q^2)/V^2$$

Kinematic Higher-Twist (target mass:TM)

$\xi_{TM} = Q^2 / [M\nu (1 + (1 + Q^2/\nu^2)^{1/2})]$

Compare complete Target-Mass calculation to simple rescaling in ξ_{TM}

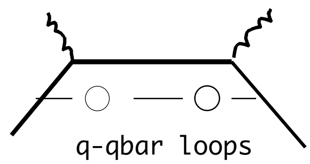


- The Target Mass Kinematic Higher Twist effects comes from the fact that the quarks are bound in the nucleon. They are important at low Q^2 and high x. They involve change in the scaling variable from x to ξ_{TM} and various kinematic factors and convolution integrals in terms of the PDFs for xF1, F_2 and xF_3
- Above x=0.9, this effect is mostly explained by a simple rescaling in ξ_{TM}.

$F_2^{pQCD+TM}(x, Q^2) = F_2^{pQCD}(\xi_{TM}, Q^2)$

Dynamic Higher Twist- Renormalon Model

- Use: Renormalon QCD model of Webber&Dasgupta- Phys. Lett. B382, 272 (1996), Two parameters a_2 and a_4. This model includes the (1/ Q^2) and (1/ Q^4) terms from gluon radiation turning into virtual quark antiquark fermion loops (from the interacting quark only, the spectator quarks are not involved).



- $F_2^{\text{theory}}(x,Q^2) = F_2^{\text{PQCD+TM}}\ [1+ D_2(x,Q^2) + D_4(x,Q^2)]$

$D_2(x,Q^2) = (1/Q^2)\ [\ a_2 / q(x,Q^2)\ ]\ \circ\ (dz/z)\ c_2(z)\ q(x/z, Q^2)$

$D_4(x,Q^2) = (1/Q^4)\ [\ a_4$ times function of x)]

In this model, the higher twist effects are different for $2xF_1$, xF_3, F_2. With complicated x dependences which are defined by only two parameters a_2 and a_4. (the $D_2(x,Q^2)$ term is the same for $2xF_1$ and , xF_3)

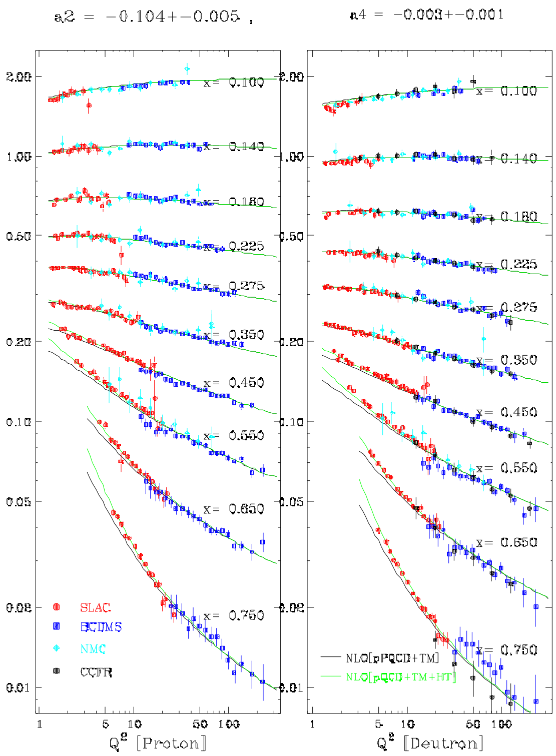
Fit a_2 and a_4 to experimental data for F_2 and $R=F_L/2xF_1$.

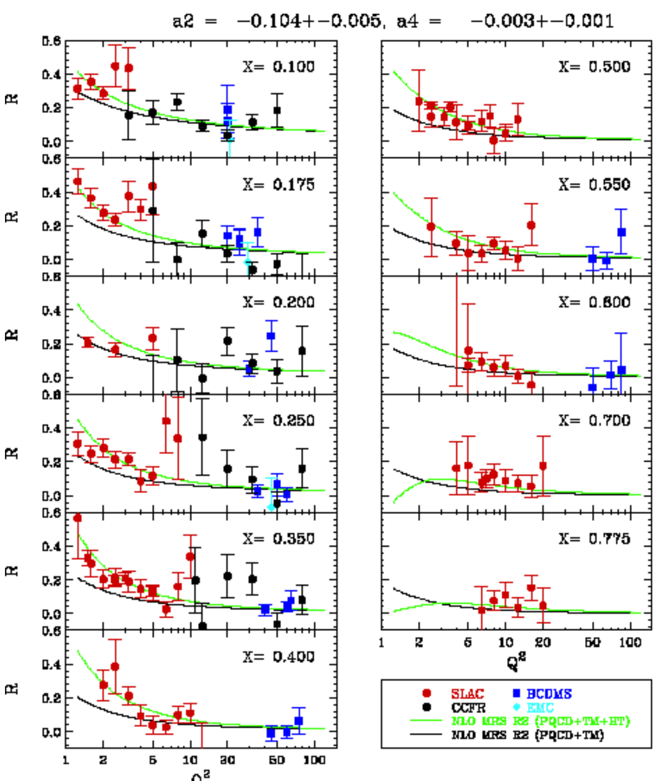
$F_2^{\text{data}}(x,Q^2) = [\ F_2^{\text{measured}} + \lambda\,\delta F_2^{\text{syst}}\ ]\ (1+ \mathbf{N})$: χ^2 weighted by errors

where **N** is the fitted normalization (within errors) and δF_2^{syst} is the is the fitted correlated systematic error BCDMS (within errors).

F_2, R comparison of QCD+TM plot (Q^2>1) vs. NLO QCD+TM+HT (use QCD Renormalon Model for HT)

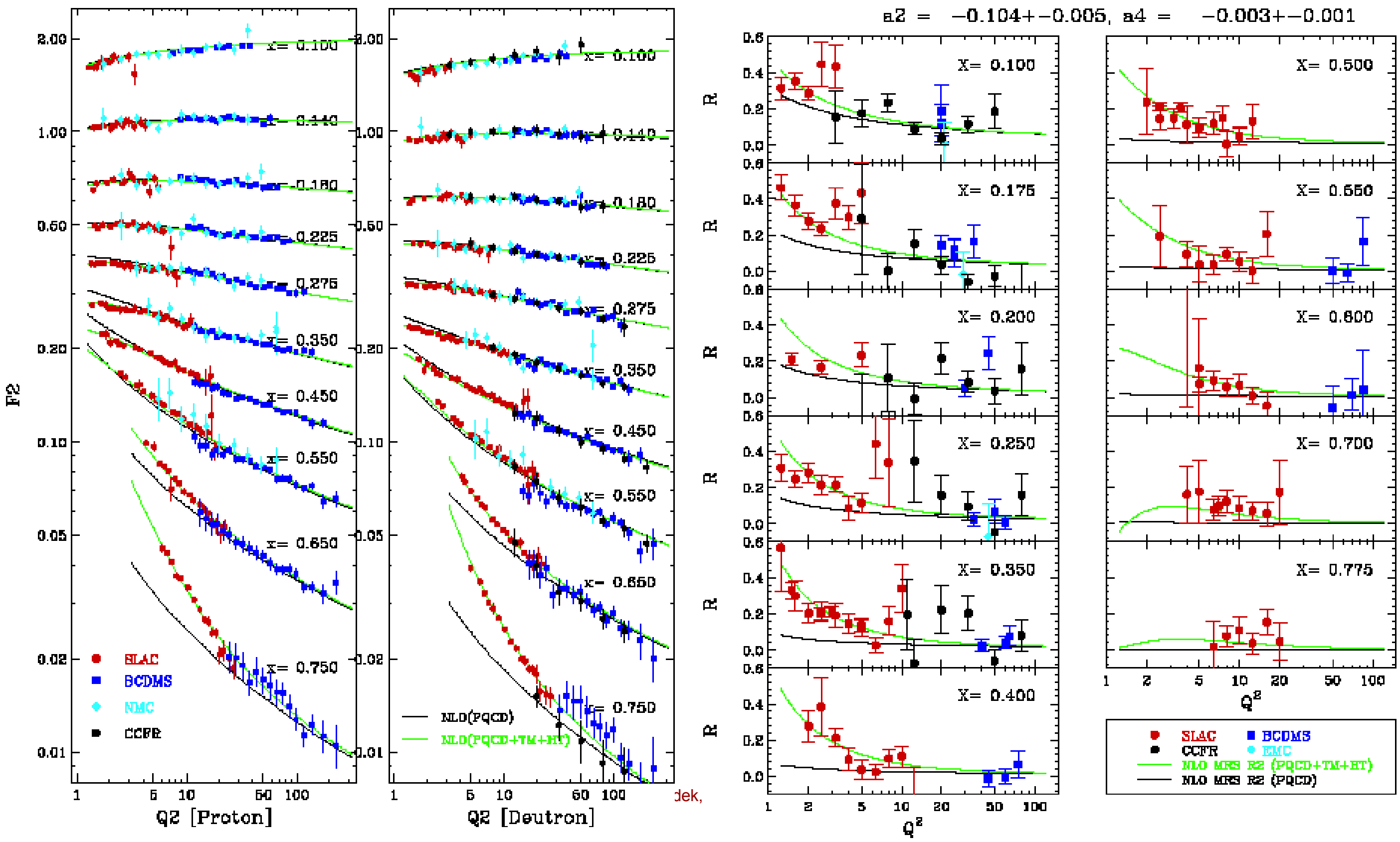
PDFs and QCD in NLO + TM + QCD Renormalon Model for Dynamic HTdescribe the F2 and R data very well, with only 2 parameters. Dynamic HT effects are there but small





Same study showing the QCD-only Plot (Q^2>1) vs. NLO QCD+TM+HT (use QCD Renormalon Model for HT)

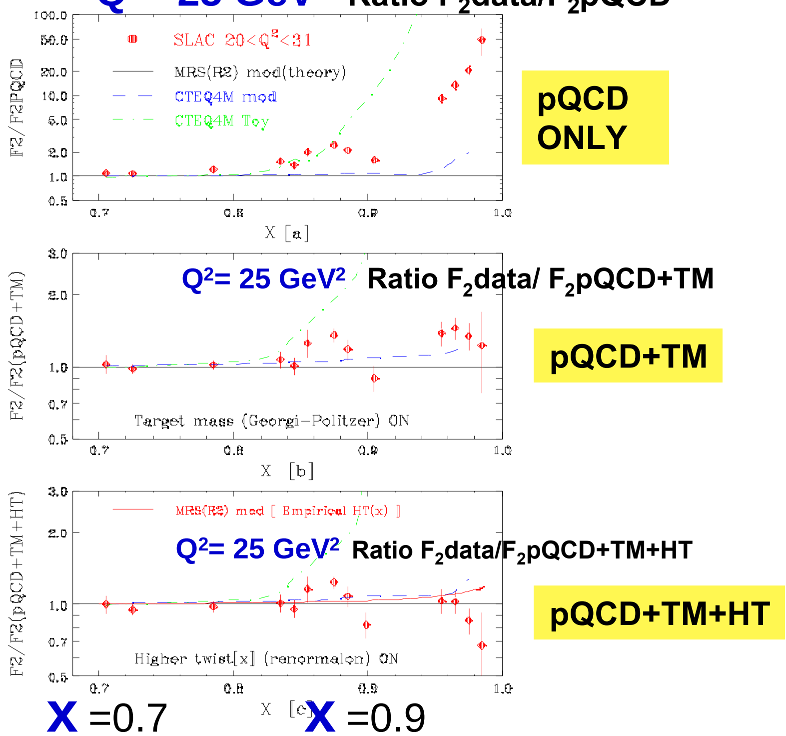
PDFs and QCD in NLO + TM + QCD Renormalon Model for Dynamic Higher Twist describe the F2 and R data reasonably well. TM Effects are LARGE

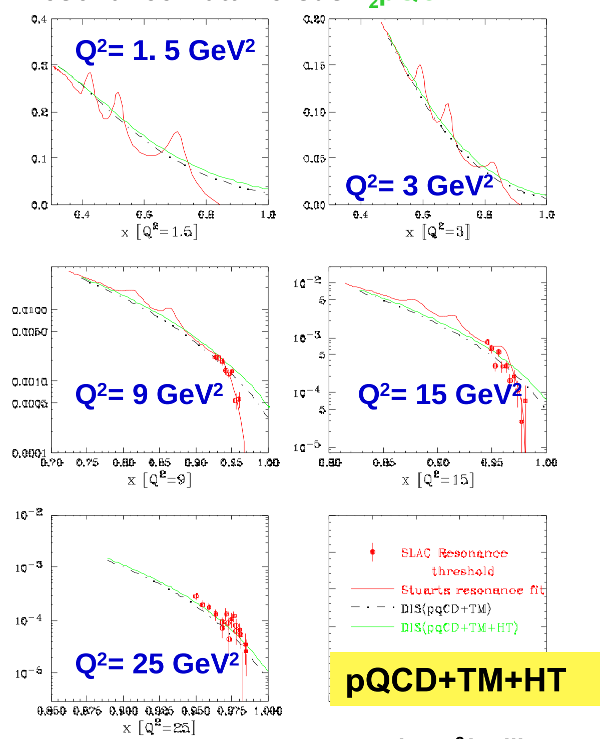


Very high x F2 proton data (DIS + resonance)
(not included in the original fits Q^2=1. 5 to 25 GeV^2)

Q^2= 25 GeV^2 Ratio F_2data/F_2pQCD

F2 resonance Data versus F_2pQCD+TM+HT

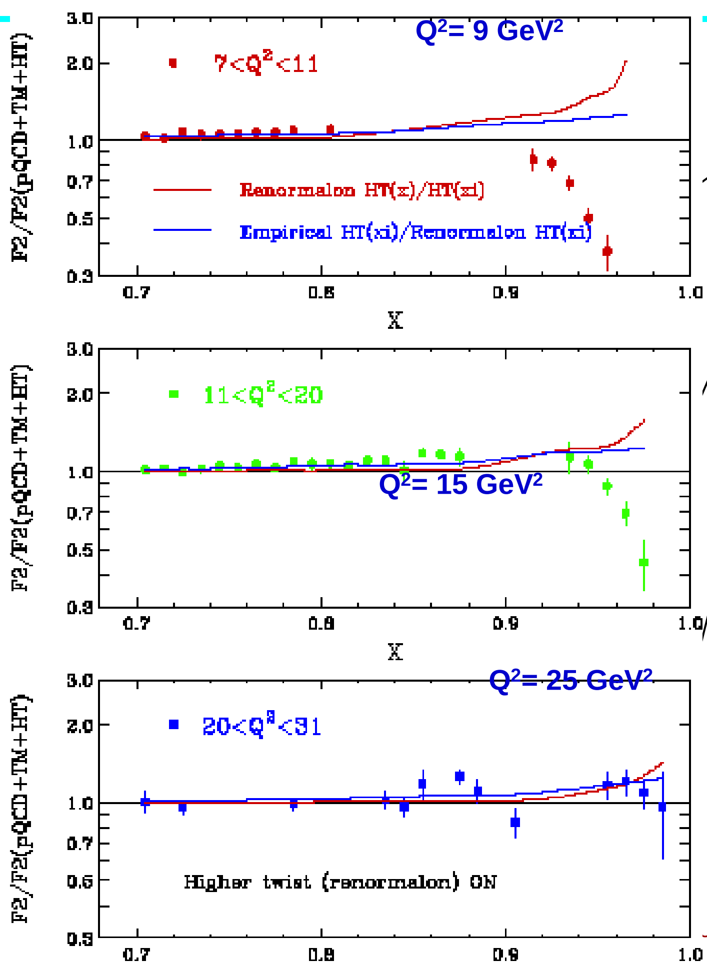




NLO pQCD + ξ TM + higher twist describes very high x DIS F_2 and resonance F_2 data well. (duality works) Q^2=1. 5 to 25 GeV^2

