Vector and Axial Form Factors Applied to Neutrino Quasi-Elastic Scattering

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We calculate the total and differential quasielastic cross sections for neutrino and antineutrino scattering on nucleons using up to date fits to the nucleon elastic electromagnetic form factors G_E^p, G_E^n, G_M^p, G_M^n, and weak and pseudoscalar form factors. We show the extraction of $F_A(q^2)$ for neutrino experiments. We show show well MINERνA the new experiment the Fermi National Accelerator Laboratory, can measure $F_A(q^2)$. We show the that $F_A(q^2)$ has a different contribution to the anti-neutrino contribution, and can be used to check the value of $F_A(q^2)$ extracted from neutino scattering. (Presented by Howard Budd at NuInt04, Mar. 2004, Laboratori Nazionali del Gran Sasso - INFN - Assergi, Italy [?])

1. INTRODUCTION

Experimental evidence for oscillations among the three neutrino generations has been recently reported [?]. Since quasielastic (QE) scattering forms an important component of neutrino scattering at low energies, we have undertaken to investigate QE neutrino scattering using the latest information on nucleon form factors.

Recent experiments at SLAC and Jefferson Lab (JLab) have given precise measurements of the vector electromagnetic form factors for the proton and neutron. These form factors can be related to the form factors for QE neutrino scattering by conserved vector current hypothesis, CVC. These more recent form factors can be used to give better predictions for QE neutrino scattering.

2. EQUATIONS FOR QE SCATTERING

The hadronic current for QE neutrino scattering is given by [?]

$$< p(p_2)|J_\lambda^+|n(p_1) >= \overline{u}(p_2)\left[\gamma_\lambda F_V^1(q^2) + \frac{i\sigma_{\lambda\nu}q^\nu \xi F_V^2(q^2)}{2M} + \gamma_\lambda\gamma_5 F_A(q^2) + \frac{q_\lambda \gamma_5 F_P(q^2)}{M}\right] u(p_1),$$

where $q = k_\nu - k_\mu$, $\xi = (\mu_p - 1) - \mu_n$, and $M = (m_p + m_n)/2$. Here, μ_p and μ_n are the proton and neutron magnetic moments. We assume that there are no second class currents, so the scalar form factor F_V^3 and the tensor form factor F_A^3 need not be included.

The form factors $F_V^1(q^2)$ and $\xi F_V^2(q^2)$ are given by:

$$F_V^1(q^2) = \frac{G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)}{1 - \frac{q^2}{4M^2}},$$

$$\xi F_V^2(q^2) = \frac{G_M^V(q^2) - G_E^V(q^2)}{1 - \frac{q^2}{4M^2}}.$$

We use the CVC to determine $G_E^V(q^2)$ and $G_M^V(q^2)$ from the electron scattering form factors $G_E^p(q^2)$, $G_E^n(q^2)$, $G_M^p(q^2)$, and $G_M^n(q^2)$:

$$G_E^V(q^2) = G_E^p(q^2) - G_E^n(q^2),$$

$$G_M^V(q^2) = G_M^p(q^2) - G_M^n(q^2).$$

Previously, many neutrino experiment have assumed that the vector form factors are described by the dipole approximation.

$$G_D(q^2) = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2}, \quad M_V^2 = 0.71\ GeV^2$$

$$G_E^p = G_D(q^2), \quad G_E^n = 0,$$

$$G_M^p = \mu_p G_D(q^2), \quad G_M^n = \mu_n G_D(q^2).$$

We refer to the above combination of form factors as 'Dipole Form Factors'. It is an approximation that has been improved by us in a previous publication [?]. We use our updated form factors to which we refer as 'BBA-2003 Form Factors' (Budd, Bodek, Arrington).

The axial form factor is given by

$$F_A(q^2) = \frac{g_A}{\left(1 - \frac{q^2}{M_A^2}\right)^2}.$$

We also use our updated value [?] of M_A 1.00 $\pm$ 0.020 GeV which is in good agreement with the theoretically corrected value from pion electroproduction [?] of 1.014 $\pm$ 0.016 GeV. For extraction of $F_A(q^2)$ we use the value of $M_A = 1.014$ since it is independent of quasielastic scattering.

