DIELECTRICS

SUMMARY

Integral form:

$$\Phi_E = \oint_{\substack{Closed\\surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{1}{\varepsilon_0} \int_{\substack{enclosed\\volume}} \rho d\tau$$

$$\Phi_B = \oint_{\substack{Closed\\surface}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = 0$$

$$\oint_{\substack{Closed\\loop\\C}} \vec{\mathbf{E}} \cdot \vec{d\mathbf{l}} = -\int_{\substack{surface\\bounded\\by\\C}} \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot \vec{\partial \mathbf{S}}$$

$$\oint_{\substack{Closed\\loop\\C}} \vec{\mathbf{B}} \cdot \vec{\mathbf{dl}} = \mu_0 \int_{\substack{Bounded\\by\\C}} (\vec{\mathbf{j}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}) \cdot d\vec{\mathbf{S}}$$

Differential form:

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{\mathbf{B}} = 0$$

$$\left(\vec{\nabla} \times \vec{\mathbf{E}}\right) = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

Lorentz force

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

Vector derivatives: Cartesian Coordinates

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

$$\textit{Gradient}: \quad \nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

$$\textit{Divergence}: \quad \nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\textit{Curl}: \quad \nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\textit{Laplacian}: \quad \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Vector derivatives: Spherical Coordinates

See Appendix A in "Introduction to Electrodynamics" by Griffiths

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

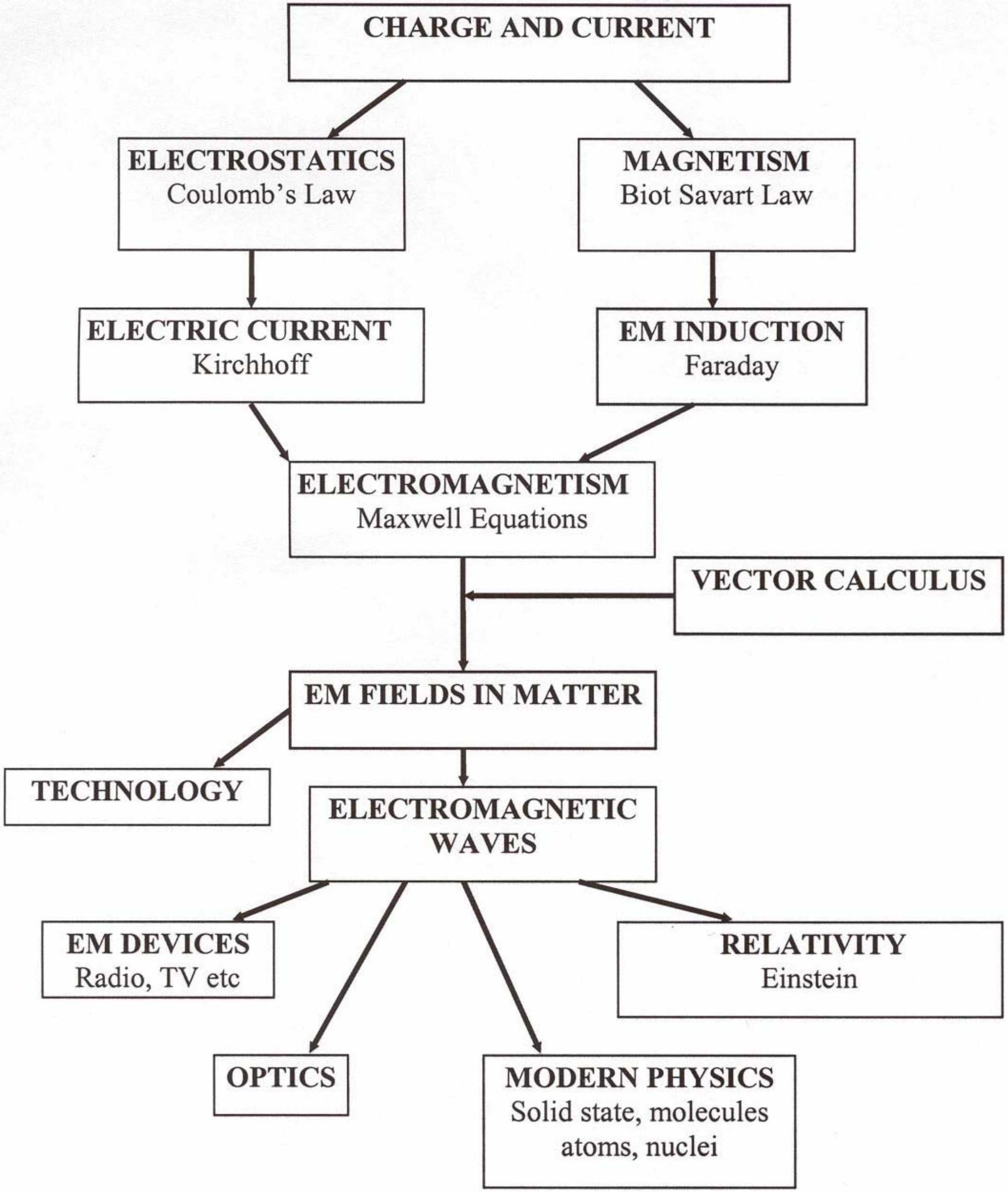
Gradient :
$$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence :
$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl :
$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian :
$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

P142 Preview; Subject Matter



P142 Dielectrics Lecture 19

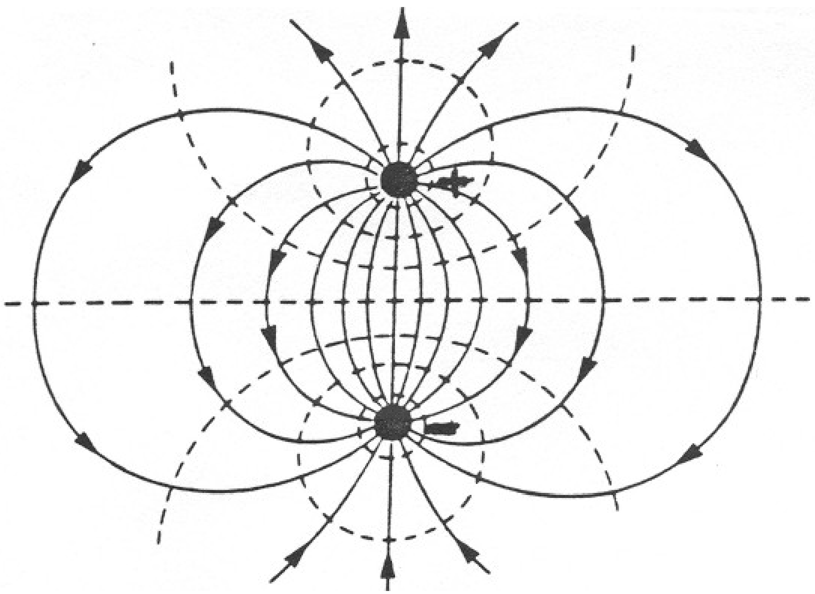
- **Electric dipole**
- **Dielectric polarization**
- **Electric fields in dielectrics**
- **Electric displacement field, D**
- **Summary**

Dielectrics: Electric dipole

The electric dipole moment **p** is defined as

$$\mathbf{p} = q\,\mathbf{d}$$

where **d** is the separation distance between the charges q pointing from the negative to positive charge.