A_w (w, Q^2) will account for interactions with spectator quarks

Look at Q^2= 8, 15, 25 GeV^2 very high x data-backup slide*

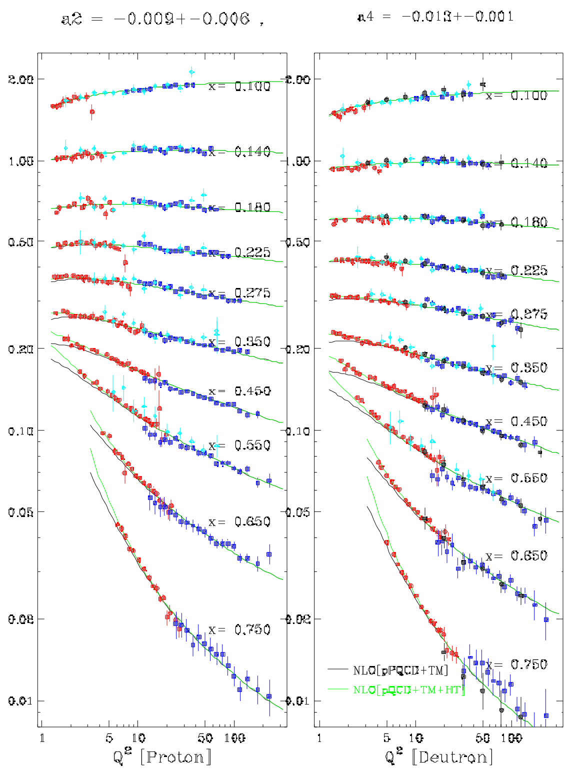


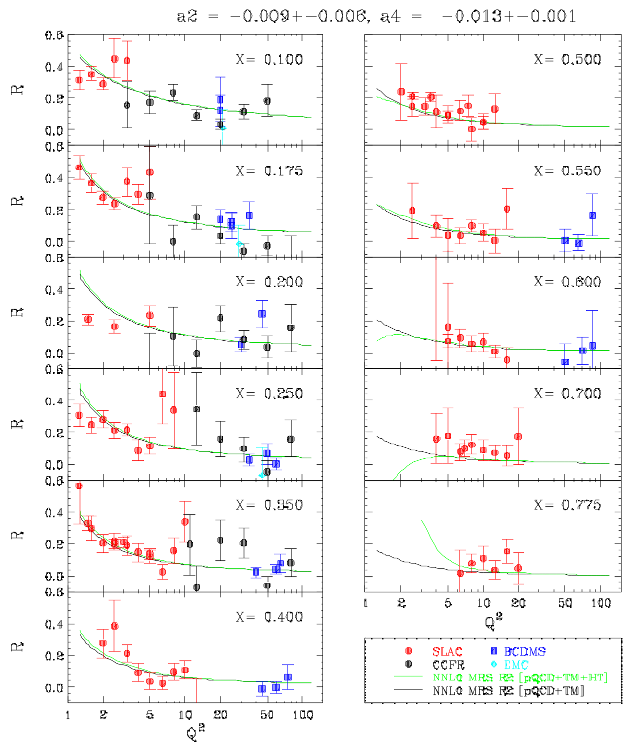
Ratio
F_2data/F_2pQCD+TM+HT

- **Pion production threshold** A_w (**w**, Q^2)
- Now Look at lower Q^2 (8,15 vs 25) DIS and resonance data for the ratio of
  F2 data/(NLO pQCD +TM +HT}
- High x ratio of F2 data to NLO pQCD +TM +HT parameters extracted from lower x data. These high x data were not included in the fit.
  - **The Very high x(=0.9**) region: It is described by NLO pQCD (if target mass and higher twist effects are included) to better than 10%

F_2, R comparison with NNLO QCD-works
=> NLO HT are missing NNLO terms (Q^2>1)

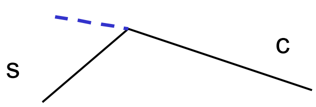
Size of the higher twist effect with NNLO analysis is really small (but not 0)
a2= -0.009 (in NNLO) versus –0.1(in NLO) - > factor of 10 smaller, a4 nonzero





At LOW x, Q^2 "NNLO terms" look similar to "kinematic final state mass higher twist" or "effective final state quark mass -> "enhanced" QCD

Charm production s to c quarks in neutrino scattering-slow rescaling



Mc (final state quark mass

$(Pi + q)^2 = Pi^2 + 2q.Pi + q^2 = Pf^2 = Mc^2$

$2\,\xi_C\, q.P = Q^2 + Mc^2 \qquad (Q^2 = -q^2)$

$2\,\xi_C\, M\nu = Q^2 + Mc^2 \qquad \xi_C$ – slow re-scaling

$\xi_c = [Q^2+Mc^2] / [2M\nu]$ (final state charm mass

At low Q2, the final state u and d quark effective mass is not zero

M*
Production of pions etc
gluon emission from the Interacting quark

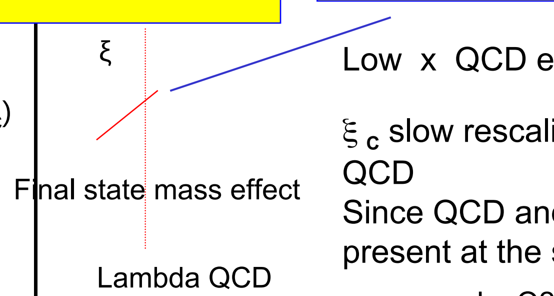
$(Pi + q)^2 = Pi^2 + 2q.Pi + q^2 = Pf^2 = M^{*2}$

$\xi_c = [Q^2+M^{*2}] / [2M\nu]$ (final state M* mass))

versus for mass-less quarks $2x\, q.P= Q^2$

$x = [Q^2] / [2M\nu]$ (compared to x]

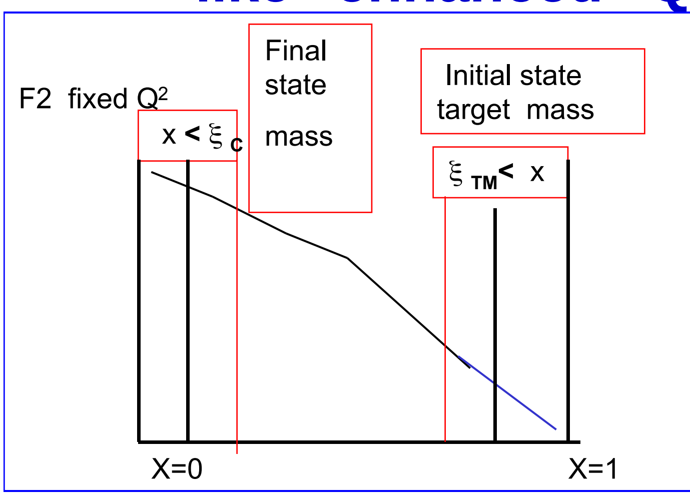
At Low x, low Q^2
$\xi_c > x$ (slow rescaling ξ_C)
(and the PDF is smaller at high x, so the low Q^2 cross section is suppressed - threshold effect.



Low x QCD evolution

ξ_c slow rescaling looks like faster evolving QCD
Since QCD and slow rescaling are both present at the same Q2

At High x, "NNLO QCD terms" have a similar form to the "kinematic -Georgi-Politzer ξ_{TM} TM effects" -> look like "enhanced" QCD evolution at low Q



At high x, Mi,Pt from multi gluon emission by initial state quark look like enhanced QCD evolution or enhance target mass effect. Add a term A

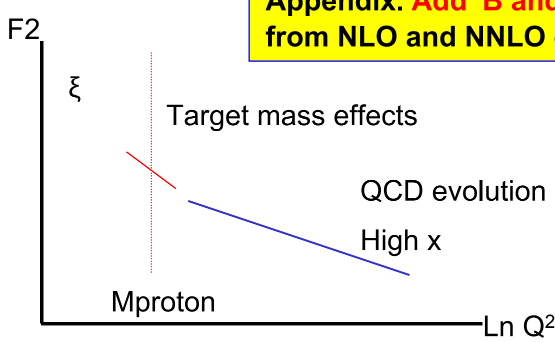
ξ_{TM} = Q^2/ **[**$M\nu$ **(**1+ (1+Q^2/ν^2)$^{1/2}$ **) +A]** proton target mass effect in Denominator plus enhancement)

ξ_C = [Q^2+M^{*2}] / [2Mν] (final state M* mass)

Combine both target mass and final state mass:

ξ_{C+TM} **= [$Q^2+M^{*2}+B$] / [Mν (1+(1+Q^2/ν^2)$^{1/2}$) +A]**
- includes both initial state target proton mass and final state M* mass effect) - Exact derivation in Appendix. Add B and A account for additional Δm^2 from NLO and NNLO effects.

At high x, low Q^2 $\xi_{TM} < x$ (tgt mass ξ) (and the PDF is higher at lower x, so the low Q^2 cross section is enhanced .



[Ref:Georgi and Politzer Phys. Rev. D14, 1829 (1976)]]

Modified LO PDFs for all Q^2 region?

Philosophy

1. We find that NNLO QCD+tgt mass works very well for Q^2 > 1 GeV^2.
2. That target mass and missing NNLO terms "explain" what we extract as higher twists in a NLO analysis. i.e. SPECTATOR QUARKS ONLY MODULATE THE CROSS SECTION AT LOW W. THEY DO NOT CONTRIBUTE TO DIS HT.
2. However, we want to go down all the way to Q^2=0. All NNLO and NLO terms blow up. However, higher twist formalism in terms of initial state target mass binding and Pt, and final state mass are valid below Q^2=1, and mimic the higher order QCD terms for Q^2>1 (in terms of effective masses, Pt due to gluon emission).
3. While the original approach was to explain the "empirical higher twists" in terms of NNLO QCD at low Q^2 (and extract NNLO PDFs), we can reverse the approach and have "higher twist" model non-perturbative QCD, down to Q^2=0, by using LO PDFs and "effective target mass and final state masses" to account for initial target mass, final target mass, and missing NLO and NNLO terms. I.e. Do a fit with:
4. $F_2(x, Q^2) = Q^2 / [Q^2+C] F_{2QCD}(\xi w, Q^2) A(w, Q^2)$ (set $A_w(w, Q^2)$ =1 for now - spectator quarks) C is the photo-production limit Non-perturbative term.
5. $\xi w = [Q^2+B] / [M\nu (1+(1+Q^2/\nu^2)^{1/2}) + A]$ or $Xw = [Q^2+B] / [2M\nu + A]$
6. B=effective final state quark mass. A=enhanced TM term, [Ref:Bodek and Yang hep-ex/0203009]

Modified LO PDFs for all Q^2 (including 0)

Construction

1. Start with GRV94 LO (Q^2_{min}=0.23 GeV^2)
   - describe F2 data at high Q^2

2A. Replace X with a new scaling, Xw
- $x = [Q^2] / [2M\nu]$
- $Xw = [Q^2+B] / [2M\nu+A]$
- A: initial binding/target mass effect plus NLO +NNLO terms)
- B: final state mass effect (but also photo production limit)

2B. Or Replace X with a new scaling, ξ w

$\xi w = [Q^2+B] / [M\nu (1+(1+Q^2/\nu^2)^{1/2}) + A]$

3. Multiply all PDFs by a factor of $Q^2/[Q^2+c]$ for photo prod. Limit+non-perturbative

$F_2(x, Q^2) = Q^2/[Q^2+C] F_{2QCD}(\xi w, Q^2) A(w, Q^2)$

4. Freeze the evolution at Q^2 = 0.24 GeV^2

$-F_2(x, Q^2 < 0.24) = Q^2/[Q^2+C] F_2(Xw, Q^2=0.24)$

A. Do a fit to SLAC/NMC/BCDMS H, D data.- Allow the normalization of the experiments and the BCDMS major systematic error to float within errors.

B. HERE INCLUDE DATA WITH Q2<1 if it is not in the resonance region

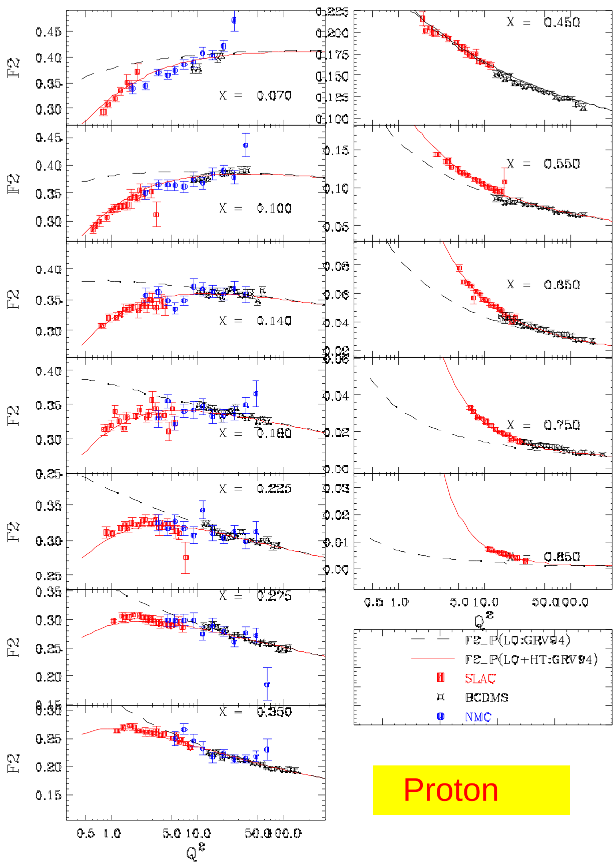
Results

- Modified LO GRV94 PDFs with three parameters (a new scaling variable, Xw, ξ w) describe DIS F2 H, D data (SLAC/BCDMS/NMC) well.
- A=1.735, B=0.624, and C=0.188 Xw (note for Xw, A includes the Proton M)
- A=0.700, B=0.327, and C=0.197 ξ w works better as expected MEASURE PROTON MASS FROM HIGHER TWIST FITTING
- *Keep final state interaction resonance modulating function A* (w, Q^2)=1 for now (will be included in the future). Fit DIS Only
- Compare with SLAC/Jlab resonance data (not used in our fit) ->*A* (w, Q^2)
- Compare with photo production data (not used in our fit)-> check on C
- Compare with medium energy neutrino data (not used in our fit)- except to the extent that GRV94 originally included very high energy data on xF_3