3. Extraction of $F_A(q^2)$

A substantial fraction of the cross section comes the form factor $F_A(q^2)$. Therefore, we can extract $F_A(q^2)$ from the differential cross section Figure **??** and **??** show the contribution of $F_A(q^2)$ to $d\sigma/dQ^2$. Figure **??** shows % change in the cross section vs % change in the form factors, i.e. $d(\%d\sigma/dQ^2)/d(\%formfactors)$. At high Q^2 F_A contributes to about 75% of the cross section. Figure **??** shows the contribution of the form factors by setting the each form factors = 0 and plotting $1 - (d\sigma/dQ^2(formfactor = 0))/(d\sigma/dQ^2)$. This method shows that F_A contributes to about 60% of the cross section. Since some terms are products of different form factors, the sum of the curves do not have be 1.

To extract F_A, we write the equation for $d\sigma/dq^2(q^2, E_\nu)$ in terms of a quadratic function

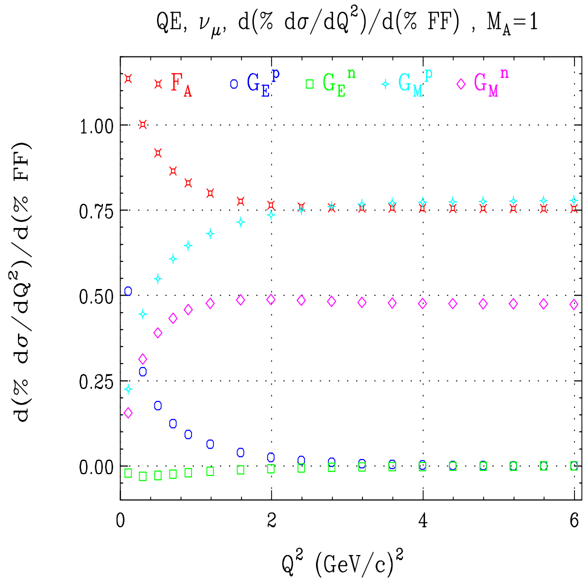

Figure 1. The percentage contribution of the cross section for a % change in the form factors, $d(\%d\sigma/dQ^2)/d(\%formfactors)$.

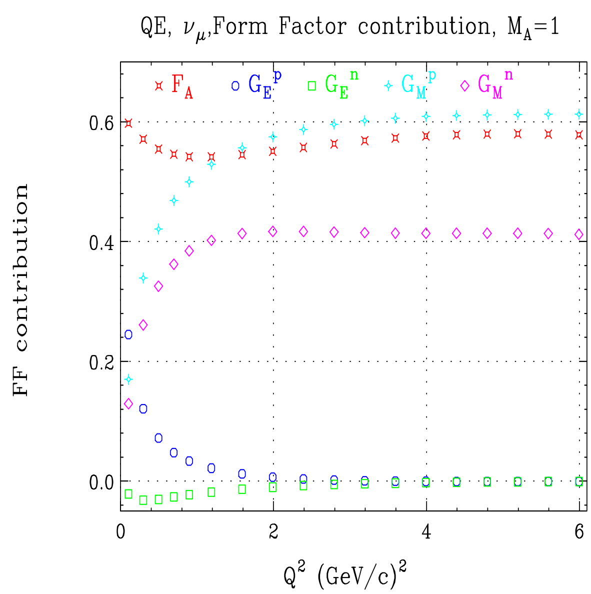

Figure 2. The contribution of the form factors determined by setting the form factors = 0, $1 - (d\sigma/dQ^2(formfactor = 0))/(d\sigma/dQ^2)$.

of $F_A(q^2)$.

$$a(q^2,E_\nu)F_A(q^2)^2 + b(q^2,E_\nu)F_A(q^2) + c(q^2,E_\nu) - \frac{d\sigma}{dq^2}(q^2,E_\nu) = 0$$

For each q^2 bin, we integrate the above equation over the q^2 bin and the neutrino flux.

$$\iint dq^2 dE_\nu \{a(q^2,E_\nu)F_A(q^2)^2 + b(q^2,E_\nu)F_A(q^2) + c(q^2,E_\nu) - \frac{d\sigma}{dq^2}(q^2,E_\nu)\} = 0$$

The above equation can be written as a quadratic equation in F_A at the bin value q_b^2.

$$\alpha F_A(q_b^2)^2 + \beta F_A(q_b^2) + \gamma - \Delta - N_{Bin}^{Data} = 0$$