Lecture 5 showed that

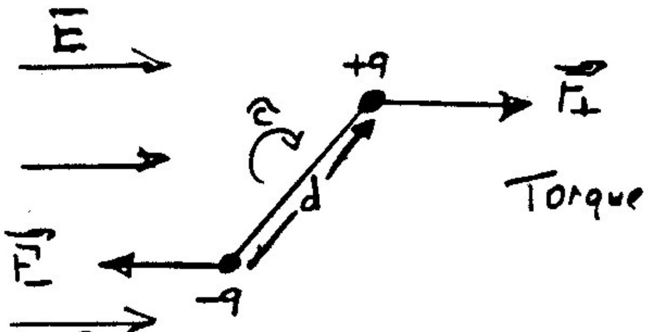
$$V_P = \frac{\overrightarrow{\mathbf{p}} \cdot \widehat{\mathbf{r}}}{4\pi\varepsilon_o r^2} \qquad r >> d$$

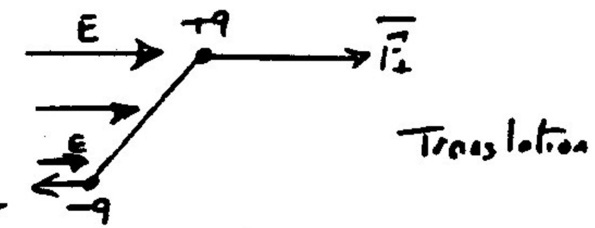
where $\widehat{\mathbf{r}}$ points from the electric dipole towards the point P. The electric field for the electric dipole, in the far-field approximation, can be derived from the above potential since $\overrightarrow{\mathbf{E}} = -\overrightarrow{\boldsymbol{\nabla}} V$ as was discussed previously.

Forces on an electric dipole in E field

Torque:

$$\overrightarrow{\boldsymbol{\tau}} = 2(\frac{d}{2}qE\sin\theta) = \overrightarrow{\mathrm{p}} \times \overrightarrow{\mathrm{E}}$$





Forces on an electric dipole in E field

Translational force:

$$\overrightarrow{\mathbf{F}_{net}} = \overrightarrow{\mathbf{p}} \cdot \overrightarrow{\nabla}\,\overrightarrow{\mathbf{E}}$$

This can be rewritten using the vector identity

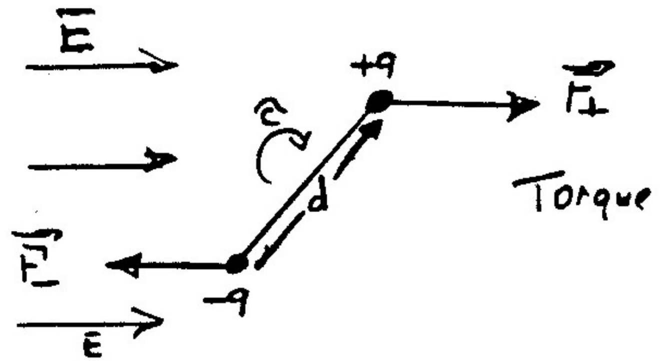
$$\overrightarrow{\mathbf{p}} \times \left(\overrightarrow{\nabla} \times \overrightarrow{\mathbf{E}}\right) = \overrightarrow{\nabla}\left(\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}}\right) - \left(\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\nabla}\right)\overrightarrow{\mathbf{E}} = 0$$

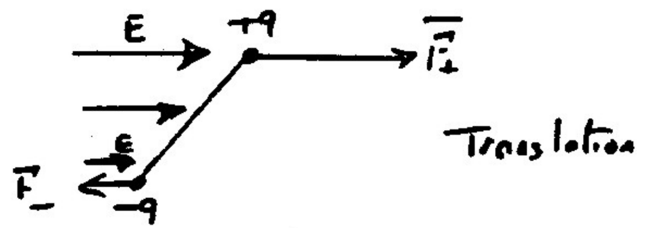
since $\overrightarrow{\nabla} \times \overrightarrow{\mathbf{E}} = 0$ for electrostatics. Thus the translational force can be rewritten as

$$\overrightarrow{\mathbf{F}_{net}} = \overrightarrow{\nabla}\left(\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}}\right)$$

This will be used following a discussion of the value of dipole polarization produced by an electric field. When the dipole and electric field are aligned then this simplifies to

$$\overrightarrow{\mathbf{F}_{net}} = qd\frac{dE}{dx}$$

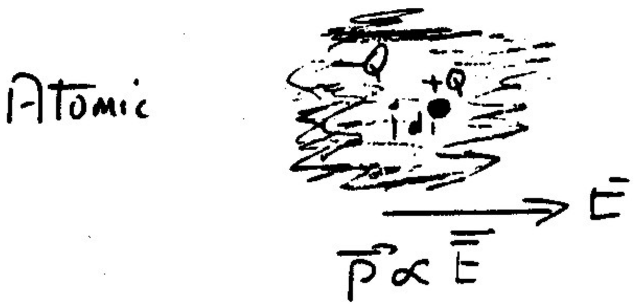




Dielectric polarization: Microscopic

Atomic polarization:

$d \sim 10^{-15}$ m



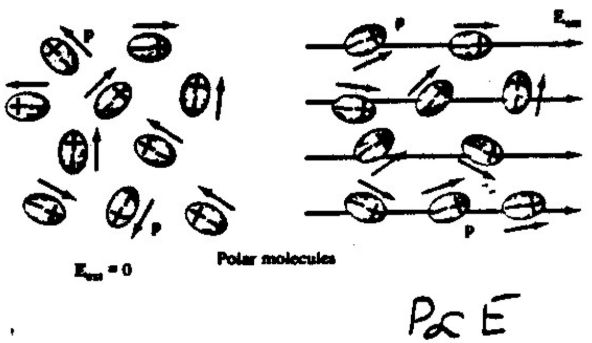
Molecular polarization:

$d \sim 10^{-14}$ m

Align polar molecules:

$d \sim 10^{-10}$ m

Competition between alignment torque and thermal motion or elastic forces



Linear dielectrics:

Three mechanisms typically lead to

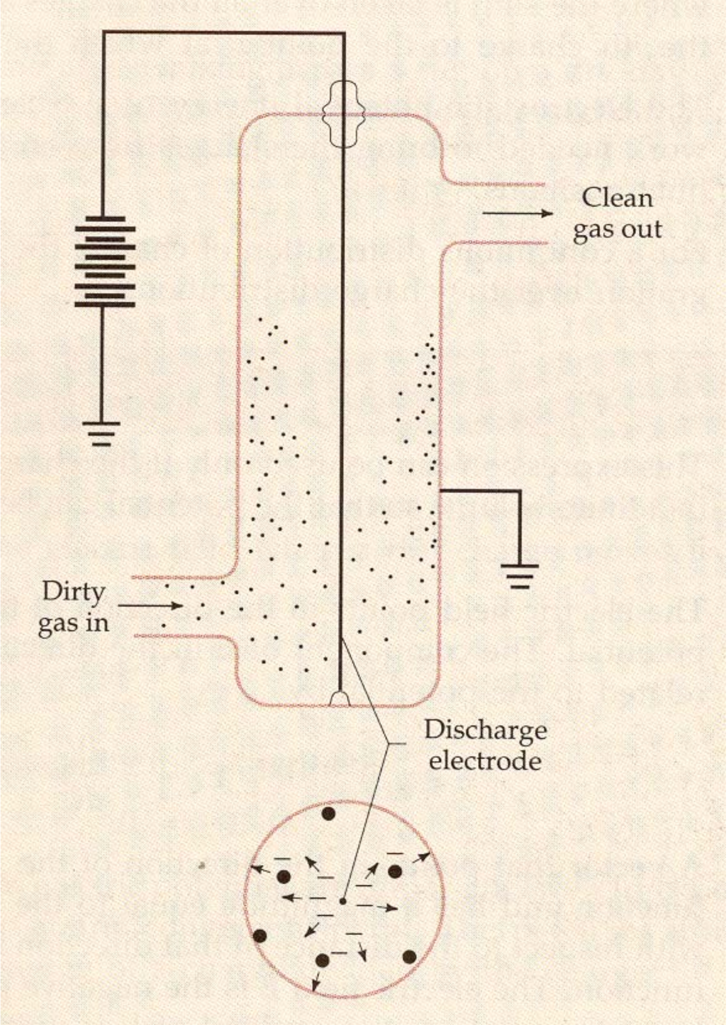
$$\vec{p} = \alpha \vec{E}$$

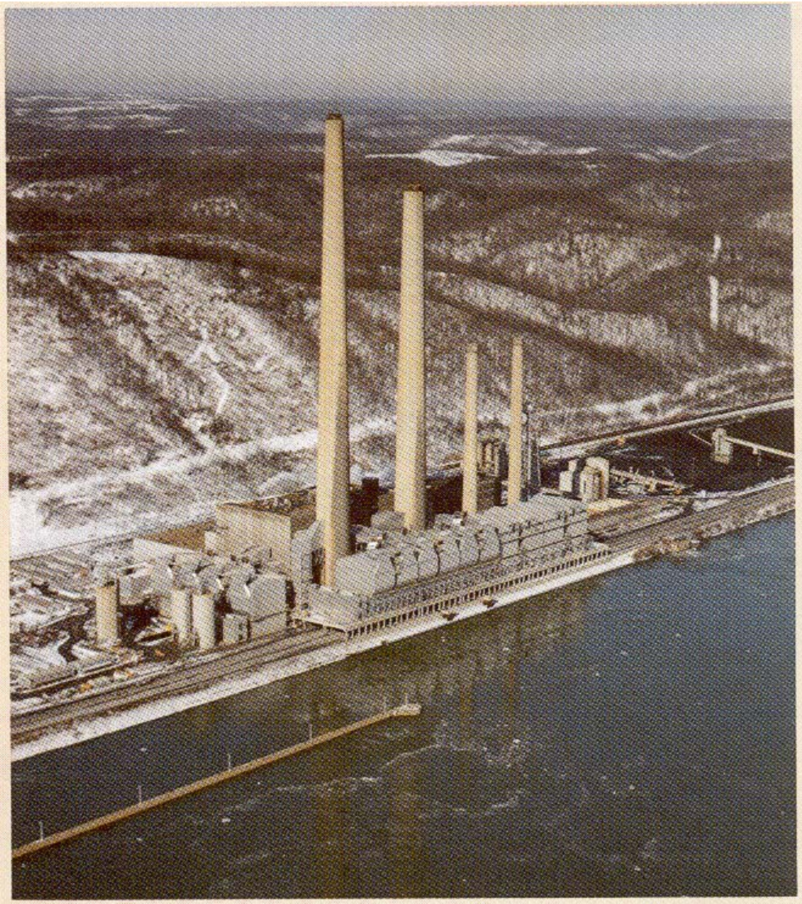
Electrostatic precipitator

Linear dielectric: $$\overrightarrow{\mathbf{p}} = \alpha \overrightarrow{\mathbf{E}}$$

Translational force: $$\overrightarrow{\mathbf{F}_{net}} = \overrightarrow{\boldsymbol{\nabla}} \left(\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}} \right) = \overrightarrow{\boldsymbol{\nabla}} \left(\alpha E^2 \right)$$

Electrostatic precipitator





Electrostatic precipitators are housed in the gray box-like structures at the base of these smokestacks.

Extract > 99% of ash and dust from gases at power, cement, and ore-processing plants

Electric field in dielectrics: Macroscopic

In the dielectric, the induced (bound) charge distribution causes an induced electric field E_{pol} that is opposite to the external electric field. Thus the net field in the dielectric is $E_{net} = E_{ext} - E_{pol}$. The induced field is zero outside of the dielectric.

The polarization, and thus the induced surface charge density σ_{pol}, of the dielectric in the parallel-plate capacitor, depends on E_{net} in the dielectric, not E_{ext}. For linear dielectrics the proportionality can be written as;

$$\sigma_{pol} = \chi_e \varepsilon_0 E_{net}$$

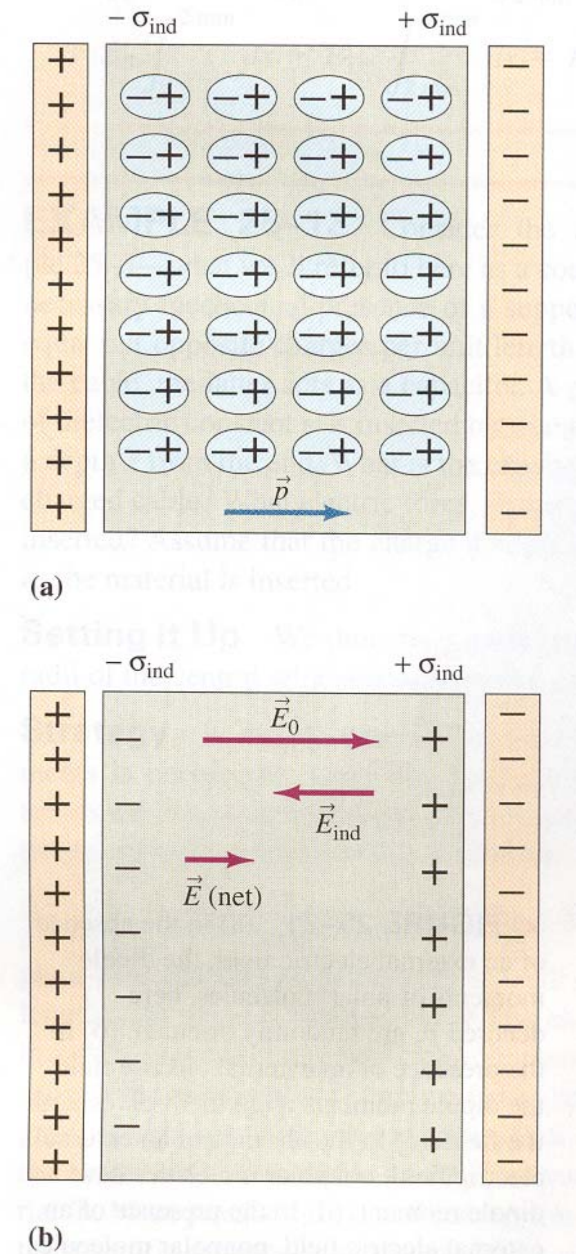
where the factor χ_e does not dependent on E_{net}.