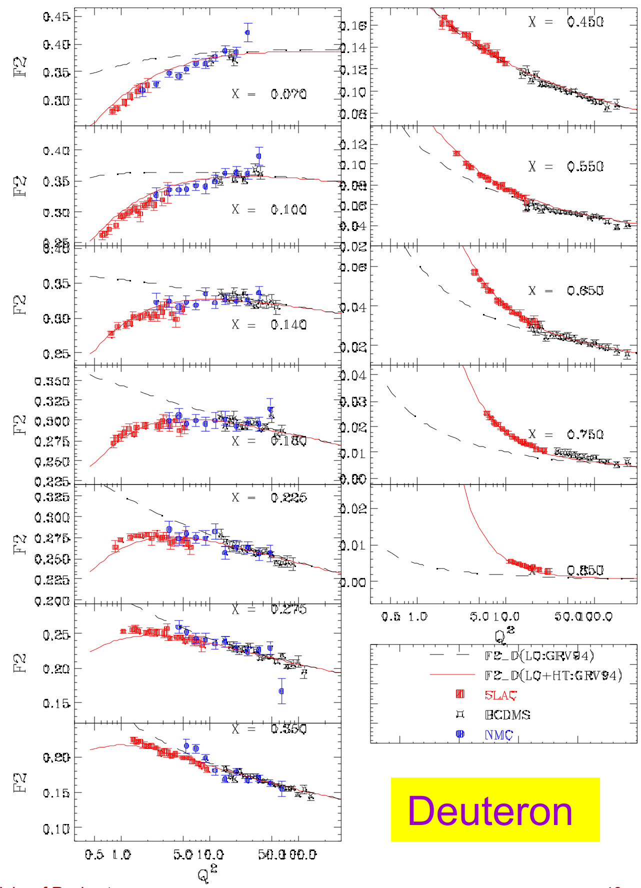
[Ref:Bodek and Yang hep-ex/0203009]

LO+HT xw fit Comparison with DIS F_2 (H, D) data

[These SLAC/BCDMS/NMC are used in this Xw fit χ^2 = 1555 /958 DOF (Q^2>0.5)]



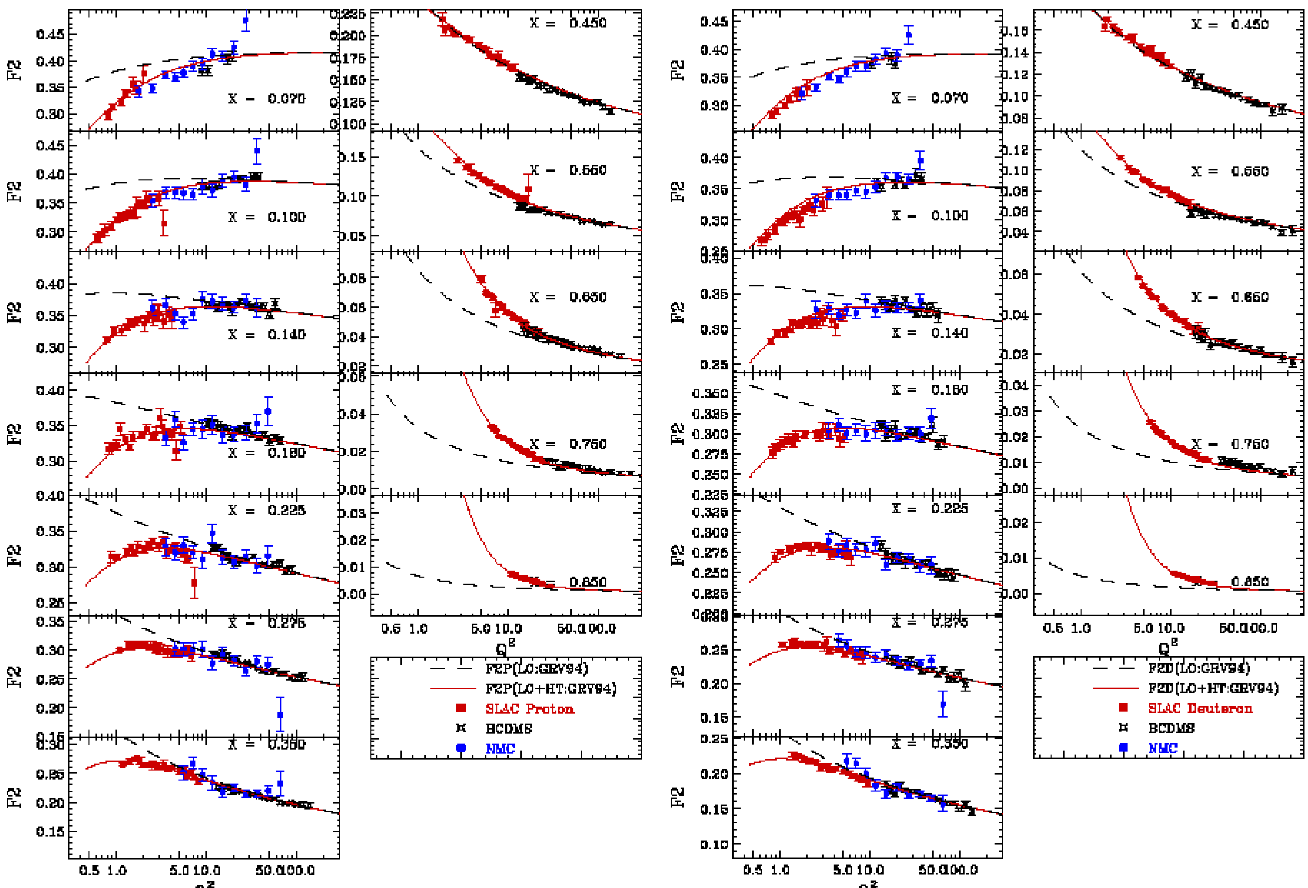
Proton



Deuteron

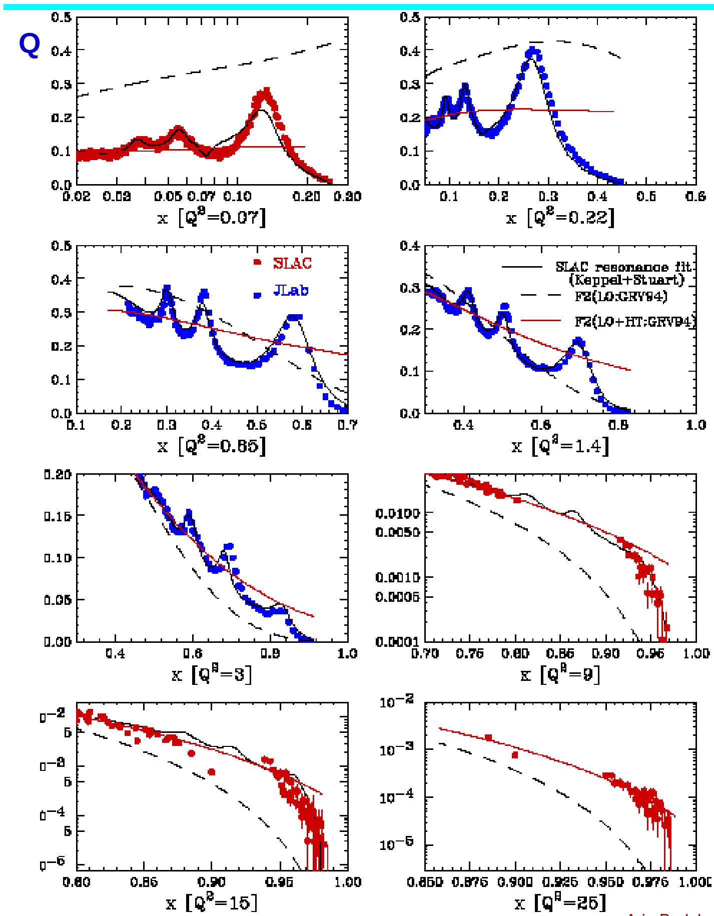
LO+HT fit Comparison with DIS F_2 (H, D) data

SLAC/BCDMS/NMC ξw works better than Xw χ^2 = 1351 /958 DOF



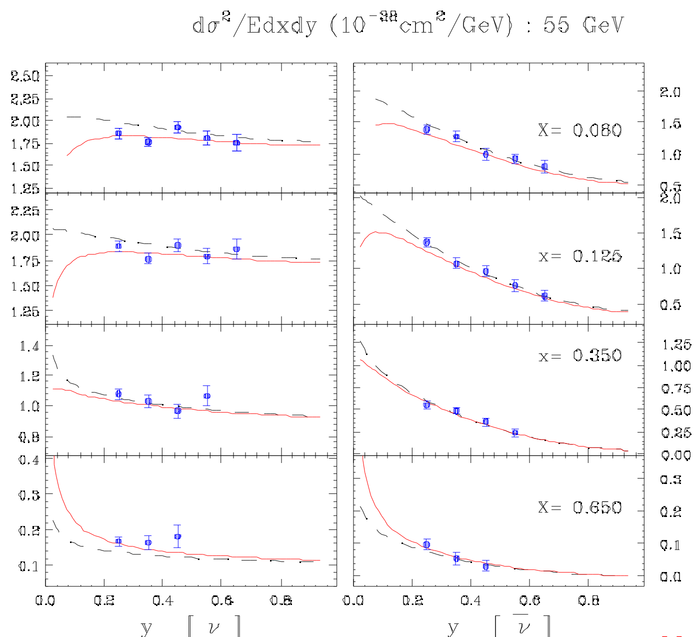
Comparison with F2 resonance data

[SLAC/ Jlab] (These data were not included in this ξw fit)



- The modified LO GRV94 PDFs with a new scaling variable, ξw describe the SLAC/Jlab resonance data very well (on average).
  - Even down to $Q^2 = 0.07$ GeV^2
  - Duality works: The DIS curve describes the average over resonance region
- ξw fit.
- For now, lets compare to neutrino data and photoproduction
- Later. repeat with other PDFs and f(x)
- Note QCD evolution between Q2=0.85 qnd Q2=0.25 small.
- Later: add the A_w (w, Q^2) modulating function. (to account for interaction with spectator quarks at low W)
- Later: check the x=1 Elasic Scattering Limit for electrons,neutrino, and 1st resonance.

Comparison of LO+HT to neutrino data on Iron [CCFR] (not used in this Xw fit)



Xw fit

Construction

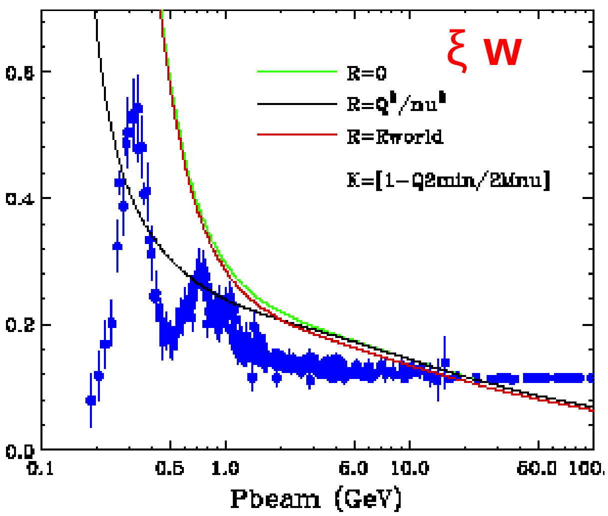
- **Apply nuclear corrections using e/μ scattering data.**
- **Calculate F_2 and xF_3 from the modified PDFs with Xw**
- **Use R=Rworld fit to get $2xF_1$ from F_2**
- **Implement charm mass effect through a slow rescaling algorithm, for F_2 $2xF_1$, and XF_3**

The modified GRV94 LO PDFs with a new scaling variable, Xw describe the CCFR diff. cross section data (Eν=30–300 GeV) well. Will repeat with ξw

Comparison with photo production data

(not included in this ξ w fit)

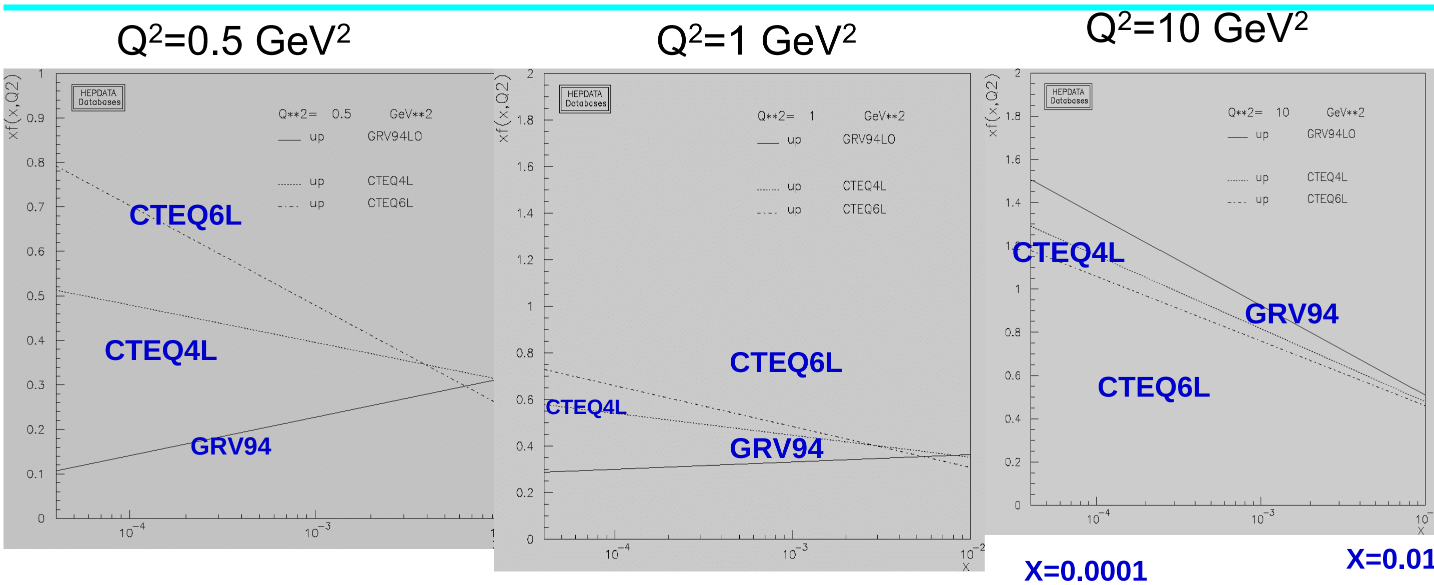
mb



- $\sigma(\gamma$-proton) = $\sigma T(Q^2=0, Xw)$
- σT = 0.112 mb $2xF_1/(KQ^2)$
- K depends on definition of virtual photon flux for usual definition K= $[1 - Q^2/ 2M\nu]$
- σT = 0.112 mb $F_2(x, Q^2)\, D(\nu, Q^2) / (KQ^2)$
- $D = (1+ Q^2/ \nu^2)/(1+R)$
- $F_2(x, Q^2)$ limit as $Q^2 \to 0$ $= Q^2/(Q^2+0.188) * F_{2\text{-GRV94}}(\xi w, Q^2 =0.24)$
- Try: R = 0
- $R= Q^2/ \nu^2$ (evaluated at Q^2 =0.24)
- R = Rw (evaluated at Q^2 =0.24)
- Note: Rw=0.034 at Q^2 =0.24 is very small (see appendix R data figure)

The modified LO GRV94 PDFs with a new scaling variable, ξ W also describe photo production data (Q^2=0) to within 15%: To get better agreement at high ν= 100 GeV (very low ξ W), the GRV94 need to be updated to fit latest HERA data at very low x and low Q^2. Can switch to other 2002 LO PDFs. If we include these photoproduction data in the fit, we will get C of about 0.22, and *agreement at the few percent level*. To evaluate $D = (1+ Q^2/ \nu^2)/(1+R)$ more precisely, we also need to look at the measured Jlab R data in the Resonance Region at Q^2 =0.24.

