

**UNIFIED APPROACH FOR MODELLING NEUTRINO AND
ELECTRON NUCLEON SCATTERING CROSS SECTIONS
FROM HIGH ENERGY TO VERY LOW ENERGY**

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We use a new scaling variable ξ_w , and add low Q^2 modifications to GRV98 leading order parton distribution functions such that they can be used to model electron, muon and neutrino inelastic scattering cross sections (and also photoproduction) at both very low and high energies.

In a previous communication¹ we used a modified scaling variable x_w and fit for modifications to the GRV94 leading order PDFs such that the PDFs describe both high energy low energy e/μ data. In order to describe low energy data down to the photoproduction limit ($Q^2 = 0$), and account for both target mass and higher twist effects, the following modifications of the GRV94 LO PDFs are need:

- (1) We increased the d/u ratio at high x as described in our previous analysis².
- (2) Instead of the scaling variable x we used the scaling variable $x_w = (Q^2 + B)/(2M\nu + A)$ (or $=x(Q^2 + B)/(Q^2 + Ax)$). This modification was used in early fits to SLAC data⁹. The parameter A provides for an approximate way to include *both* target mass and higher twist effects at high x , and the parameter B allows the fit to be used all the way down to the photoproduction limit ($Q^2=0$).
- (3) In addition as was done in earlier non-QCD based fits¹⁰ to low energy data, we multiplied all PDFs by a factor $K=Q^2 / (Q^2 +C)$. This was done in order for the fits to describe low Q^2 data in the photoproduction limit, where F_2 is related to the photoproduction

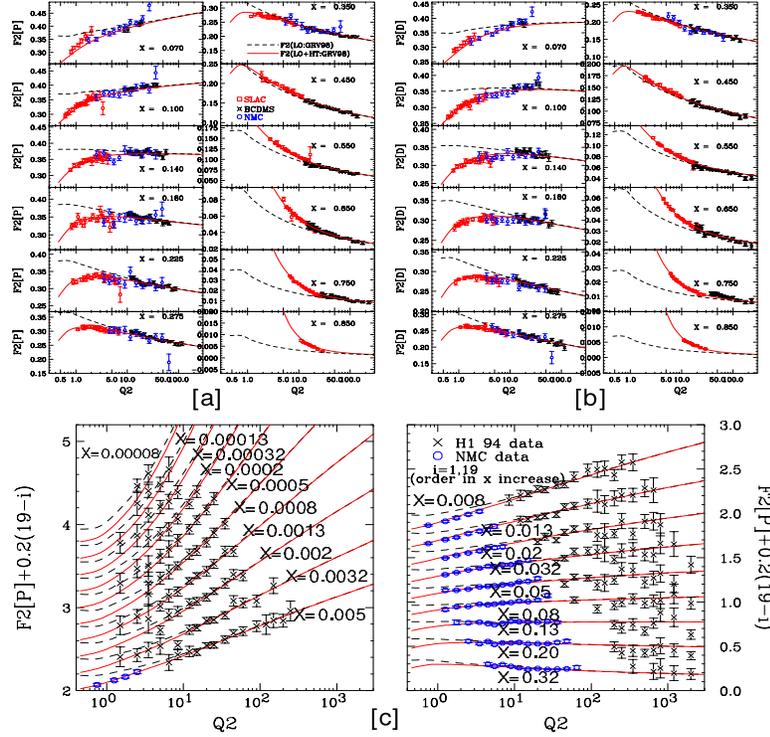


Figure 1. Electron and muon F_2 data (SLAC, BCDMS, NMC, H1 94) used in our GRV98 ξ_w fit compared to the predictions of the unmodified GRV98 PDFs (LO, dashed line) and the modified GRV98 PDFs fits (LO+HT, solid line); [a] for F_2 proton, [b] for F_2 deuteron, and [c] for the H1 and NMC proton data at low x .

cross section according to

$$\sigma(\gamma p) = \frac{4\pi^2\alpha_{EM}}{Q^2} F_2 = \frac{0.112mb GeV^2}{Q^2} F_2$$

- (4) Finally, we froze the evolution of the GRV94 PDFs at a value of $Q^2 = 0.24$ (for $Q^2 < 0.24$), because GRV94 PDFs are only valid down to $Q^2 = 0.23$ GeV².

In our analyses, the measured structure functions were corrected for the BCDMS systematic error shift and for the relative normalizations between the SLAC, BCDMS and NMC data ^{2,3}. The deuterium data were corrected for nuclear binding effects ^{2,3}.