Gauss's law can be used to relate the surface charge distributions to the electric fields. Taking an infinitessimal pillbox shaped Gaussian surface enclosing an element of the capacitor plate, gives that ;

$$E_{ext} = \frac{\sigma_{free}}{\varepsilon_0}$$

Similarly a infinitessimal pillbox-shaped Gaussian surface enclosing an element of the surface of the dielectric gives:

$$E_{pol} = \frac{\sigma_{pol}}{\varepsilon_0}$$



Electric field in dielectrics: Macroscopic

The net field in the dielectric is:

$$E_{net} = E_{ext} - E_{pol} = \frac{1}{\varepsilon_0}(\sigma_{free} - \sigma_{pol})$$

The problem with this dependence is that it is necessary to know E_{net} to compute σ_{pol} in order to calculate E_{net}. This problem can be resolved by rewriting this equation using the dependence of σ_{pol} on E_{net}:

$$E_{net}(1 + \chi_e) = \frac{\sigma_{free}}{\varepsilon_0} = E_{ext}$$

Define the *dielectric constant*

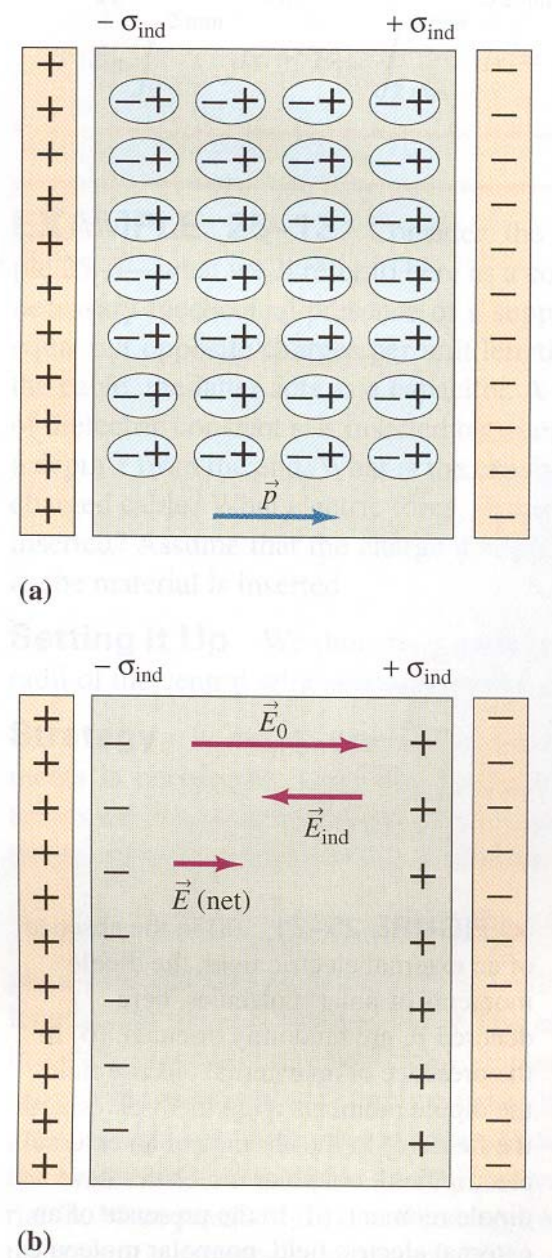
$$\kappa_e \equiv 1 + \chi_e$$

then we get:

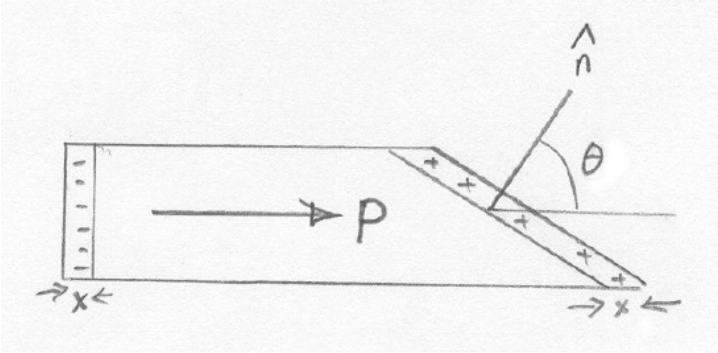
$$E_{net} = \frac{\sigma_{free}}{\kappa_e \varepsilon_0} = \frac{E_{ext}}{\kappa_e}$$

Thus E_{net} in the dielectric is a factor κ_e weaker than when there is no dielectric. Knowing E_{net} and the known applied charge distribution σ_{free} gives the induced charge density σ_{pol}:

$$\sigma_{pol} = \frac{\kappa_e - 1}{\kappa_e}\sigma_{free}$$



Polarization density P in dielectrics:



Knowledge of the polarization charge surface charge density σ_{pol} requires discussion of the *Polarization density* $\overrightarrow{\mathbf{P}}$, which is the dipole moment per unit volume, defined by

$$\overrightarrow{\mathbf{P}} \equiv N < \overrightarrow{\mathbf{p}} >$$

where there are N dipoles/unit volume each with statistical average individual dipole moment of $< \overrightarrow{\mathbf{p}} >$.