Comparison of u quark PDF for GRV94 and CTEQ4L and CTEQ6L (more modern PDFs)



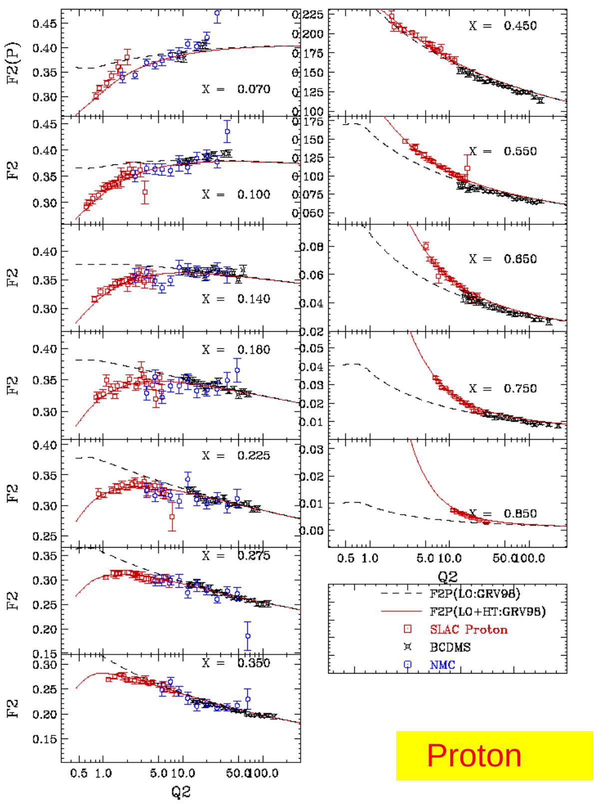
GRV94 LO PDFs need to be updated.at very low x, but this is not important in the few GeV region

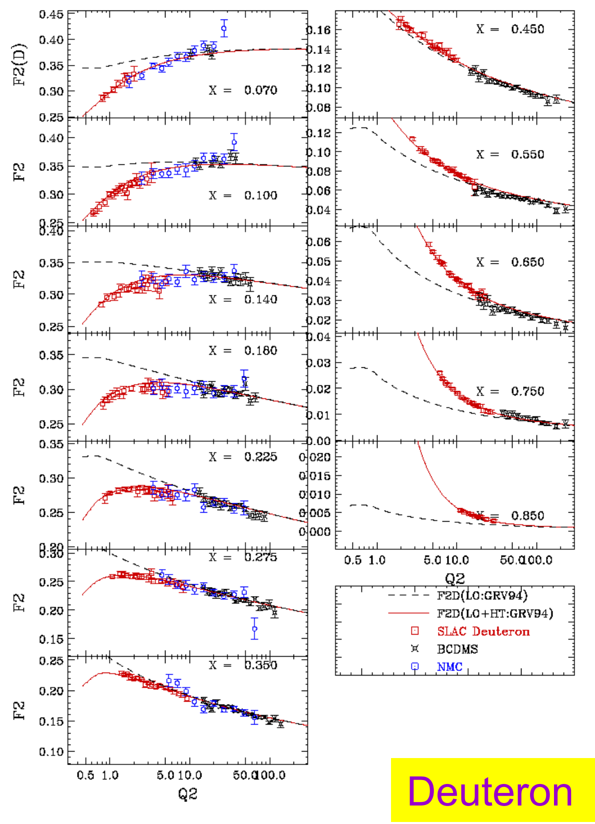
The GRV LO need to be updated to fit latest HERA data at very low x and low Q^2.

We used GRV94 since they are the only PDFs to evolve down to Q2=0.24 GeV^2 . All other PDFs (LO) e.g. GRV98 stop at 1 GeV^2 or 0.8 GeV^2. Now it looks like we can freeze at Q2=0.8 and have no problems. So switch to modern PDFs.

LO+HT fit GRV98 DIS F_2 (H, D) data

SLAC/BCDMS/NMC ξ_w GRV98 works even better χ^2 = 1017 /958 DOF





Fitting results with F2 proton and deuteron data.Much better fit with GRV98 Chi2/DOF = 1017/958 using ξ_w.

Chi2/DOF = 1017/958	1351/958	1555/958
GRV98 ξ_w **(includes TM)**	GRV94 ξ_w **(includes TM)**	GRV94 x_w **(no TM)**
*Ehanced tgt mass a = 0.25 +- 0.02	0.70	1.74
Final state mass b = 0.10 +- 0.01	0.33	0.62
Photolimit c = 0.18 +- 0.004	0.20	0.19

nslacP = 0.9852 +- 0.002

nslacD = 0.9804 +- 0.002

nbcdmsP = 0.9488 +- 0.002

nbcdmsD = 0.9677 +- 0.002

nmcP = 0.9813 +- 0.003

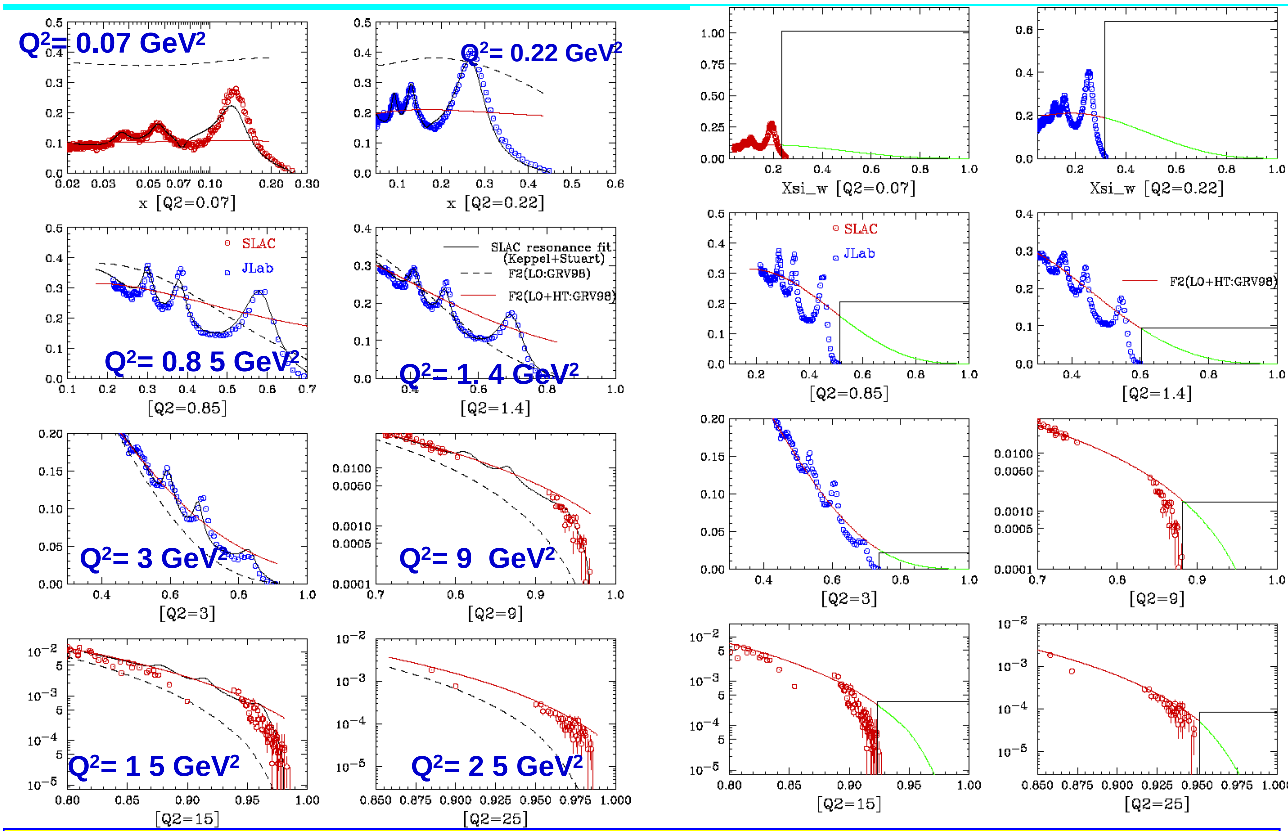
nmcD = 0.9835 +- 0.003

BCDMS Lambda = 2.21 +- 0.16

Note: Free parameters a, b,c are already very small. GeV2.

GRV98 Comparison with F2 resonance data

[SLAC/ Jlab] (These data were not included in this ξw fit)

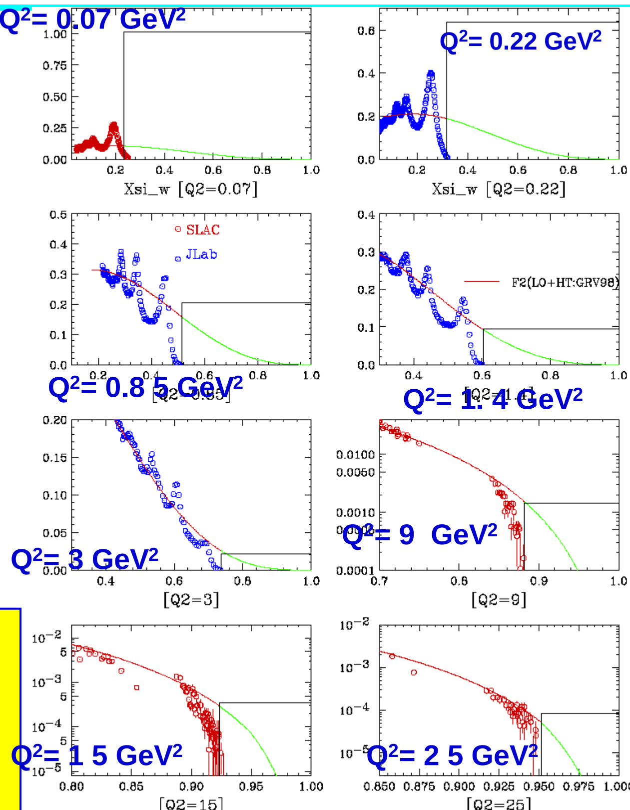


- The modified LO GRV98 PDFs with a new scaling variable, ξw describe the SLAC/Jlab resonance data very well (on average). **Better than GRV94**

GRV98 When does duality break down

[SLAC/ Jlab] (These data were not included in this ξ_W fit)

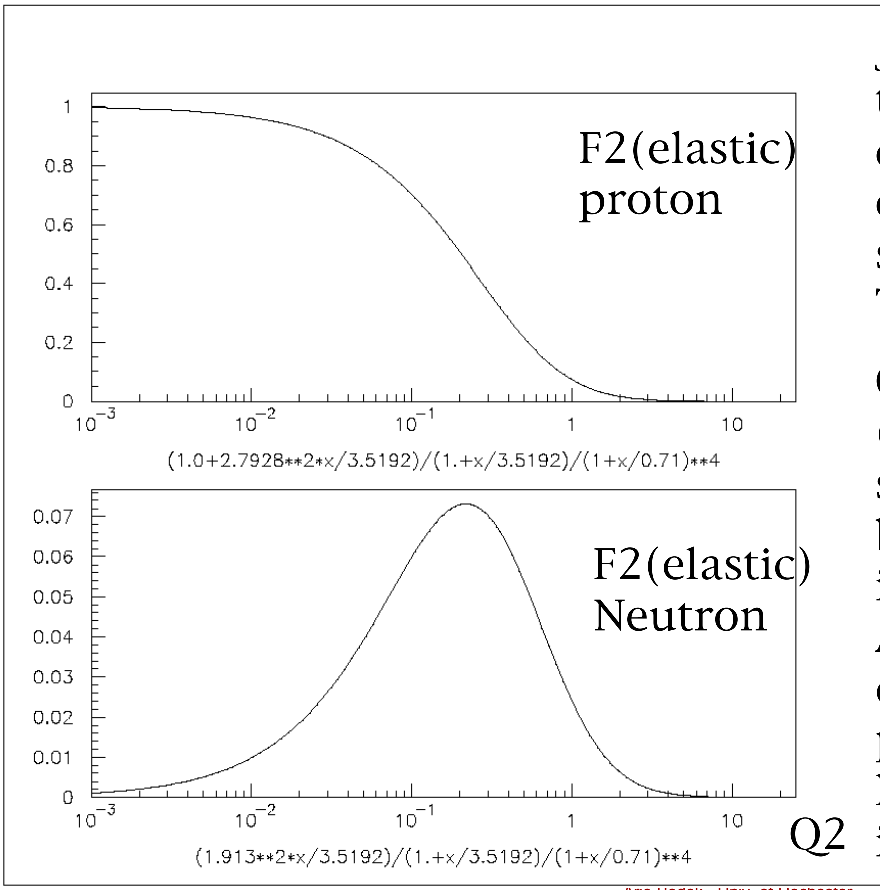
Int F2P	Q2
Elastic peak	
1.0000000	0
0.7775128	0.07
0.4340529	0.25
0.0996406	0.85
0.0376200	1.4
0.0055372	3
0.0001683	9
0.0000271	15
0.0000040	25
DIS high Q2 Integral F2p 0.17	



- **In proton :**
- **QPM Integral of F2p** =
- 0.17*(1/3)^2+0.34*(2/3)^2 = 0.17 (In neutron=0.11)
- Where we use the fact that
- 50% carried by gluon
- 34% u and 17% d quarks

Very low Q2: Revenge of the Spectator Quarks

F2(elastic) versus Q2 (GeV2)



Just like in p-p scattering there is a strong connection between elastic and inelastic scattering (Optical Theorem).

Quantum Mechanics (Closure) requires a strong connection between elastic and inelastic scattering. Although spectator quarks were ignored in pQCD - they rebel at lowQ2 and will not be ignored.