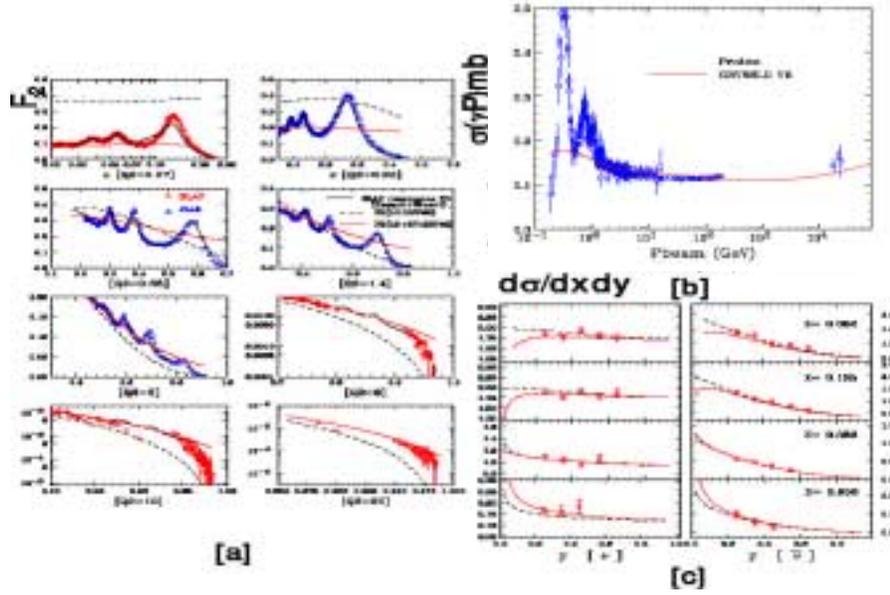


Figure 2. Comparisons to data not included in the fit. (a) Comparison of SLAC and JLab (electron) F_{2p} data in the resonance region (or fits to these data) and the predictions of the GRV98 PDFs with (LO+HT, solid) and without (LO, dashed) our modifications. (b) Comparison of photoproduction data on protons to predictions using our modified GRV98 PDFs. (c) Comparison of representative CCFR ν_μ and $\bar{\nu}_\mu$ on iron at 55 GeV and the predictions of the GRV98 PDFs with (LO+HT, solid) and without (LO, dashed) our modifications.

In this publication we update our previous studies,⁸ which were done with a new improved scaling variable ξ_w , and fit for modifications to the more modern GRV98 LO PDFs such that the PDFs describe both high energy and low energy electron/muon data. We now also include NMC and H1 94 data at lower x . Here we freeze the evolution of the GRV98 PDFs at a value of $Q^2 = 0.8$ (for $Q^2 < 0.8$), because GRV98 PDFs are only valid down to $Q^2 = 0.8$ GeV². In addition, we use different photoproduction limit multiplicative factors for valence and sea. Our proposed new scaling variable is based on the following derivation. Using energy momentum conservation, it can be shown that the fractional momentum $\xi = (p_z + p_0)/(P_z + P_0)$ carried by a quark of 4-momentum p in a proton target of mass M and 4-momentum P is given by $\xi = xQ'^2/[0.5Q^2(1+[1+(2Mx)^2/Q^2]^{1/2})]$,

where

$$2Q'^2 = [Q^2 + M_f^2 - M_i^2] + [(Q^2 + M_f^2 - M_i^2)^2 + 4Q^2(M_i^2 + P_T^2)]^{1/2}.$$

Here M_i is the initial quark mass with average initial transverse momentum P_T and M_f is the mass of the quark in the final state. The above expression for ξ was previously derived ⁶ for the case of $P_T = 0$. Assuming $M_i = 0$ we use instead:

$$\xi_w = x(Q^2 + B + M_f^2)/(0.5Q^2(1 + [1 + (2Mx)^2/Q^2]^{1/2}) + Ax)$$

Here $M_f=0$, except for charm-production processes in neutrino scattering for which $M_f=1.5$ GeV. For ξ_w the parameter A is expected to be much smaller than for x_w since now it only accounts for the higher order (dynamic higher twist) QCD terms in the form of an *enhanced* target mass term (the effects of the proton target mass are already taken into account using the exact form in the denominator of ξ_w). The parameter B accounts for the initial state quark transverse momentum and final state quark *effective* ΔM_f^2 (originating from multi-gluon emission by quarks).