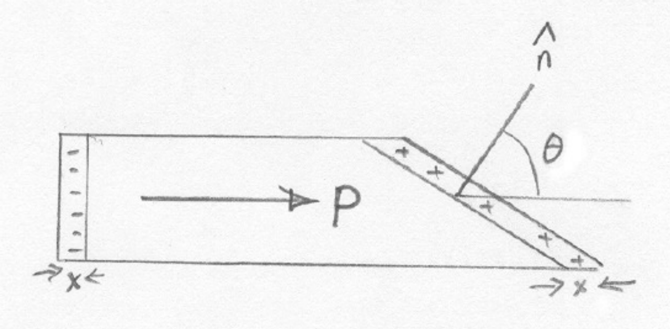
As illustrated in figure 5, polarization causes a net bound charge distribution σ_{pol} at the surfaces where the equal and opposite positive and negative charge distributions do not cancel. Note that if the polarization is due to a shift x of the dipole charges, q, where $\overrightarrow{\mathbf{p}} = q\overrightarrow{\mathbf{x}}$, then the net polarization charge for a surface area dS is

$$Nqx\cos\theta dS = N\overrightarrow{\mathbf{p}} \cdot \overrightarrow{d\mathbf{S}} = \overrightarrow{\mathbf{P}} \cdot \overrightarrow{d\mathbf{S}} = \sigma_{pol} dS.$$

That is, we have a direct relation between the polarization surface charge density σ_{pol}, and the Polarization density $\overrightarrow{\mathbf{P}}$. That is;

$$\sigma_{pol} = \overrightarrow{\mathbf{P}} \cdot \widehat{\mathbf{n}}$$

Polarization density in dielectrics:



$$\sigma_{pol} = \overrightarrow{\mathbf{P}} \cdot \widehat{\mathbf{n}}$$

The polarization, and thus the induced surface charge density σ_{pol}, of the dielectric in the parallel-plate capacitor, depends on E_{net} in the dielectric, not E_{ext}. For linear dielectrics

$$\begin{aligned} \overrightarrow{\mathbf{P}} &\equiv N < \overrightarrow{\mathbf{p}} > \\ &= N\alpha \overrightarrow{\mathbf{E}} \end{aligned}$$

The constant of proportionality can be written as;

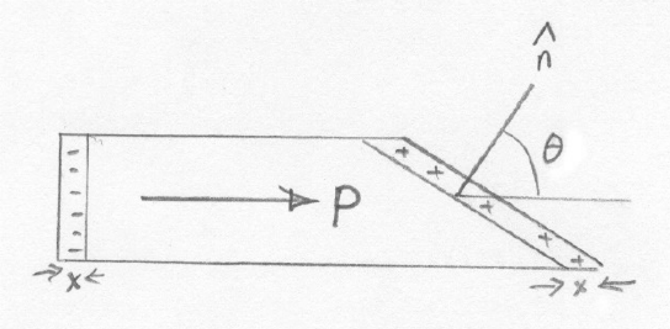
$$\overrightarrow{\mathbf{P}} = N\alpha \overrightarrow{\mathbf{E}} = \chi_e \varepsilon_0 \overrightarrow{\mathbf{E}}$$

where inside the dielectric $\overrightarrow{\mathbf{E}} = \overrightarrow{\mathbf{E}}_{net}$. For the parallel-plate capacitor shown above the polarization charge density simplifies to

$$\begin{aligned} \sigma_{pol} &= \overrightarrow{\mathbf{P}} \cdot \widehat{\mathbf{n}} \\ &= \chi_e \varepsilon_0 \overrightarrow{\mathbf{E}}_{net} \cdot \hat{\mathbf{n}} \end{aligned}$$

Polarization density in dielectrics:

$$E_{net} = E_{ext} - E_{pol} = \frac{1}{\varepsilon_0}(\sigma_{free} - \sigma_{pol})$$



The problem with this dependence is that it is necessary to know E_{net} to compute σ_{pol} in order to calculate E_{net}. This problem can be resolved by rewriting this equation using the dependence of σ_{pol} on E_{net}:

$$E_{net} + \frac{\sigma_{pol}}{\varepsilon_0} = \frac{\sigma_{free}}{\varepsilon_0} = E_{ext}$$

$$E_{net}(1 + \chi_e) = \frac{\sigma_{free}}{\varepsilon_0} = E_{ext}$$

Define the *dielectric constant*

$$\kappa_e \equiv 1 + \chi_e$$

then we get:

$$E_{net} = \frac{\sigma_{free}}{\kappa_e \varepsilon_0} = \frac{E_{ext}}{\kappa_e}$$

The dielectric constant κ_e

- Net field in a dielectric $\mathbf{E}_{net} = \mathbf{E}_{Ext}/\kappa_e$

- Most dielectrics linear up to dielectric strength

Dielectric constants (at 20° C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Rubber, neoprene	6.7	12×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

Volume polarization density

From charge conservation

$$\oint \sigma_{pol} dS + \int_{\substack{Enclosed\\volume}} \rho_{pol} d\tau = 0$$

But

$$\sigma_{pol} = \overrightarrow{\mathbf{P}} \cdot \widehat{\mathbf{n}}.$$

Thus

$$\oint \overrightarrow{\mathbf{P}} \cdot \overrightarrow{d\mathbf{S}} = -\int_{\substack{Enclosed\\volume}} \rho_{pol} d\tau$$

This can be rewritten using the Divergence theorem to give

$$\int_{\substack{Enclosed\\volume}} \overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{P}} d\tau = -\int_{\substack{Enclosed\\volume}} \rho_{pol} d\tau$$

That is:

$$\overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{P}} = -\rho_{pol}$$

It is stressed that ρ_{pol} is a real volume charge distribution due to polarization of bound charges in the dielectric.

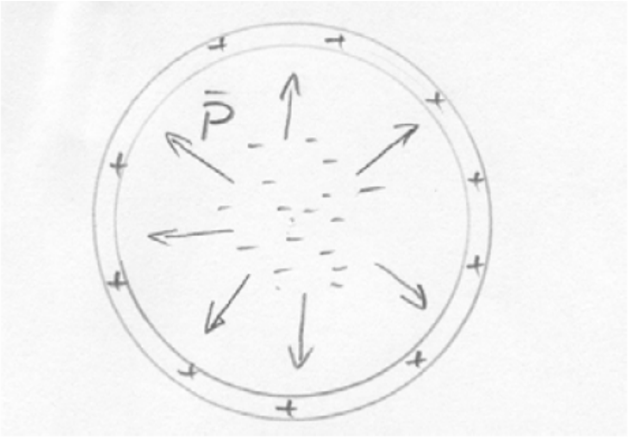


Figure 7 Surface and volume bound charge distributions ˈue to radial polarization of a sphere of dielectric.