Revenge of the Spectator Quarks

Stein et al PRD 12, 1884 (1975)-1

$$\nu W_{2p}(q^2,\nu) = [1 - W_2^{el}(q^2)]\,F_{2p}(\omega')\,, \qquad (13)$$

where $F_{2p}(\omega')$ is the scaling limit structure function and

$$W_2^{el}(q^2) = \frac{G_E{}^2(q^2) + \tau G_M{}^2(q^2)}{1+\tau}\,, \quad \tau = \frac{q^2}{4M^2} \qquad (14)$$

is the counterpart of W_2 for elastic scattering (see Appendix B), where G_E and G_M are, respectively, the elastic electric and magnetic form factors for the proton. This form satisfies the constraint that W_2 vanish at $q^2=0$. Integrating W_{2p} over all values of ν yields

$$\int_{\text{inelastic}} d\nu\, W_{2p}(q^2,\nu) = [1 - W_2^{el}(q^2)] \int_{\text{inelastic}} \frac{d\nu}{\nu} F_{2p}(\omega')\,. \qquad (15)$$

But this is the Gottfried sum rule[27] for the proton,

where

$$\int_{\text{inelastic}} \frac{d\nu}{\nu} F_{2p}(\omega') = \sum_i q_i{}^2 \qquad (16)$$

is the sum of the parton charges squared.

2. Application

We can now apply these results to the proton and neutron if we consider them as being made of constituents. These yield immediately

$$\int_{\text{inel}} d\nu\, W_{2p}(q^2,\nu) = \left(\sum_{i=1}^{N} e_i{}^2\right)_p [1 - |F_{el}^p(q^2)|^2] + C_p(q^2)\left(\sum_{i\neq j}^{N}\sum e_i e_j\right)_p\,, \qquad \text{(B15)}$$

$$\int_{\text{inel}} d\nu\, W_{2n}(q^2,\nu) = \left(\sum_{i=1}^{N} e_i{}^2\right)_n [1 - |F_{el}^N(q^2)|^2] + C_n(q^2)\left(\sum_{i\neq j}^{N}\sum e_i e_j\right)_n\,. \qquad \text{(B16)}$$

F_{el}^p and F_{el}^n would be equal if the momentum distributions of the constituents were the same in the proton and neutron, so if the correlation terms were negligible, one might expect W_{2n}/W_{2p} to scale to lower values of q^2 than either W_{2p} or W_{2n} alone. Gottfried noted that in the simple quark model the charge sum in the correlation contribution vanishes for the proton, but not for the neutron.[27]

For the case of particles with spin, magnetic moments, and more realistic ground states, the results get much more complicated. There are several more detailed accounts in the case of nuclear scattering in the literature.[41] However, the simple approach stated here agrees with the spirit of the more complex analyses.

Revenge of the Spectator Quarks

Stein etal PRD 12, 1884 (1975)-2

[41]For more detailed treatment of closure, see, for example O. Kofoed-Hanson and C. Wilkin, Ann. Phys. (N.Y.) 63, 309 (1971); K. W. McVoy and L. Van Hove, Phys. Rev. 125, 1034 (1962).

[27]K. Gottfried, Phys. Rev. Lett. 18, 1174 (1967).

$$G_{el}(q^2) = \left|\sum_{i=1}^{N} e_i\right|^2 |F_{el}(q^2)|^2 ,$$

$$G_{inel}(q^2) = \sum_{i=1}^{N} e_i^2[1 - |F_{el}(q^2)|^2] + C(q^2)\sum_{i\neq j}^{N}\sum e_i e_j . \quad \text{(B14)}$$

$$\nu W_{2p}(q^2, \nu) = [1 - W_2^{el}(q^2)] F_{2p}(\omega') , \quad (13)$$

where $F_{2p}(\omega')$ is the scaling limit structure function and

$$W_2^{el}(q^2) = \frac{G_E^2(q^2) + \tau G_M^2(q^2)}{1+\tau} , \quad \tau = \frac{q^2}{4M^2} \quad (14)$$

$$G_{Ep} = P(q^2)/(1+q^2/0.71)^2 ,$$

P is close to 1 and gives deviations

From Dipole form factor (5%)

Note: at low Q2

$[1 - W_2^{el}] = 1 - 1/(1+Q^2/0.71)^4$

$= 1-(1-4Q^2/0.71) =$

$= 1- (1-Q^2/0.178) =$

$\to Q^2/0.178$ as $Q^2 \to 0$

Versus Our GRV98 fit with

$Q^2/(Q^2+C) \to Q^2/C$

c = 0.1797 +- 0.0036

Revenge of the Spectator Quarks -3 - History of Inelastic Sum rules C. H. Llewellyn Smith hep-ph/981230

Talk given at the Sid Drell Symposium
SLAC, Stanford, California, July 31st, 1998

Gottfried noted that in the '*breathtakingly crude*' naïve three-quark model the second term in the following equation vanishes for the proton (it also vanishes for the neutron, but neutrons are not mentioned):

$$\sum_{i,j} Q_i Q_j \equiv \sum_i Q_i^2 + \sum_{i \neq j} Q_i Q_j \; . \qquad (5)$$

Thus for any charge-weighted, flavour-independent, one-body operator all correlations vanish, and therefore using the closure approximation the following sum rule can be derived:

$$\int_{\nu_0} W_2^{ep}(\nu, q^2) d\nu = 1 - \frac{G_E^2 - q^2 G_M^2/4m^2}{1 - q^2/4m^2} \; , \qquad (6)$$

where ν_0 is the inelastic threshold (the methods used to derive this sum rule are those that have long been used to derive sum rules in atomic and nuclear physics, for example the sum rule [13] derived in 1955 by Drell and Schwarz). After observing that this sum

Revenge of the Spectator Quarks -4 - History of Inelastic Sum rules C. H. Llewellyn Smith hep-ph/981230

rule appears to be oversaturated in photoproduction (we now know that the integral is actually infinite in the deep inelastic region), Gottfried asked whether it was '*idiotic*', and stated that if, on the contrary there is some truth in it, one would want a '*derivation that a well-educated person could believe*'.

In his talk at the 1967 SLAC conference Bj quoted Gottfried's paper and stated that diffractive contributions should presumably be excluded from the integral, which could be done by taking the difference between protons and neutrons, leading to the following result, in modern notation:

$$\int \left(F_2^{ep}(x,q^2) - F_2^{en}(x,q^2) \right) \frac{dx}{x} = \frac{1}{3} \,. \tag{7}$$

This result, which is generally known as the Gottfried sum rule, is not respected by the data which give the value [14] 0.235 ± 0.026. In parton notation, the left-hand side can be written

$$\frac{1}{3}(n_u + n_{\bar{u}} - n_d - n_{\bar{d}}) = \frac{1}{3} + \frac{2}{3}(n_{\bar{u}} - n_{\bar{d}}) \,, \tag{8}$$

S. Adler, Phys. Rev. 143, 1144 (1966) Exact Sum rules from Current Algebra. Valid at all Q2 from zero to infinity. - 5

Strangeness-Conserving Case

The kinematic analysis of Sec. 3 shows that we may write the reaction differential cross section in the form

$$d^2\sigma\left(\binom{\nu}{\bar{\nu}}+p\to\binom{l}{\bar{l}}+\beta(S=0)\right)\Big/ d\Omega_l dE_l=\frac{G^2\cos^2\theta_C}{(2\pi)^2}\frac{E_l}{E_\nu}$$

$$\times[q^2\alpha^{(\pm)}(q^2,W)+2E_\nu E_l\cos^2(\tfrac{1}{2}\phi)\beta^{(\pm)}(q^2,W)\mp(E_\nu+E_l)q^2\gamma^{(\pm)}(q^2,W)]. \quad (13)$$

By measuring $d^2\sigma/d\Omega_l dE_l$ for various values of the neutrino energy E_ν, the lepton energy E_l, and the lepton-neutrino angle ϕ, we can determine the form factors $\alpha^{(\pm)}$, $\beta^{(\pm)}$, and $\gamma^{(\pm)}$ for all $q^2>0$ and for all W above threshold.

In Sec. 4 we prove that:

(i) the local commutation relations of Eq. (1a) and Eq. (1c) imply

$$2=g_A(q^2)^2+F_1^V(q^2)^2+q^2F_2^V(q^2)^2+\int_{M_N+M_\pi}^{\infty}\frac{W}{M_N}dW[\beta^{(-)}(q^2,W)-\beta^{(+)}(q^2,W)]; \quad (14)$$

Strangeness-Changing Case

$$(4,2)=\int\frac{W}{M_N}dW[\beta_{(p,n)}{}^{(-)}(q^2,W)-\beta_{(p,n)}{}^{(+)}(q^2,W)]; \quad (18)$$

The integrals of Eqs. (18)–(20) have discrete contributions at $W=M_\Lambda$ and/or M_Σ and a continuum extending from $W=M_\Lambda+M_\pi$ or from $W=M_\Sigma+M_\pi$ to $W=\infty$. We have not explicitly separated off the discrete contributions to the integrals, as was done in Eqs. (14)–(16) for the strangeness-conserving case. It would, of course, be straightforward to do this.

F. Gillman, Phys. Rev. 167, 1365 (1968)- 6

Adler like Sum rules for electron scattering.

$\alpha = W_1/M_N$,

$\beta = W_2/M_N$.

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)]=1. \qquad (18)$$

The functions $\beta^{(\pm)}(q_0,q^2)$ are defined just as in Eq. (7) except that in place of the electromagnetic currents $J_\mu(0)$ and $J_\mu(0)$ we have put the isospin raising or lowering F-spin currents $\mathfrak{F}_{(1\pm i2)\mu}(0)$ [recall that $\mathfrak{F}_{3\mu}(0)$ is just the isovector part of the electromagnetic current]. If we explicitly separate out the nucleon Born term in Eq. (18), we have

$$[F_1^V(q^2)]^2+q^2\left(\frac{\mu^V}{2M_N}\right)^2[F_2^V(q^2)]^2 + \int_{M_\pi+(q^2+M_\pi^2)/2M_N}^\infty dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(8)}(q_0,q^2)]=1, \qquad (19)$$

where the superscript V denotes the fact that we are dealing with the isovector part of the current; the isovector anomalous magnetic moment $\mu^V=\mu_p'-\mu_n'$ $=3.70$. As $q^2\to 0$, we see from Eq. (10) or (17) that only the first term, $[F_1^V(q^2)]^2$, on the left-hand side of Eq. (19) survives, and as $q^2\to 0$ it goes to 1, in agreement with the left-hand side.

In the derivation[3] of Eq. (18) only two assumptions enter: (1) the commutation relation Eq. (3a) of the F-spin densities, and (2) an unsubtracted dispersion relation for the forward Compton scattering amplitudes (which are the coefficients of $p_\mu p_\nu$ and $q_\mu q_\nu$ in the expansion of $T_{\mu\nu}$) corresponding to $\beta(q_0,q^2)$. It is of course the second assumption which is most open to question. However, we note the following:

(a) The fact that as $q^2\to 0$ the left- and right-hand sides of Eq. (19) as it now stands automatically become equal rules out a q^2-independent subtraction. This just means we have done nothing grossly wrong, e.g., introduced a kinematic singularity in q^2 in one of our amplitudes.

F. Gillman, Phys. Rev. 167, 1365 (1968)- 7
Adler like Sum rules for electron scattering.

$\alpha = W_1/M_N$,
$\beta = W_2/M_N$.

Therefore the factor

$[1 - W_2^{el}] = 1 - 1/(1+Q^2/0.71)^4$

$= 1-(1-4Q^2/0.71) =$

$= 1- (1-Q^2/0.178) =$

$\to Q^2/0.178$ as $Q^2 \to 0$

Is valid for VALENCE QUARKS FROM THE ADLER SUM RULE FOR the Vector part of the interaction

Versus Our GRV98 fit with

$Q^2/(Q^2+C) \to Q^2/C$

c = 0.1797 +- 0.0036

And C is probably somewhat different for the sea quarks.

F2nu-p(vector)= d+ubar

F2nubar-p(vector) =u+dbar

1=F2nubar-p-F2nu-p= (u+dbar)-(d+ubar)

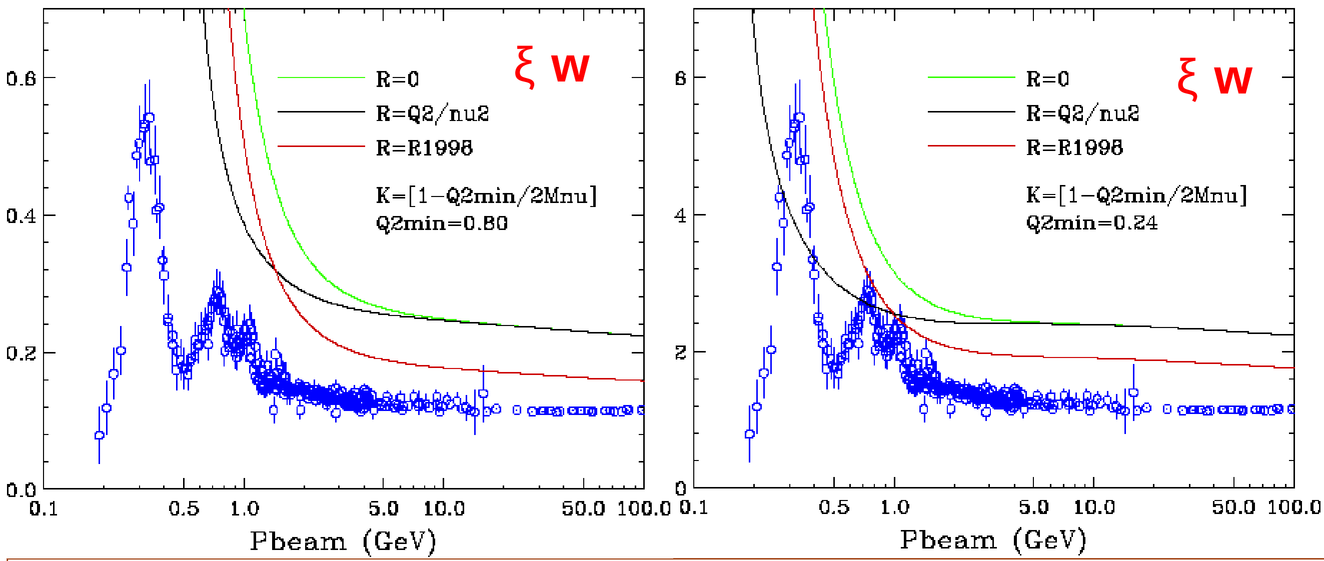
= (u-ubar)- (d-dbar) = 1

INCLUDING the

x=1 Elastic contribution

Therefore, the inelastic part is reduced by the elastic x=1 term.

Comparison of GRV98 photo production data (not included in this ξ w fit) Note K=1 also valid (virtual photon flux definition is mb ewhat arbitrary except at Q2=0, so overall normalization can change)



Here use GRV98 with R1998. Note that GRV98 freeze at Q2=0.8 GeV2. So this Is an extrapolation all the way from Q2=0.8 to Q2=0. Higher precision is not really needed, but could use (1-F2(elastic) instead if we want better agreement for Q2=0.