Using closure considerations ¹¹ (*e.g.* the Gottfried sum rule) it can be shown that, at low Q^2 , the scaling prediction for the *valence* quark part of F_2 should be multiplied by the factor $K=[1-G_D^2(Q^2)][1+M(Q^2)]$ where $G_D = 1/(1+Q^2/0.71)^2$ is the proton elastic form factor, and $M(Q^2)$ is related to the magnetic elastic form factors of the proton and neutron. At low Q^2 , $[1-G_D^2(Q^2)]$ is approximately $Q^2/(Q^2 + C)$ with $C = 0.71/4 = 0.178$. In order to satisfy the Adler Sum rule ¹² we add the function $M(Q^2)$ to account for terms from the magnetic and axial elastic form factors of the nucleon). Therefore, we try a more general form $K_{valence}=[1-G_D^2(Q^2)][Q^2+C_{2v}]/[Q^2+C_{1v}]$, and $K_{sea}=Q^2/(Q^2+C_{sea})$. Using this form with the GRV98 PDFs (and now also including the very low x NMC and H1 94 data in the fit) we find $A=0.419$, $B=0.223$, and $C_{1v}=0.544$, $C_{2v}=0.431$, and $C_{sea}=0.380$ (all in GeV², $\chi^2 = 1235/1200$ DOF). With these modifications, the GRV98 PDFs must also be multiplied by $N=1.011$ to *normalize* to the SLAC F_{2p} data. The fit (Figure 1) yields the following normalizations relative to the SLAC F_{2p} data ($SLAC_D=0.986$, $BCDMS_P=0.964$, $BCDMS_D=0.984$, $NMC_P=1.00$, $NMC_D=0.993$, $H1_P=0.977$, and BCDMS systematic error shift of 1.7). (Note, since the GRV98 PDFs do not include the charm sea, for $Q^2 > 0.8$ GeV² we also include charm production using the photon-gluon fusion model in order to fit the very high ν HERA data. This is not needed for any of the low energy comparisons but is only needed to describe the highest ν HERA electro and photoproduction data).

Comparisons of *predictions* using these modified GRV98 PDFs to other data which were *not included* in the fit is shown in Figures 2 and 3. From

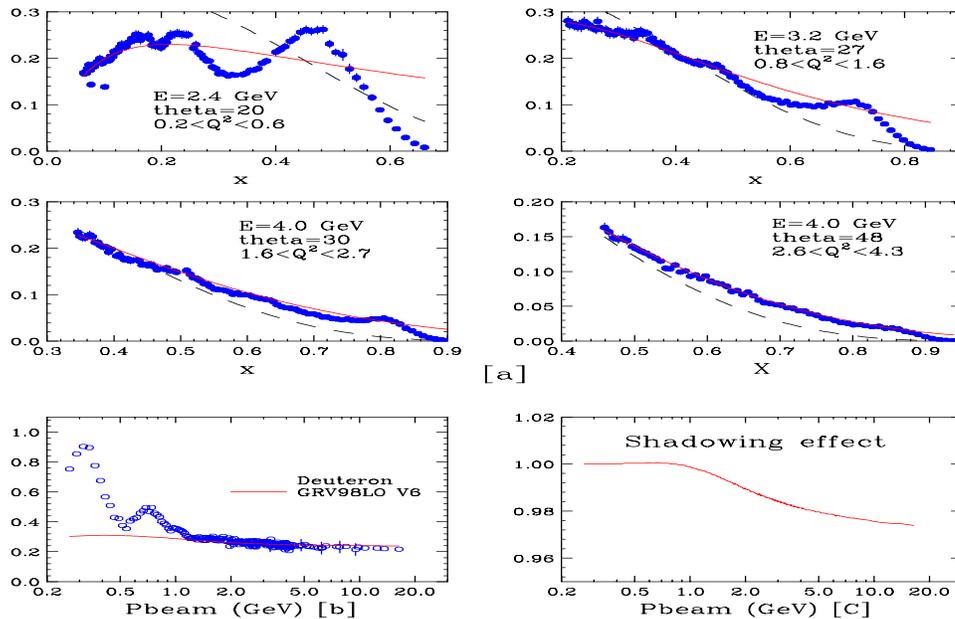


Figure 3. Comparisons to data on deuterium which were not included in our GRV98 ξ_w fit. (a) Comparison of SLAC and JLab (electron) F_{2d} data in the resonance region and the predictions of the GRV98 PDFs with (LO+HT, solid) and without (LO, dashed) our modifications. (b) Comparison of photoproduction data on deuterium to predictions using our modified GRV98 PDFs (including shadowing corrections). (c) The shadowing corrections that were applied to the PDFs for predicting the photoproduction cross section on deuterium.