Electric Displacement Field

Maxwell's equations that pertain to electrostatics are

$$\overrightarrow{\nabla} \cdot \overrightarrow{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$$

and

$$\overrightarrow{\nabla} \times \overrightarrow{\mathbf{E}} = 0$$

The charge distribution ρ includes *all* charge distributions, both bound and free. That is

$$\rho = \rho_{free} + \rho_{pol}$$

which can be written as

$$\rho = \rho_{free} - \overrightarrow{\nabla} \cdot \overrightarrow{\mathbf{P}}$$

Electric Displacement Field

$$\rho = \rho_{free} - \overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{P}}$$

Gauss's law is

$$\overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{E}} = \frac{\rho}{\varepsilon_0} = \frac{\rho_{free} - \overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{P}}}{\varepsilon_0}$$

This can be rewritten as

$$\varepsilon_0 \overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{E}} + \overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{P}} = \overrightarrow{\boldsymbol{\nabla}} \cdot \left(\varepsilon_0 \overrightarrow{\mathbf{E}} + \overrightarrow{\mathbf{P}} \right) = \rho_{free}$$

A new vector field $\overrightarrow{\mathbf{D}}$ called the *electric displacement* is defined to be

$$\overrightarrow{\mathbf{D}} \equiv \varepsilon_0 \overrightarrow{\mathbf{E}} + \overrightarrow{\mathbf{P}}$$

Using this definition gives

$$\overrightarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\mathbf{D}} = \rho_{free}$$

Electric Displacement Field

The usefulness of introducing this new field $\overrightarrow{\mathbf{D}}$ is because the polarization of linear dielectrics is proportional to the effective field in the dielectric. That is, it can be written as

$$\overrightarrow{\mathbf{P}} = \chi_e \varepsilon_0 \overrightarrow{\mathbf{E}}$$

where $\chi_e \varepsilon_0 = N\alpha$. Thus this gives

$$\overrightarrow{\mathbf{D}} \equiv \varepsilon_0 \overrightarrow{\mathbf{E}} + \overrightarrow{\mathbf{P}} = \varepsilon_0 \left(1 + \chi_e\right) \overrightarrow{\mathbf{E}}$$

Using the definition of the dielectric constant

$$\kappa_e = 1 + \chi_e$$

gives

$$\overrightarrow{\mathbf{D}} = \varepsilon_0 \kappa_e \overrightarrow{\mathbf{E}}$$

It is fortunate that many dielectrics are linear, that is κ_e is independent of $\overrightarrow{\mathbf{E}}$. The polarization of the dielectric then can be buried in the dielectric constant κ_e.

Electric Displacement Field

The problem of handling the polarization charges for dielectric materials in Maxwell's Equations is greatly simplified using the concept of electric displacement field

$$\overrightarrow{\mathbf{D}} \equiv \varepsilon_0 \overrightarrow{\mathbf{E}} + \overrightarrow{\mathbf{P}} = \varepsilon_0 \left(1 + \chi_e\right) \overrightarrow{\mathbf{E}}$$

where χ_e is the electric susceptibility. Fortunately many dielectrics are linear and thus χ_e is a constant. Using the definition of the dielectric constant

$$\kappa_e = 1 + \chi_e$$

gives

$$\overrightarrow{\mathbf{D}} = \varepsilon_0 \kappa_e \overrightarrow{\mathbf{E}}$$

The infulence of the dielectric can be absorbed into the displacement field $\overrightarrow{\mathbf{D}}$ allowing a modified form of maxwell's Equations to be written. Namely

$$\overrightarrow{\nabla} \cdot \overrightarrow{\mathbf{D}} = \rho_{free}$$

$$\overrightarrow{\nabla} \times \overrightarrow{\mathbf{E}} = 0$$

Summary; 1

This lecture has focussed on the influence of matter on electric fields. The electric dipole $\overrightarrow{\mathbf{p}} = q\,\overrightarrow{\mathbf{d}}$, plays a pivotal role in this discussion. The forces on an electric dipole in an electric field are:

$$Torque \qquad \overrightarrow{\tau} = \overrightarrow{\mathbf{p}} \times \overrightarrow{\mathbf{E}}$$

$$Translation \qquad \overrightarrow{\mathbf{F}} = \overrightarrow{\mathbf{p}}.\overrightarrow{\nabla}\,\overrightarrow{\mathbf{E}}$$

For linear dielectrics $\overrightarrow{\mathbf{p}} = \alpha\,\overrightarrow{\mathbf{E}}$.

The electric field inside a dielectric, $\overrightarrow{\mathbf{E}_{net}}$ for a dielectric constant κ_e, is related to the applied field $\overrightarrow{\mathbf{E}_{free}}$ by

$$\overrightarrow{\mathbf{E}_{net}} = \frac{\overrightarrow{\mathbf{E}_{free}}}{\kappa_e}$$

Where the induced charge density σ_{pol} is related to the induced electric field $\overrightarrow{\mathbf{E}_{ind}}$ by:

$$\overrightarrow{\mathbf{E}_{pol}} = \frac{\sigma_{pol}}{\varepsilon_0}$$

Summary; 2

A modified form of Maxwell's equations for electrostatics can be written that is especially useful for solving problems of electric fields in linear dielectrics.

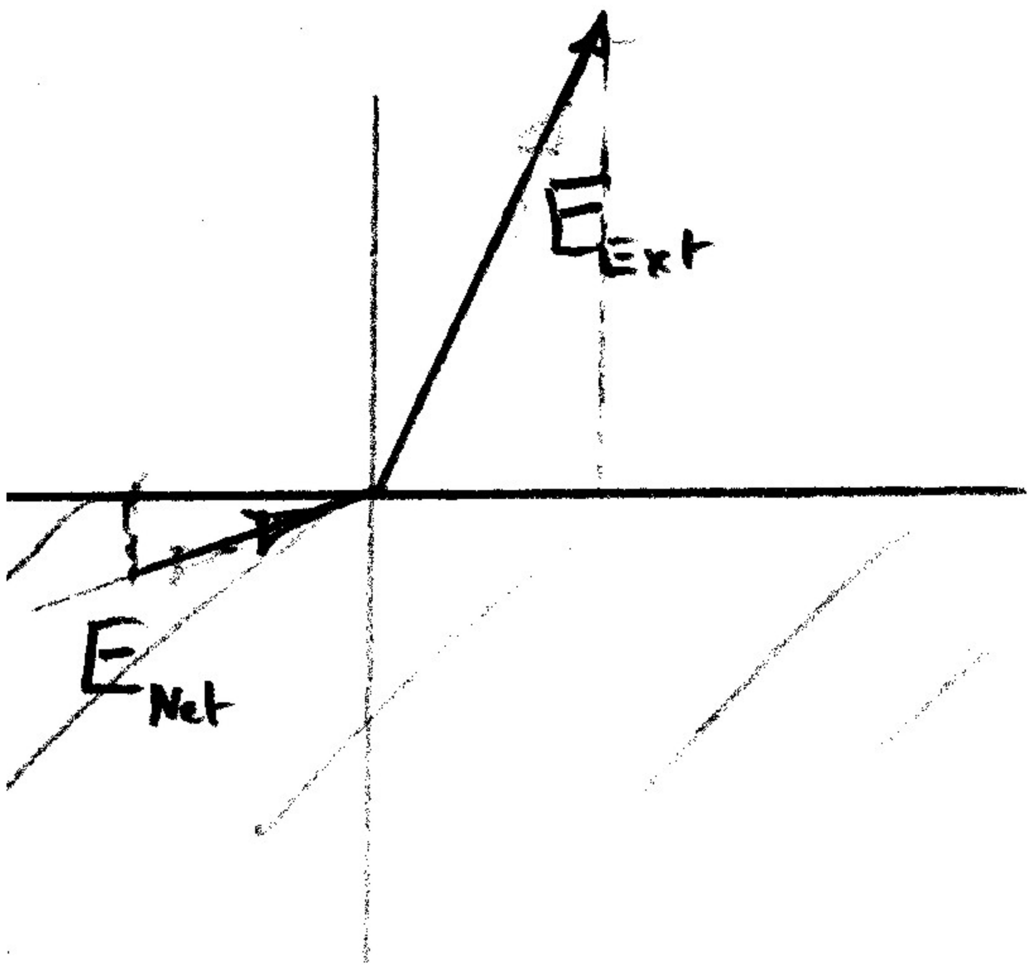
$$\overrightarrow{\nabla} \cdot \overrightarrow{\mathbf{D}} = \rho_{free}$$