- **Try: R = 0. -> It is important now to get all the Q^2/ν^2 terms**
- **R= Q^2/ν^2 (evaluated at Q^2 =0.24, 0.8) Will work on this next**
- **R = R1998 (evaluated at Q^2 =0.24, 0.8) LOOKS like K=1 works better**
- **Note: F2 is zero at Q2=0, this is just the SLOPE of F2 at Q2=0.**

1st Summary

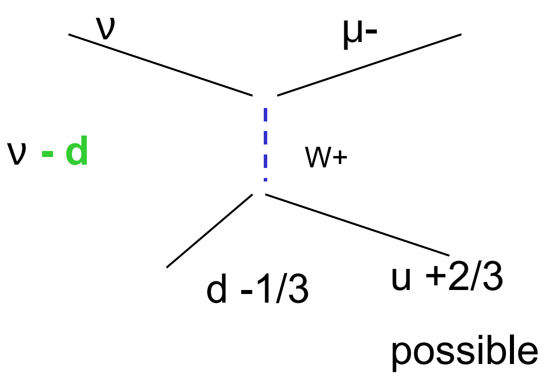
- Our modified GRV LO PDFs with a modified scaling variables, Xw and ξw describe all SLAC/BCDMS/NMC DIS data. GRV98 ξw works best
- The modified PDFs also yields the average value over the resonance region as expected from duality argument, ALL THE WAY TO $Q^2 = 0$
- Could get all the **Q^2/ν^2** terms in - for exact photoproduction prediction, and more refined form with W2elastic. Now good to 20% at Q2=0.
- Also good agreement with high energy neutrino data.
- Therefore, this model should also describe a low energy neutrino cross sections reasonably well- to be tested next.
- For Now, ONLY USE this model for W above quasielastic and First resonance (Use old form factor picture for 1st resonance and quasi.).
- We will investigate further refinements to ξw, What are the further improvement in ξw - Mostly to reduce the size of the three free parameters a, b, c as more theoretically motivated terms are added into the formalism (mostly intellectual curiosity, since the model is already good enough). E.g. Add Pt^2 from Drell Yan data.
- Also As we did with our NLO and NNLO studies, can introduce f(x) to make PDF fit better (can easily be done for x>0.1 since all QCD sum rules dominated by low x).Right now assume LO PDFs are perfect
- This work is continuing… focus on further improvement to ξw (although very good already) and *Ai,j,k* (W, Q^2) (low W + spectator quark modulating function). Test in limit of x=1, 1st resonance, electron vs neutrino etc..

Summary continued

- Do some more work on the Q2=0 limit (terms in Q2/Nu2, PT2 need to be included)
- Future studies involving both neutrino and electron scattering including new experiments are of interest.
- As x gets close to 1, local Duality is very dependent on the spectator quarks (e.g. different for Gep. Gen, Gmp, Gmn, Gaxial, Gvector neutrinos and antineutrinos
- In DIS language it is a function of Q2 and is different for W1, W2 , W3 (or transverse (--left and right, and longitudinal cross sections for neutrinos and antineutrinos on neutrons and protons.
- This is why the present model is probably good in the 2nd resonance region and above, and needs to be further studied in the region of the first resonance and quasielastic scattering region.
- Nuclear Fermi motion studies are of interest, best done at Jla with electrons.
- Nuclear dependence of hadronic final state of interest.
- Nuclei of interest, C12, P16, Fe56. (common materials for neutrino detectors).

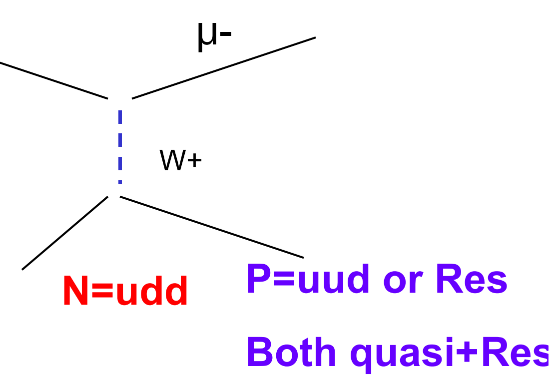
NEUTRINOS **On neutrons** ***both quasielastic***

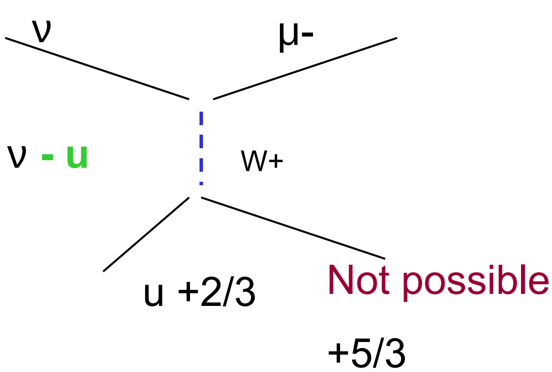
On quarks **And resonance+DIS production possible.**



NEUTRINOS

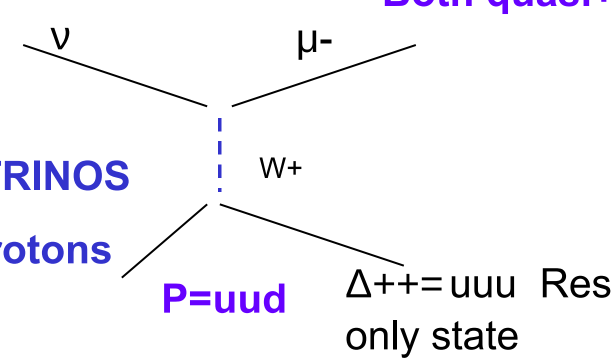
On Neutrons





NEUTRINOS

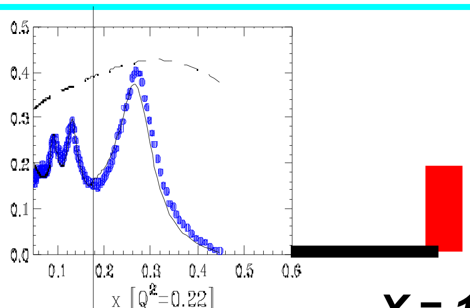
On Protons



On protons only resonance+ DIS production possible.

NEUTRINOS **On neutrons** ***both quasielastic***

On nucleons **And resonance+DIS production possible.**



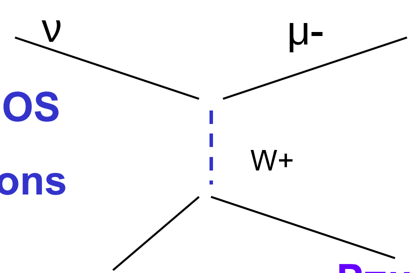
1st reson

X = 1

quasielastic

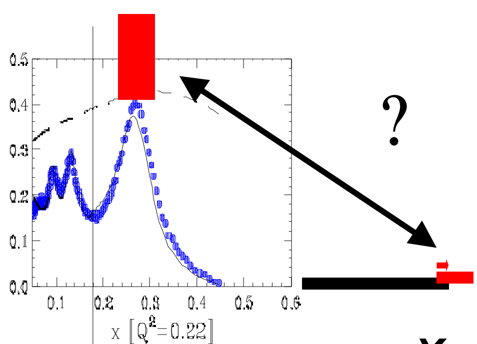
NEUTRINOS

On Neutrons



N=udd **P=uud or Res**

Both quasi+Res



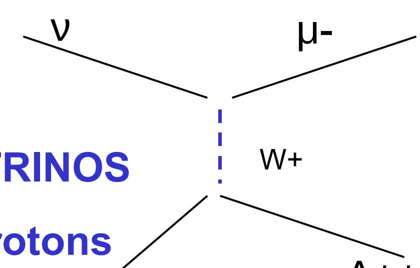
1st reson

X = 1

zero

NEUTRINOS

On Protons



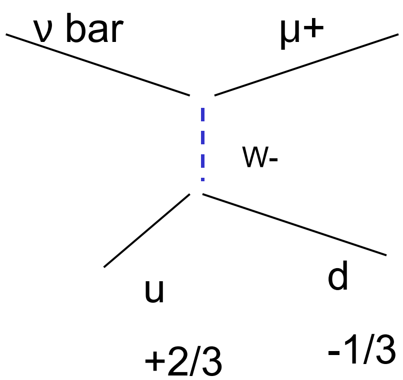
P=uud Δ++=uuu Res only state

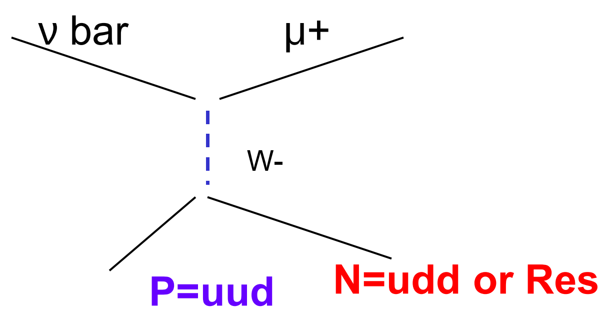
On protons only resonance+ DIS production possible.

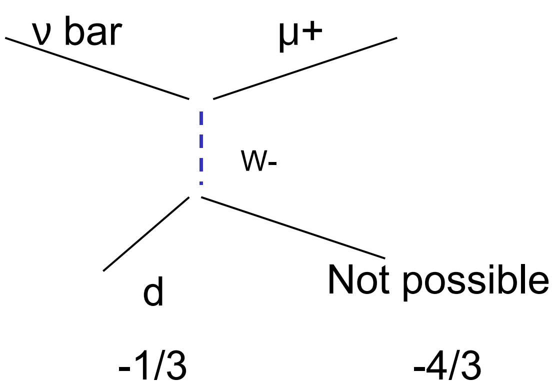
ANTI-NEUTRINOS

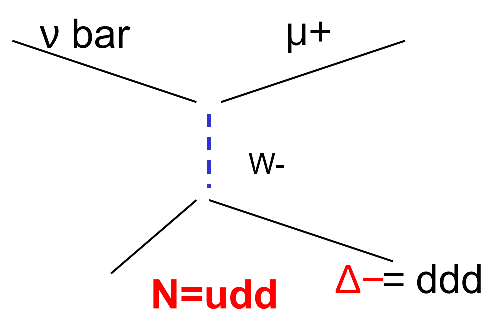
On Protons both quasielastic

And resonance+DIS production possible.









On Neutrons only

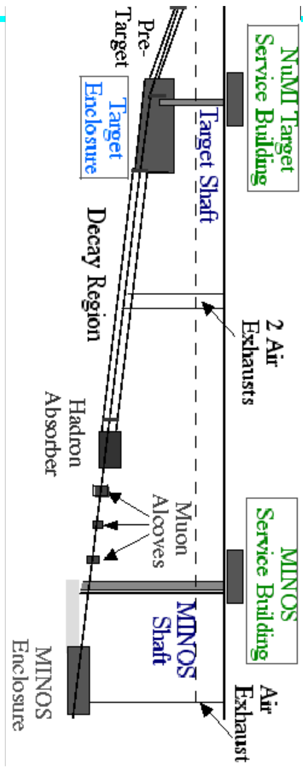
resonance+ DIS

production possible.

NUMI Off-Axis Near Detector

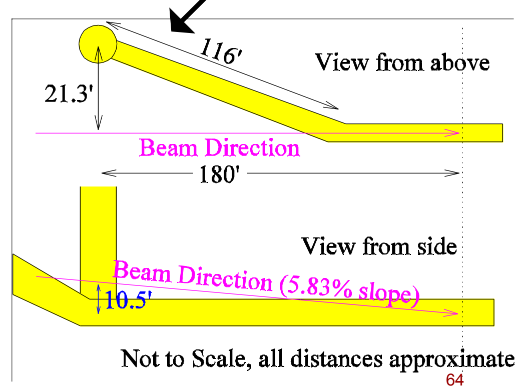
Rochester EOI

(with Jlab and others)

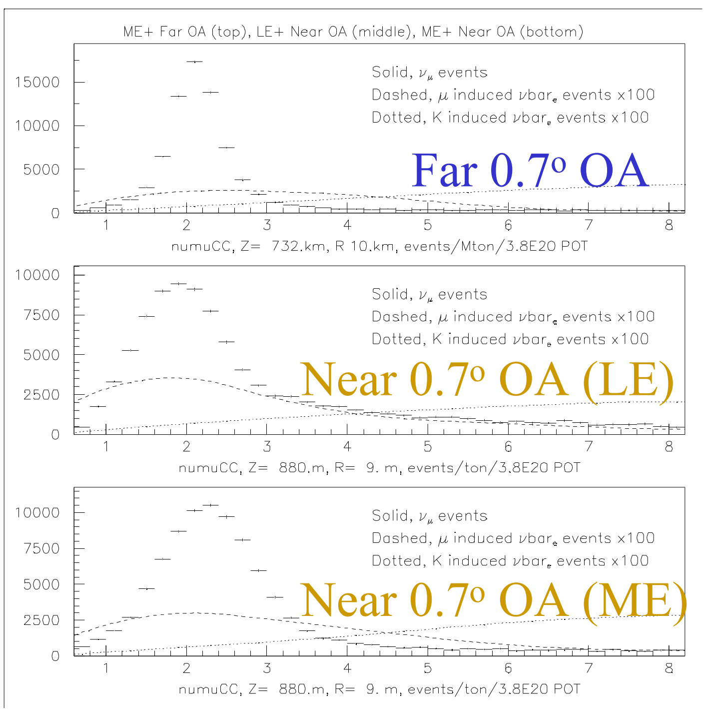


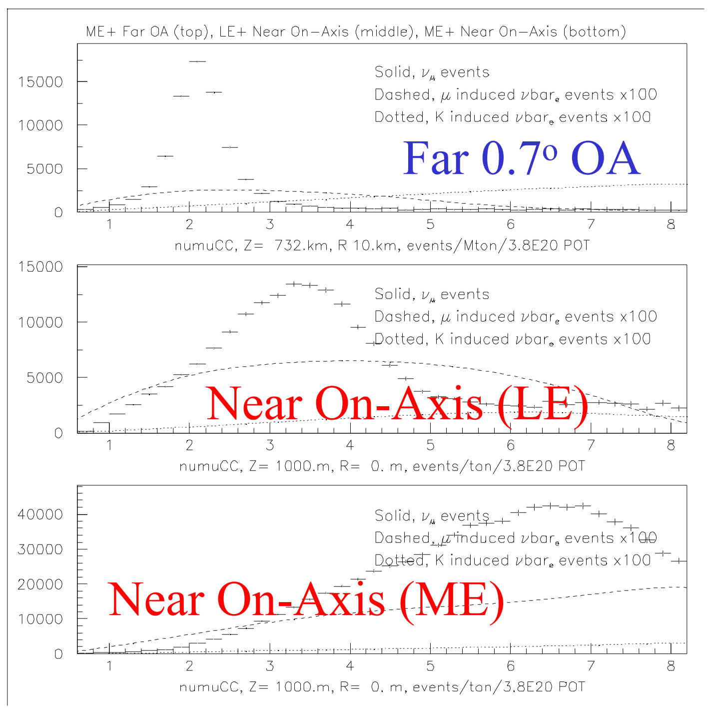
- Narrow band beam, similar to far detector
  - Can study cross-sections (NBB)
  - Near/far for ν_μ –> ν_μ;
  - backgrounds for ν_μ –>ν_e