The terms of this equation are given below:

$$\alpha = \iint dq^2 dE_\nu a(q^2,E_\nu)$$

$$\beta = \iint dq^2 dE_\nu b(q^2,E_\nu)$$

$$\gamma = \iint dq^2 dE_\nu c(q^2,E_\nu)$$

To find q_b^2, we assume a nominal $F_A(q^2)$, written $F_A^N(q^2)$. We determine q_b^2 from

$$\alpha F_A^N(q_b^2)^2 - \iint dq^2 dE_\nu a(q^2,E_\nu)F_A^N(q^2)^2 = 0.$$

Δ is a bin center correction term which uses the nominal $F_A^N(q^2)$. Δ is determined by

$$\Delta = \iint dq^2 dE_\nu \left[\frac{\beta F_A^N(q^2)^2}{\alpha} - b(q^2,E_\nu)F_A^N(q^2)\right].$$

The number of events in the bin is given by N_{Bin}^{Data}. The number of events in the bin from theory is

$$N_{Bin}^{Thy} = \iint dq^2 dE_\nu \frac{d\sigma}{dq^2}(q^2,E_\nu).$$

The errors in the points are given by

$$\frac{\sqrt{N_{Bin}^{Thy}}}{2\alpha F_A^N(q_b^2) + \beta}.$$

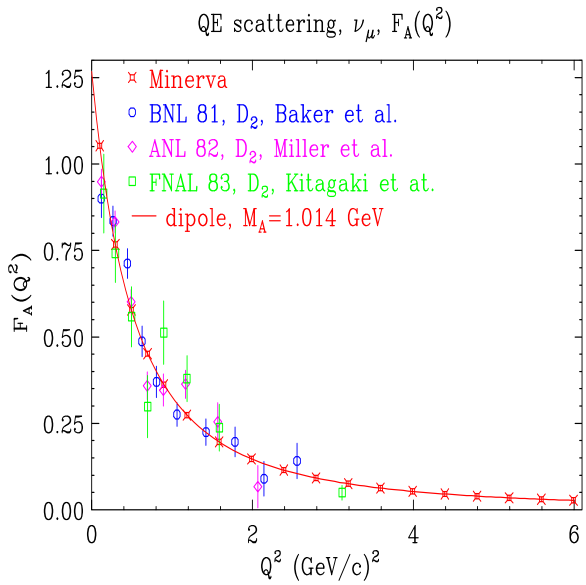

Figure 3. Extracted values of $F_A(q^2)$ for the three deuterium bubble chamber experiments Baker *et al.* [?], Miller *et al.* [?], and Kitagaki *et al.* [?]. Also shown are the expected errors for MINERνA assuming a dipole form factor for $F_A(q^2)$ with M_A=1.014

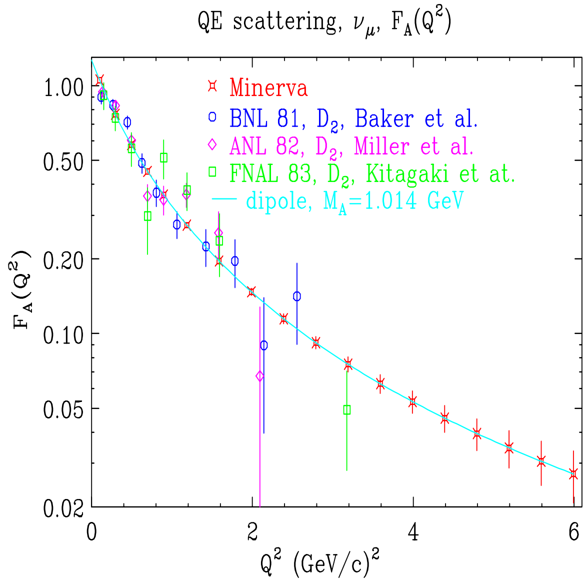

Figure 4. Same as Figure ?? with a logarithmic scale.