$$\overrightarrow{\nabla} \times \overrightarrow{\mathbf{E}} = 0$$

where for linear dielectrics

$$\overrightarrow{\mathbf{D}} = \varepsilon_0 \kappa_e \overrightarrow{\mathbf{E}} = \varepsilon \overrightarrow{\mathbf{E}}$$

Refraction of the electric field at boundaries with dielectrics



Capacitance with dielectrics

Consider a parallel-plate capacitor, spacing t, partially filled with a dielectric slab of thickness d.

The surface charge distribution σ_{free} on the surface of the capacitor plate is related to $E_{\perp}^{air}$ by Gauss's law, that is

$$E_{\perp}^{air} = \frac{\sigma_{free}}{\varepsilon_o}$$

Inside the dielectric this field is reduced by the factor κ_e, that is

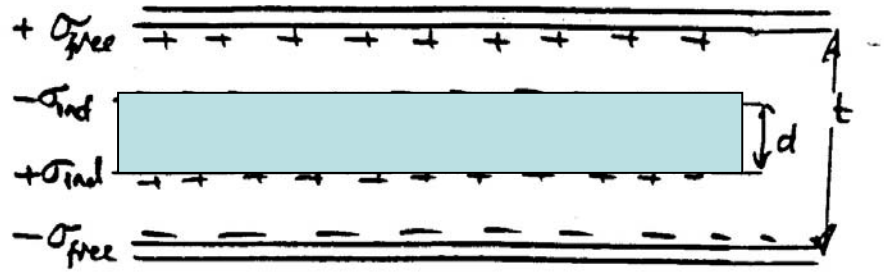
$$E_{\perp}^{dielectric} = \frac{\sigma_{free}}{\kappa_e \varepsilon_o}$$

Thus the potential difference between the capacitor plates is given by the line integral:

$$\Delta V = -\int \vec{\mathbf{E}} \cdot \vec{d\mathbf{l}} = (t-d)E_{\perp}^{air} + d.E_{\perp}^{dielectric}$$

$$\Delta V = (t-d)E_{\perp}^{air} + \frac{d}{\kappa_e} E_{\perp}^{air}$$

$$\Delta V = \frac{\sigma_{free}}{\varepsilon_0}\left[t - (\frac{\kappa_e - 1}{\kappa_e})d\right]$$



Since $\sigma_{free} = \frac{Q}{A}$ we get that the net capacitance of the partially-filled parallel-plate capacitor is:

$$C = \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{\left[t - (\frac{\kappa_e - 1}{\kappa_e})d\right]}$$

A useful special case is when the space between the plates is filled completely with dielectric, then $d = t$ and:

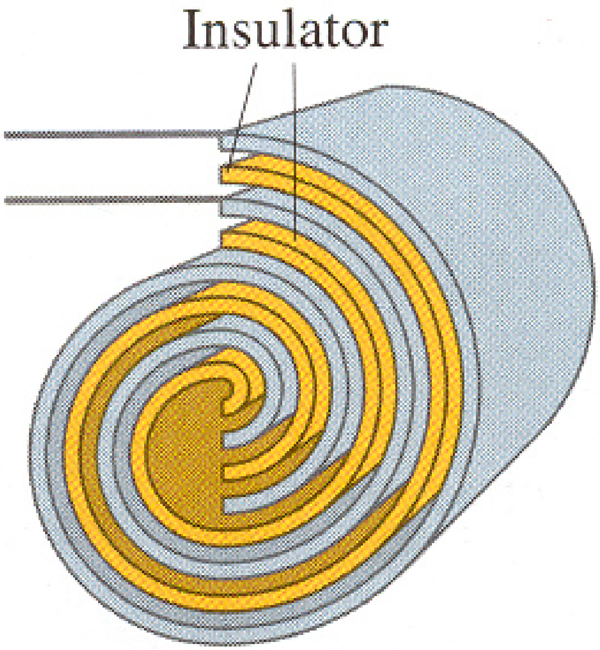
$$C = \frac{A\varepsilon_0 \kappa_e}{d}$$

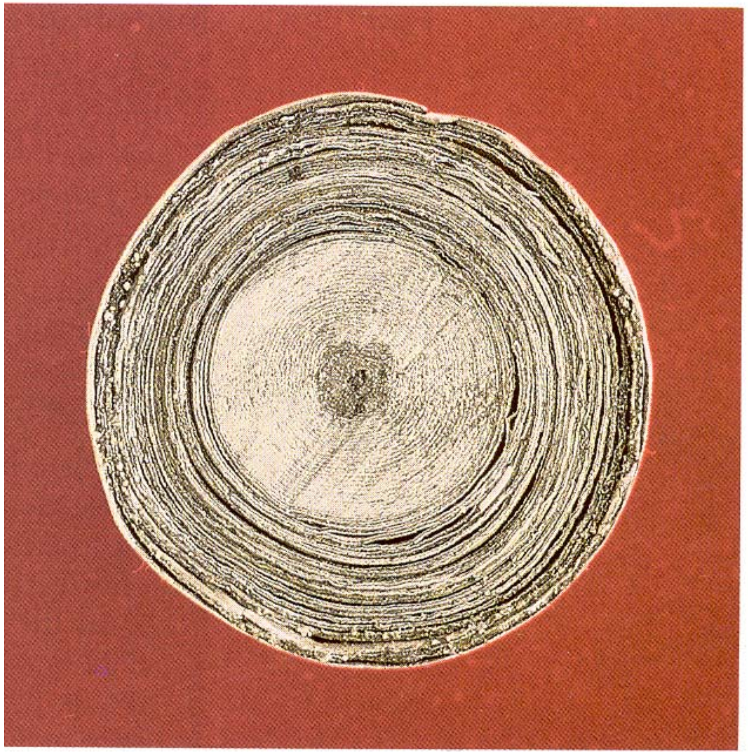
The capacitance of a capacitor filled with dielectric of constant κ_e, is a factor κ_e times larger than when the region between the capacitor plates is a vacuum. That is;

$$C_{dielectric} = \kappa_e C_{vacuum}$$

Capacitance with dielectrics

$$C_{dielectric} = \kappa_e C_{vacuum}$$





Electric energy storage

- Showed last lecture that the energy stored in a capacitor is given by

$$W = U = \frac{1}{2}\frac{Q_o^2}{C} = \frac{1}{2}CV_o^2 = \frac{1}{2}Q_oV_o$$

where for a dielectric $C_{dielectric} = \kappa_e C_{vacuum}$

- Typically it is more useful to express energy in terms of magnitude of the electric field **E.**