Locate off of access drift

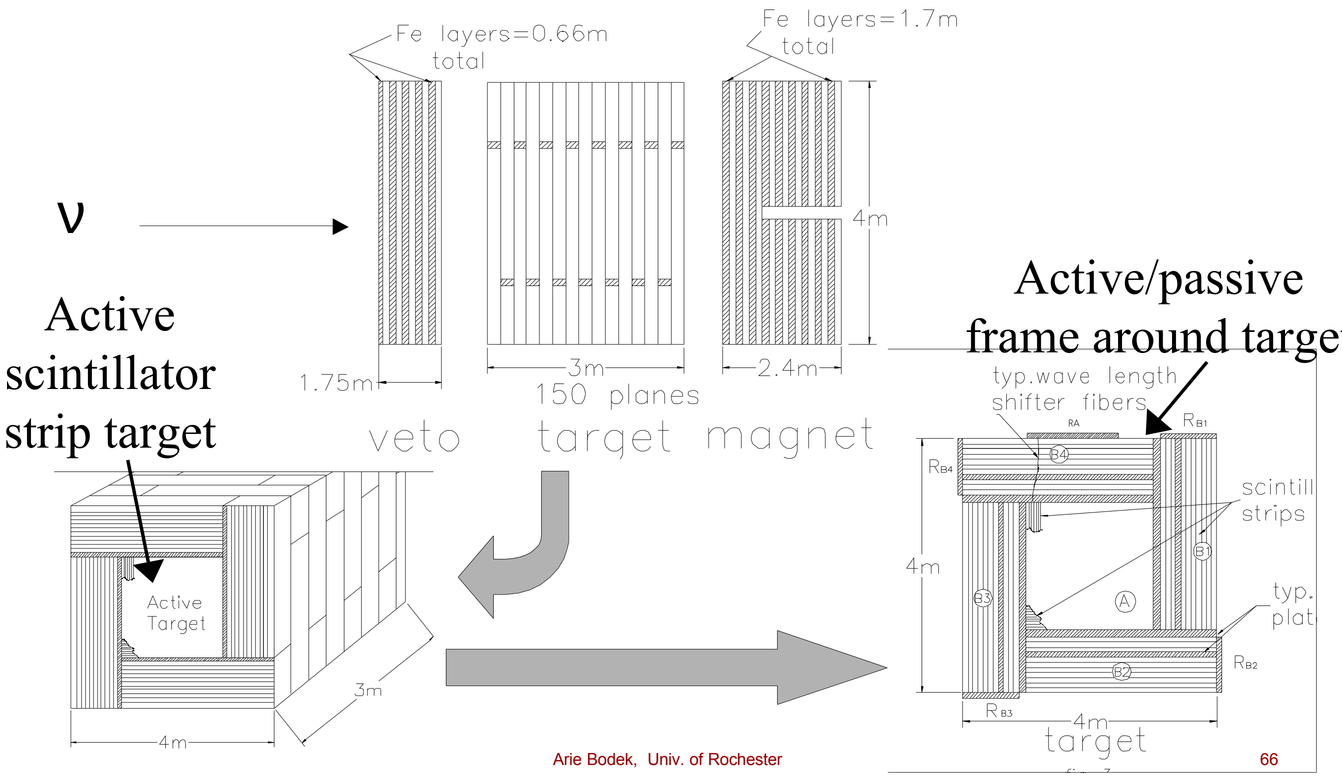


Event Spectra in Near Off-Axis, Near On-Axis and Far Detectors





Fully-Active Off-Axis Near Detector (Conceptual)



Future Progress

Next Update on this Work, NuInt02, Dec. 15,2002
At Irvine. Finalize modified PDFs and do duality tests with electron scattering data and Whatever neutrino data exists.

Also --> Get A(w,Q2) for electron proton and deuteron scattering cases (collaborate with Jlab Physicists on this next stage).

Meanwhile, Rochester and Jlab/Hampton physicists Have formed the nucleus of a collaboration to expand the present Rochester EOI to a formal NUMI Near Detector off-axis neutrino proposal (Compare Neutrino data to existing and future data from Jlab).

--contact person, Kevin McFarland.

Tests of Local Duality at high x, How local Electron Scattering Case

- INELASTIC High Q^2 x-->1.
- QCD at High Q^2 Note d refers to d quark in the proton, which is the same as u in the neutron. d/u=0.2; x=1.
- F2 (e-P) = (4/9)u+(1/9)d = (4/9+1/45) u = (21/45) u
- F2(e-N) = (4/9)d+(1/9)u = (4/45+5/45) u = (9/45) u

- F2(e-N) /F2 (e-P) = 9/21=0.43

- Elastic/quasielastic +resonance at high Q^2 dominated by magnetic form factors which have a dipole form factor times the magnetic moment
- F2 (e-P) = A G^2mP(el) +BG^2mN(res c=+1)
- F2 (e-N) = AG^2mN(el) +BG^2mN(res c=0)
- TAKE ELASTIC TERM ONLY
- F2(e-N) /F2 (e-P) (elastic) =

μ^2(N)/ μ^2(P) = $(1.913/2.793)^2$ =0.47

Close if we just take the elastic/quasielastic x=1 term.

Different at low Q2, where Gep,Gen dominate.

Since Gep=0.

Tests of Local Duality at high x, How local Neutrino Charged current Scattering Case

- INELASTIC High Q2, x-->1. QCD at High Q2: Note d refers to d quark in the proton, which is the same as u in the neutron. d/u=0.2; x=1.
- F2 (ν -P) = 2d
- F2(ν -N) = 2u
- F2 (ν bar -P) = 2u
- F2(ν bar-N) = 2d
- F2(ν -P) /F2 (ν -N) =d/u= 0.2
- F2(ν -P) /F2 (ν bar-P) =d/u=0.2
- F2(ν -P) / F2(ν bar-N) =1
- F2(ν -N) /F2 (ν bar-P) =1

- Elastic/quasielastic +resonance at high Q^2 dominated by magnetic form factors which have a dipole form factor times the magnetic moment
- F2 (ν -P) = A 0 (quasiel) + B(Resonance c=+2)
- F2(ν -N) = A Gm (ν quasiel) + B(Resonance c=+1)
- F2 (ν bar -P) = A Gm (ν quasiel) + B(Resonance c=0)
- F2(ν bar-N) = A 0(ν quasiel) + B(Resonance c=-1)

TAKE quasi ELASTIC TERM ONLY

- F2(ν -P) /F2 (ν -N) =0
- F2(ν -P) /F2 (ν bar-P) =0
- F2(ν -P) / F2(ν bar-N) =0/0
- F2(ν -N) /F2 (ν bar-P) =1

FAILS TEST MUST TRY TO COMBINE Quasielastic and first resonance)

Comparison of Xw Fit and ξw Fit backup slide *

Same construction for Xw and ξ w fits

$Xw = [Q^2+B] / [2M\nu+A]$ used in 1972

$\xi w = [Q^2+B] / [M\nu (1+(1+Q^2/\nu^2)^{1/2}) + A]$

(theoretically derived)

Multiply all PDFs by a factor of $Q^2/[Q^2+C]$

Fitted normalizations

HT fitting with **Xw**

	p	d
SLAC	0.979 +-0.0024	0.967 +- 0.0025
NMC	0.993 +-0.0032	0.990 +- 0.0028
BCDMS	0.956 +-0.0015	0.974 +- 0.0020

BCDMS Lambda = 1.01 +-0.156

HT fitting with ξw

	p	d
SLAC	0.982 +-0.0024	0.973 +- 0.0025
NMC	0.995 +-0.0032	0.994 +- 0.0028
BCDMS	0.958 +-0.0015	0.975 +- 0.0020

BCDMS Lambda = 0.976 +- 0.156.

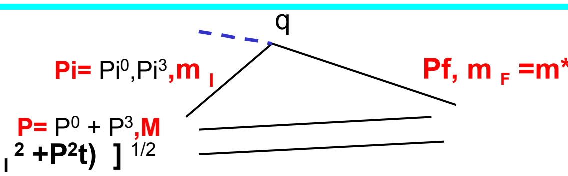
Comparison

- **Modified LO GRV94 PDFs with three parameters and the scaling variable, Xw, describe DIS F2 H, D data (SLAC/BCDMS/NMC) reasonably well.**
- **A=1.735, B=0.624, and C=0.188 (+-0.022) (+-0.014) (+-0.004)**
- **χ^2 = 1555 /958 DOF**
- ***<u>With ξ w A and B are smaller</u>* Modified LO GRV94 PDFs with three parameters and the scaling variable, ξ w describe DIS F2 H, D data (SLAC/BCDMS/NMC) *<u>EVEN BETTER</u>***
- **A=0.700, B=0.327, and C=0.197**
- **(+-0.020) (+-0.012) (+-0.004)**
- **χ^2 = 1351 /958 DOF**
- **Note: No systematic errors (except for normalization and BCDMS B field error) were included. GRV94 Assumed to be PEFECT (no f(x) floating factors). Better fits expected with GRV98 and floating factors f(x)**

Pseudo Next to Leading Order Calculations

Use LO : Look at PDFs(Xw) times (Q^2/Q^2+C) And PDFs (ξ w) times (Q^2/Q^2+C)

Xw= [Q +B] / [2Mν +A]
ξ w= [Q'^2 +B] / [Mν (1+(1+Q^2/ν^2) $^{1/2}$) +A]



Where $2Q'^2 = [Q^2 + m_F{}^2 - m_I{}^2] + [(Q^2 + m_F{}^2 - m_I{}^2)^2 + 4Q^2 (m_I{}^2 + P^2t)]^{1/2}$
(for now set P^2t =0, masses =0 excerpt for charm.

Add B and A account for effects of additional Δ m^2 from NLO and NNLO effects.

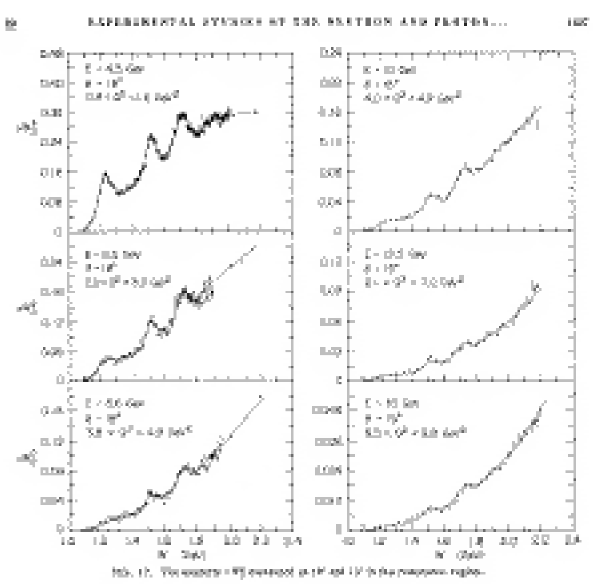
There are many examples of taking Leading Order Calculations and correcting them for NLO and NNLO effects using external inputs from measurements or additional calculations: e.g.

2. **Direct Photon Production - account for initial quark intrinsic Pt and Pt due to initial state gluon emission in NLO and NNLO processes by smearing the calculation with the MEASURED Pt extracted from the Pt spectrum of Drell Yan dileptons as a function of Q2 (mass).**
3. **W and Z production in hadron colliders. Calculate from LO, multiply by K factor to get NLO, smear the final state W Pt from fits to Z Pt data (within gluon resummation model parameters) to account for initial state multi-gluon emission.**
4. **K factors to convert Drell-Yan LO calculations to NLO cross sections. Measure final state Pt.**
3. **K factors to convert NLO PDFs to NNLO PDFs**
4. **Prediction of 2xF1 from leading order fits to F2 data , and imputing an empirical parametrization of R (since R=0 in QCD leading order).**
5. **THIS IS THE APPROACH TAKEN HERE. i.e. a Leading Order Calculation with input of effective initial quark masses and Pt and final quark masses, all from gluon emission.**

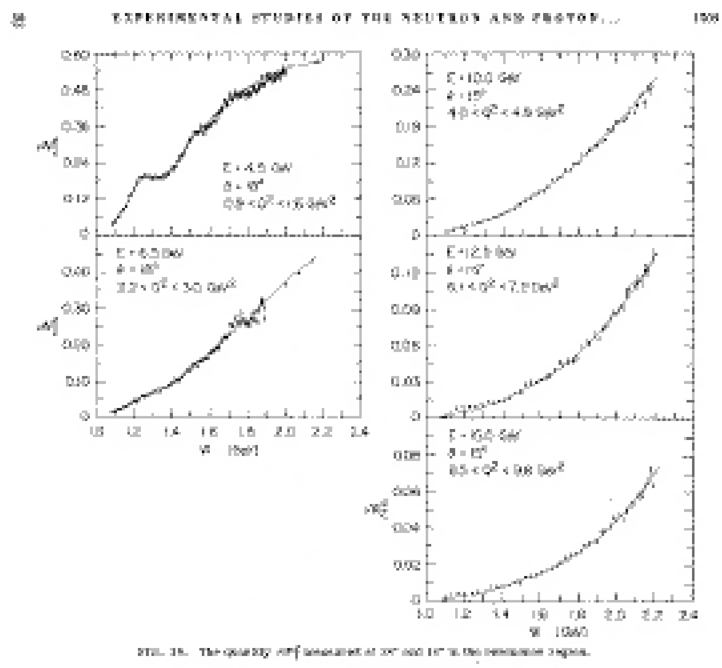
Model	χ^2 / DOF	Data Fit	PDF used	Scaling Variable	Power Param	Photo limit	A(W,Q2) Reson.	Ref.
QPM-0 Published 1979	--	e-N DIS/Res Q2>0	F2p F2d * f(x)	Xw= (Q2+B)/ (2Mν+A)	A=1.64 B=0.38	X/Xw C=B =0.38	A_P(W,Q) A_D(W,Q)	Bodek et al PRD-79
NLO-2 Published 1999	1470 /928 DOF	e/ μ --N, DIS Q2>1	MRSR2 * f(x)	ξ_{TM}=Q2/TM+ Renormalon model for 1/Q2	a2= -0.104 a4= - 0.003	Q2>1 NA	1.0- average	Yang/ Bodek PRL -99
NNLO-3 Published 2000	1406 /928 DOF	e/ μ --N, DIS Q2>1	MRSR2 * f(x)	ξ_{TM}=Q2/TM+ Renormalon model for 1/Q2	a2= -0.009 a4= -0.013	Q2>1 NA	1.0- average	Yang/ Bodek EPJC -00
LO-1 published 2001	1555 /958 DOF	e/μ--N, DIS Q2>0	GRV94 f(x)=1	Xw= (Q2+B)/ (2Mν+A)	A=1.74 B=0.62	Q2/ (Q2+C) C=0.19	1.0- average	Bodek/ Yang NuInt01
LO-1- Current 2002	1351 /958 DOF	e/ μ --N, DIS Q2>0	GRV94 f(x)=1	ξ w= (Q2+B)/ (TM+A)	A=0.70 B=0.33	Q2/ (Q2+C) C=0.20	1.0- average	Bodek/ Yang NuFac02
LO-1- Future work 2002-3	TBA	e/ μ --N, γ--N, ν--N, DIS/Res Q2>0	GRV? or other * f(x)	ξ 'w= (Q2+B..Pt^2)/ (TM+A)	A=TBA B=TBA Pt^2 = TBA	Q2/ (Q2+C) C= TBA	Au(W,Q) Ad(W,Q) ? Spect. Quark dependent	Bodek/ Yang Nutin02 +PRD

e-P, e-D: Xw scaling MIT SLAC DATA 1972 Low Q2
QUARK PARTON MODEL 0TH order (Q^2>0.5)

e-P scattering Bodek PhD thesis 1972
[PRD 20, 1471(1979)] Proton Data
Q^2 from 1.2 to 9 GeV^2 versus
$\nu W2= (x/x_w)^* F_2(X_w)^* A_P (W,Q^2)$-- QPM fit.