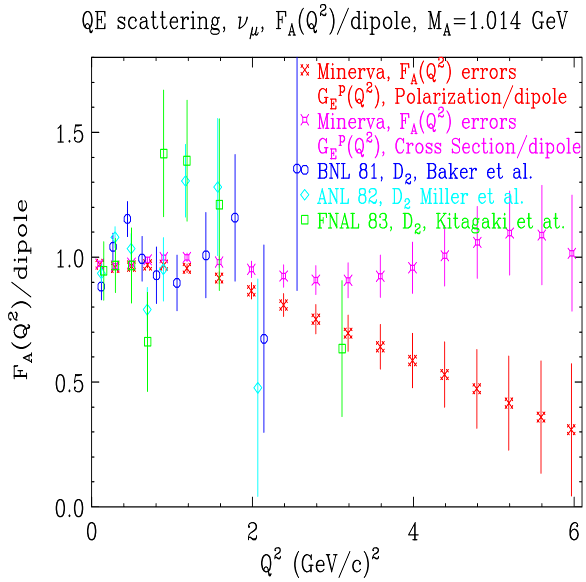

Figure 5. Extracted values of $F_A(q^2)/dipole$ for deuterium bubble chamber experiments Baker *et al.* [?], Miller *et al.* [?], and Kitagaki *et al.* [?]. For MINERνA we show $G_E^p/dipole$ for G_E^p determined from the polarization and cross section data and G_E^p determined from the cross section data. MINERνA errors are for a 4 year run.

Figure ?? and ?? show our extracted values of $F_A(q^2)$ for the three deuterium bubble chamber exteriments. For these plots we assume $F_A^N(q^2)$ is a dipole with m_A=1.014, the value extracted from pion-electro production. The data and fluxes given in their papers are used in the extraction of $F_A(q^2)$. These plots show the previous data is not sufficent to determine the form for $F_A(q^2)$.

In addition, we have shown the expected values for MINERνA and its errors. We have plotted MINERνA assuming it is a dipole. We have included the effects of inefficiencies and background. Resolution smearing and systematic errors are not included.

Figure ?? plots $F_A(q^2)$/dipole to show how well MINERνA can measure $F_A(q^2)$. $G_E^p(q^2)$ from electron scattering experiments depends upon the measuring technique. For the MINERνA $F_A(q^2)$ points, we show $G_E^p(q^2)$ derived from the cross section technique (Rosenbuth seperation) and the polarization transfer technique. The MINERνA errors are plotted assuming the plotted $F_A(q^2)$ is the nominal $F_A(q^2)$. We see that MINERνA can distinquish between these to the two possible forms for $G_E^p(q^2)$. In addition, MINERνA will be able to determine whether $F_A(q^2)$ is a dipole or not.

4. Extraction of $F_A(q^2)$ from anti-neutrinos

The determination of $F_A(q^2)$ will have systematic errors from the flux, nuclear effects, quasi-elastic identifications, background determination, etc. Anti-neutrino data can provide a check on $F_A(q^2)$. Figure ?? and ?? show the contribution of $F_A(q^2)$ to the cross section vs Q^2 for anti-neutrinos. Figure ?? shows % change in the cross section vs % change in the form factors, i.e., $d(\%d\sigma/dQ^2)/d(\%formfactors)$. The plot shows that $F_A(q^2)$ has a different contribution to the cross section for anti-neutrinos than neutrinos. At $Q^2 \sim 3GeV^2$, F_A is not contributing to the

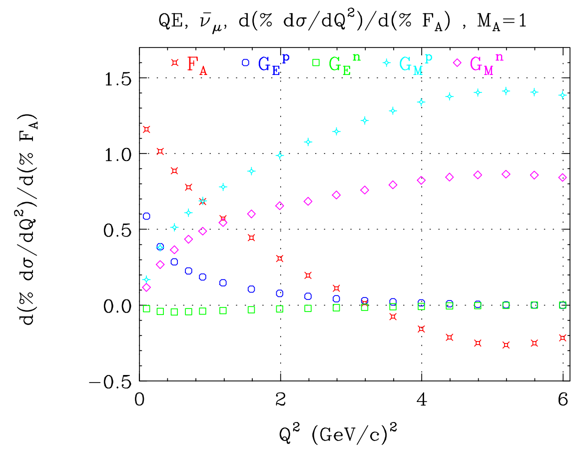

Figure 6. $d(\%d\sigma/dQ^2)/d(\%formfactors)$. The percentage contribution of the cross section for a % change in the form factors.