duality¹⁴ considerations, with the ξ_w scaling variable, the modified GRV98 PDFs should also provide a reasonable description of the average value of F_2 in the resonance region. Figures 2(a) and 3(a) show a comparison between resonance data (from SLAC and Jefferson Lab, or parametrizations of these data¹⁵) on protons and deuterons versus the predictions with the standard GRV98 PDFs (LO) and with our modified GRV98 PDFs (LO+HT). The modified GRV98 PDFs are in good agreement with SLAC and JLab resonance data down to $Q^2 = 0.07$ (although resonance data were not included in our fits). There is also very good agreement of the *predictions* of our modified GRV98 in the $Q^2 = 0$ limit with photoproduction data on protons and deuterons as shown in Figure 2(b) and 3(b). In predicting the photoproduction cross sections on deuterium, we have applied shadowing corrections[?] as shown in Figure 3(c). We also compare the *predictions* with our modified GRV98 PDFs (LO+HT) to a few representative high

energy CCFR ν_μ and $\bar{\nu}_\mu$ charged-current differential cross sections^{4,13} on iron (neutrino data were not included in our fit). In this comparison we use the PDFs to obtain F_2 and xF_3 and correct for nuclear effects in iron¹. The structure function $2xF_1$ is obtained by using the R_{world} fit from reference⁵. There is very good agreement of our *predictions* with these neutrino data on iron at 55 GeV (assuming that vector and axial structure functions are the same). We are currently working on further corrections to account for the fact that at low energies, the vector and axial structure functions are different.

References

1. A. Bodek and U. K. Yang, (hep-ex/0203009) Nucl.Phys.Proc.Suppl.112:70-76,2002
2. U. K. Yang and A. Bodek, Phys. Rev. Lett. **82**, 2467 (1999).
3. U. K. Yang and A. Bodek, Eur. Phys. J. **C13**, 241 (2000).
4. U. K. Yang, Ph.D. thesis, Univ. of Rochester, UR-1583 (2001).
5. L. W. Whitlow *et al.* (SLAC-MIT) , Phys. Lett. **B282**, 433 (1995); A. C. Benvenuti *et al.* (BCDMS) , Phys. Lett. **B237**, 592 (1990); M. Arneodo *et al.* (NMC), Nucl. Phys. **B483**, 3 (1997).
6. H. Georgi and H. D. Politzer, Phys. Rev. **D14**, 1829 (1976); R. Barbieri *et al.*, Phys. Lett. **B64**, 171 (1976), and Nucl. Phys. **B117**, 50 (1976); J. Pestieau and J. Urias, Phys.Rev.**D8**, 1552 (1973)
7. A.L. Kataev *et al.*, Phys. Lett. **B417**, 374 (1998), and also hep-ph/0106221; J. Bluemlein and A. Tkabladze, Nucl. Phys. **B553**, 427 (1999).
8. A. Bodek, U. K. Yang, hep-ex/0210024 , J. Phys. G. Nucl. Part. Phys. **29**, 1 (2003); A. Bodek and U. K. Yang, To be published in Proceeding of NUINT02 2nd Workshop on Neutrino - Nucleus Interactions in the Few GeV Region (Nulnt01), Irvine CA ,2002; A. Bodek and U. K. Yang, hep-ex/0301036
9. A. Bodek *et al.*, Phys. Rev. **D20**, 1471 (1979).
10. A. Donnachie and P. V. Landshoff, Z. Phys. **C 61**, 139 (1994); B. T. Fleming *et al.*(CCFR), Phys. Rev. Lett. **86**, 5430 (2001).
11. S. Stein *et al.*, Phys. Rev. **D12**, 1884 (1975); K. Gottfried, Phys. Rev. Lett. **18**, 1174 (1967).
12. S. Adler, Phys. Rev. **143**, 1144 (1966); F. Gillman, Phys. Rev. **167**, 1365 (1968).
13. U. K. Yang *et al.*(CCFR), Phys. Rev. Lett. **87**, 251802 (2001).
14. E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. **25**, 1140 (1970).
15. C. Keppel, Proc. of the Workshop on Exclusive Processes at High P_T , Newport News, VA, May (2002).]