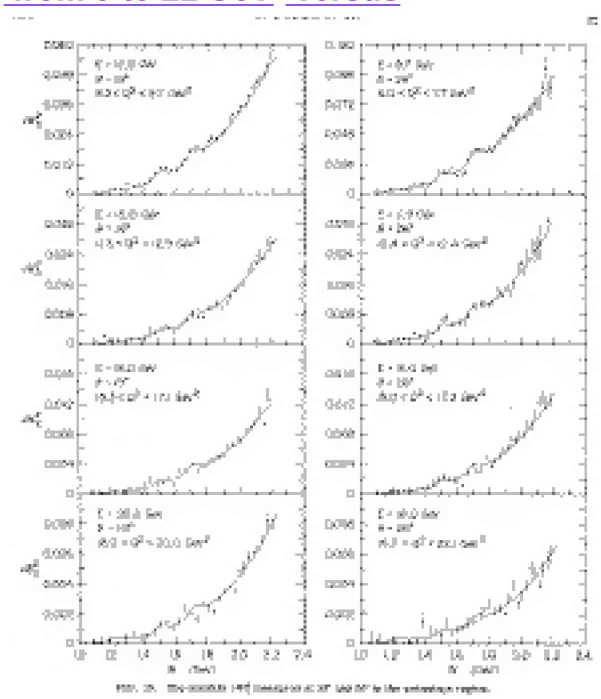


e-D scattering from same publication.
NOTE Deuterium Fermi Motion
Q^2 from 1.2 to 9 GeV^2 versus
$\nu W2= (x/x_w)^* F_2(X_w)^* A_D(W,Q^2)$ --QPM fit.

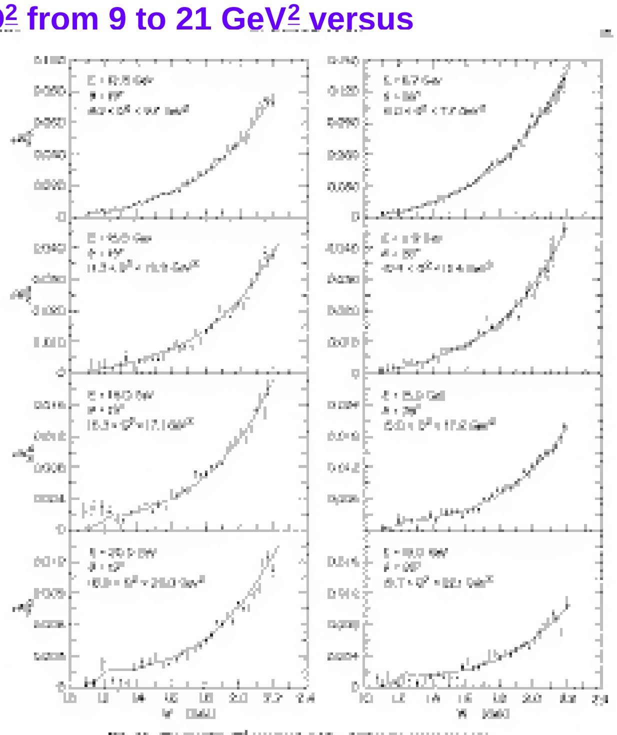


e-P, e-D: Xw scaling MIT SLAC DATA 1972 High Q2
QUARK PARTON MODEL 0TH order (Q^2>0.5)

e-P scattering Bodek PhD thesis 1972
[PRD 20, 1471(1979)] Proton Data
$\nu W2 = (x/x_w)^* F_2(X_w)^* A_P (W,Q^2)$-- QPM fit
Q^2 from 9 to 21 GeV^2 versus

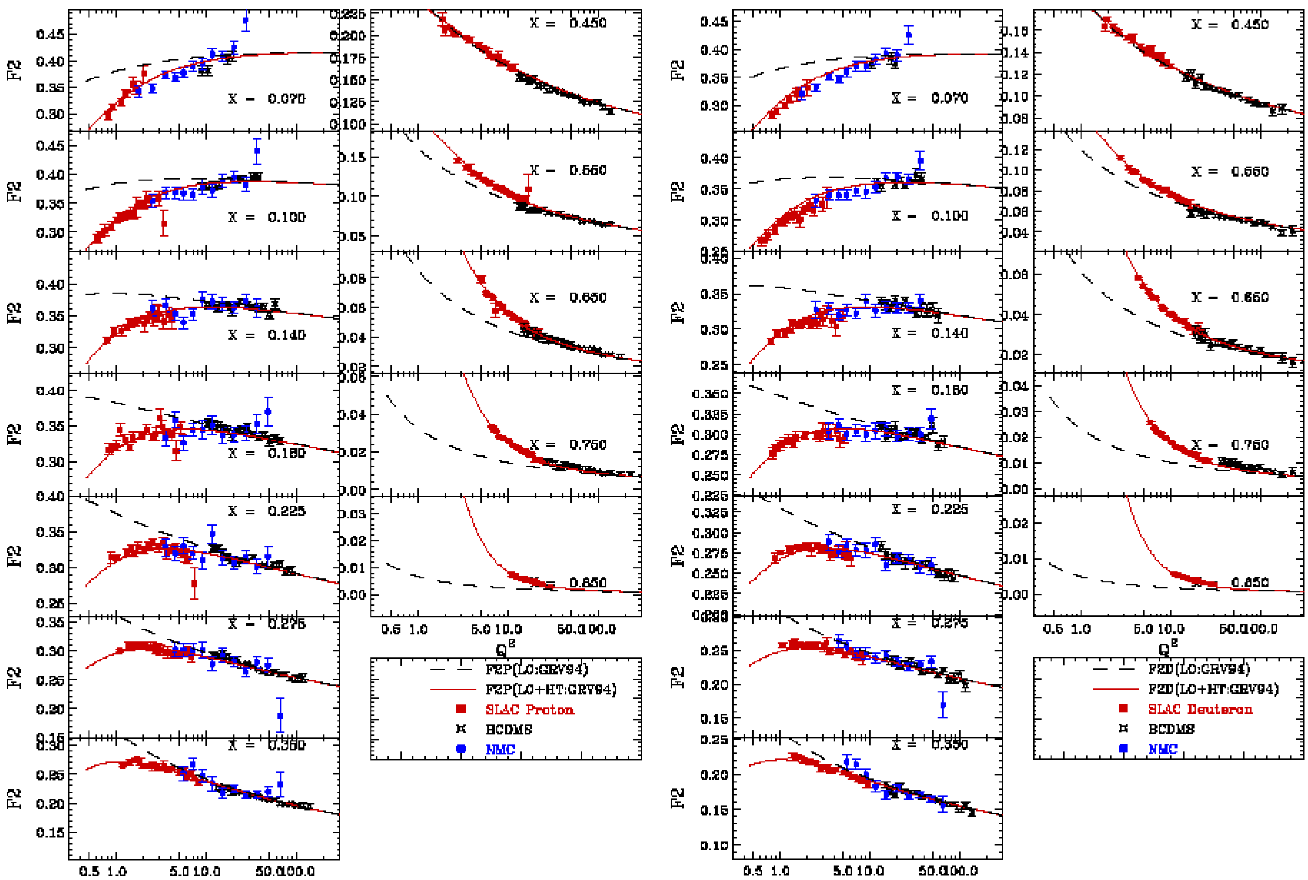


e-D scattering from same publication.
NOTE Deuterium Fermi Motion
$\nu W2 = (x/x_w)^* F_2(X_w)^* A_D(W,Q^2)$ --QPM fit.
Q^2 from 9 to 21 GeV^2 versus



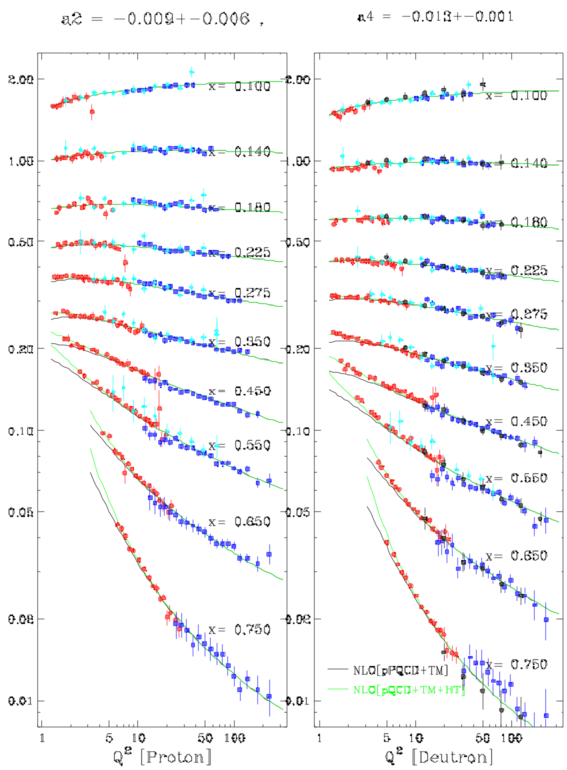
LO+HT ξ_w fit Comparison with DIS F_2 (H, D) data

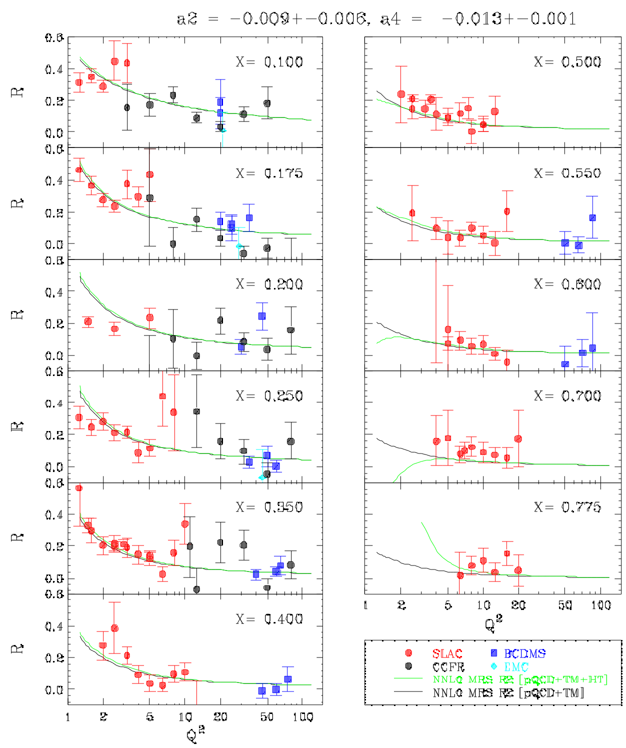
SLAC/BCDMS/NMC ξ_w works better χ^2 = 1351 /958 DOF **(Q^2>0.5)**



F_2, R comparison with NNLO QCD-works => NLO HT are missing NNLO terms (Q^2>1)

Size of the higher twist effect with NNLO analysis is really small (but not 0)
a2= -0.009 (in NNLO) versus –0.1(in NLO) - > factor of 10 smaller, a4 nonzero





Future Work - part 1

- Implement $A_{e/\mu}(W,Q^2)$ resonances into the model for F_2 with ξ_w scaling.
- For this need to fit all DIS and SLAC and JLAB resonance date and Photo-production H and D data and CCFR neutrino data.
- Check for local duality between ξ_w scaling curve and elastic form factors Ge, Gm in electron scattering. - Check method where its applicability will break down.
- Check for local duality of ξ_w scaling curve and quasielastic form factors Gm. Ge, G_A, G_V in quasielastic electron and neutrino and antineutrino scattering.- Good check on the applicability of the method in predicting exclusive production of strange and charm hyperons
- Compare our model prediction with the Rein and Seghal model for the 1st resonance (in neutrino scattering).
- Implement differences between ν and e/μ final state resonance masses in terms of
- A (i,j, k) (W,Q^2) (i is the interacting quark, and j,k are spectator quarks).
- Look at Jlab and SLAC heavy target data for possible Q^2 dependence of nuclear dependence on Iron.
- Implementation for R (and $2xF_1$) is done exactly - use empirical fits to R (agrees with NNLO+GP tgt mass for $Q^2>1$); Need to update Rw $Q^2<1$ to include Jlab R data in resonance region.
- Compare to low-energy neutrino data (only low statistics data, thus new measurements of neutrino differential cross sections at low energy are important).
- Check other forms of scaling e.g. $F_2=(1+ Q^2/\nu^2)^{1/2}\nu W_2$ (for low energies)

Future Work - part 2

- Investigate different scaling variable parameters for different flavor quark masses (u, d, s, u_v, d_v, u_{sea}, d_{sea} in initial and final state) for F_2. ,
- **Note:** $\xi_w = [Q^2+B] / [M\nu (1+ (1+Q^2/\nu^2)^{1/2}) +A]$ **assumes** $m_F = m_i = 0$, $P^2t=0$
- **More sophisticated General expression (see derivation in Appendix):**
- $\xi_w' = [Q'^2+B] / [M\nu (1+ (1+Q^2/\nu^2)^{1/2}) +A]$ **with**
- $2Q'^2 = [Q^2+ m_F^2 - m_I^2] + [(Q^2+ m_F^2 - m_I^2)^2 + 4Q^2 (m_I^2 +P^2t)]^{1/2}$
- **or** $2Q'^2 = [Q^2+ m_F^2 - m_I^2] + [Q^4 +2 Q^2(m_F^2 + m_I^2 +2P^2t) + (m_F^2 - m_I^2)^2]^{1/2}$
  Here B and A account for effects of additional Δm^2 from NLO and NNLO effects. However, one can include P^2t, as well as m_F , m_i as the current quark masses (e.g. Charm, production in neutrino scattering, strange particle production etc.). In ξ_w, B and A account for effective masses+initial Pt. When including Pt in the fits, we can constrain Pt to agree with the measured mean Pt of Drell Yan data..
- Include a floating factor f(x) to change the x dependence of the GRV94 PDFs such that they provide a good fit to all high energy DIS, HERA, Drell-Yan, W-asymmetry, CDF Jets etc, for a global PDF QCD LO fit to include Pt, quark masses A, B for ξ_w **scaling and the $Q^2/(Q^2+C)$ factor, and** $A_{e/\mu}(W,Q^2)$ as a first step towards modern PDFs. (but need to conserve sum rules).
- Put in fragmentation functions versus W, Q2, quark type and nuclear target