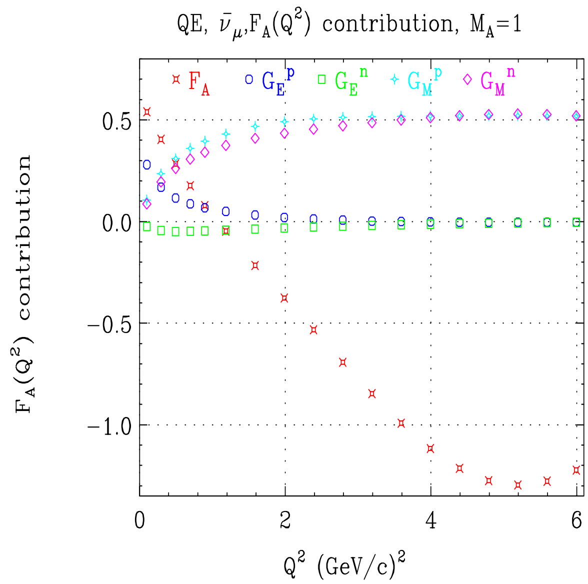

Figure 7. $1 - d\sigma/dQ^2(formfactor = 0)/d\sigma/dQ^2$. The contribution of the form factors determined by setting the form factors = 0.

cross section, and the cross section becomes independent of $F_A(q^2)$. Hence, at higher Q^2 the cross section can be predicted and compared to the data to determine errors to the neutrino extraction. Figure **??** shows the contribution of the form factors by setting the form factors = 0, $1 - d\sigma/dQ^2(formfactor = 0)/d\sigma/dQ^2$. Note, since some terms are products of different form factors the sum of the curves do not have to sum to 1.

Figure **??** shows the errors on F_A for anti-neutrinos. The errors are shown for $F_A(Q^2)/dipole$. The overall errors scale is arbitrary. As we expect, the errors on $F_A(q^2)$ become large at Q^2 around 3 GeV^2 when the derivative of the cross section with respect to $F_A(q^2)$ goes to 0. Figure **??** shows the percentage reduction in the cross section if $F_A(q^2)$ is reduced by 10%. At $Q^2 = 3GeV^2$ the cross section is independent of $F_A(q^2)$.

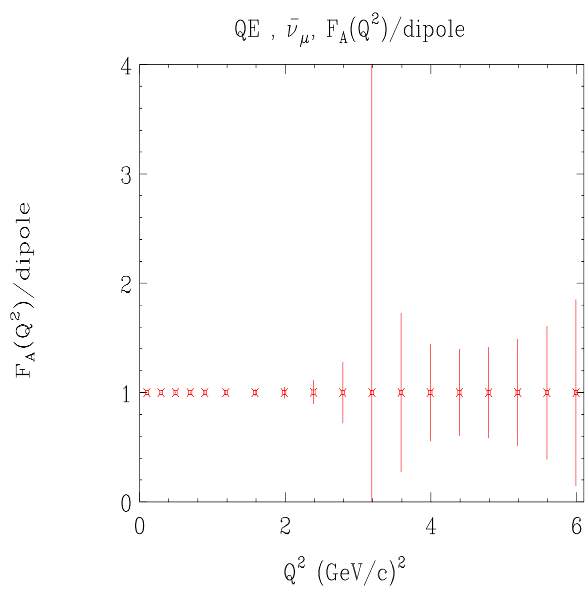

Figure 8. The relative bin by bin errors for an extraction of F_A using anti-neutrinos. The errors are shown for $F_A(Q^2)/dipole$. The flux is arbitrary.

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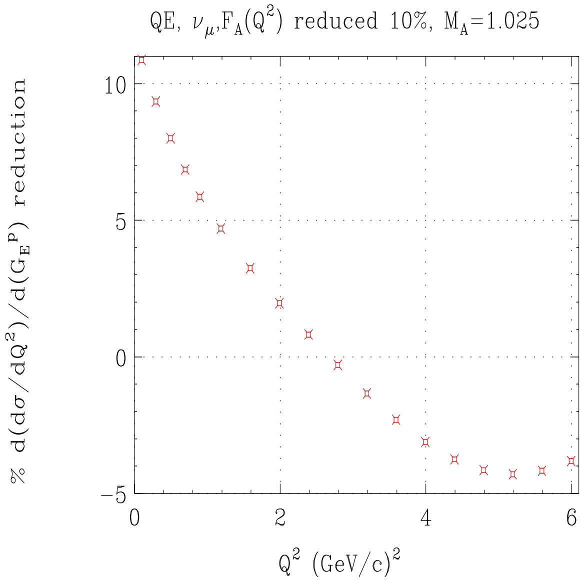

Figure 9. Percentage reduction in the differential anti-neutrino cross section if F_A is reduced by 10%.

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