- It can be shown that the energy density in the **E** field is $\eta = \frac{1}{2}\kappa_e\varepsilon_0 E^2 \; joules/m^3$

- Thus the total stored energy is given by integrating the energy density over all space

$$U = \int_{\substack{all \\ space}} \frac{1}{2}\kappa_e\varepsilon_0 E^2 d\tau$$

Note that the energy stored is proportional to κ_e

Electric energy storage

- Showed last lecture that the energy stored in a capacitor is given by

$$W = U = \frac{1}{2}\frac{Q_o^2}{C} = \frac{1}{2}CV_o^2 = \frac{1}{2}Q_oV_o$$

where for a dielectric $C_{dielectric} = \kappa_e C_{vacuum}$

- Typically it is more useful to express energy in terms of magnitude of the electric field **E.**

- It can be shown that the energy density in the **E** field is $\eta = \frac{1}{2}\kappa_e\varepsilon_0 E^2 \ joules/m^3$

- Thus the total stored energy is given by integrating the energy density over all space

$$U = \int_{\substack{all \\ space}} \frac{1}{2}\kappa_e\varepsilon_0 E^2 d\tau$$

Note that the energy stored is proportional to κ_e

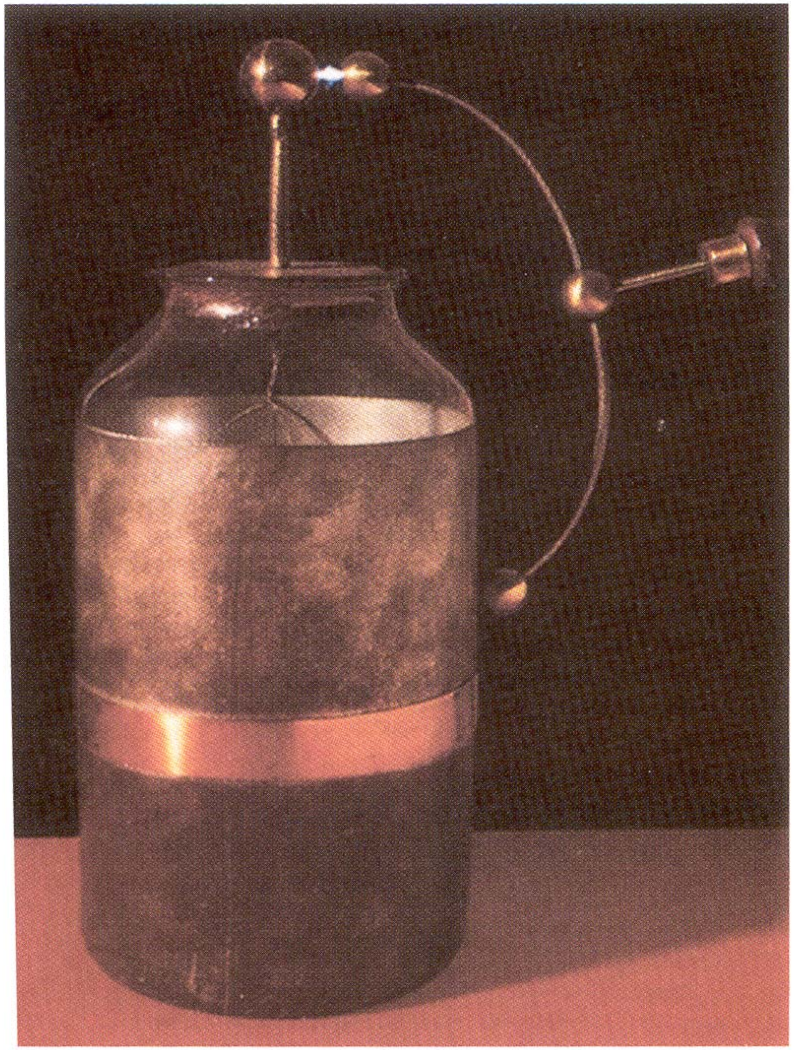
Electric energy storage

Leyden jar:

• Dielectric between concentric cylindrical capacitor plates

• Store a charge Q on the capacitor

• Demonstrate the stored energy



Concept Quiz 2

Energy storage

Attraction of dielectric between capacitor plates

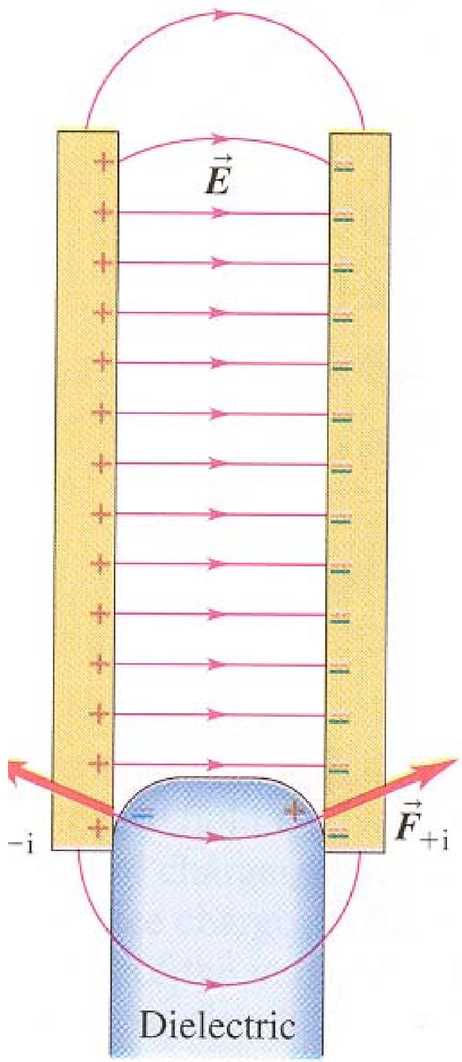
If Q on plates is fixed then the stored energy in the capacitor is

$$U = \tfrac{1}{2}Q^2/C$$

If dielectric is inside the plates then

$$C_{dielec} = \kappa_e \, C_{vac}$$

Therefore the stored energy is lower if the capacitor is filled with dielectric. The increase in energy when the dielectric is removed equals the work done pulling the dielectric slab out of the capacitor, that is, it equals the force them the distance moved.



Summary

This lecture has focussed on the influence of matter on electric fields. The electric dipole $\overrightarrow{\mathbf{p}} = q\overrightarrow{\mathbf{d}}$ plays a pivotal role in this discussion. The torque on an electric dipole in an electric field is:

$$\overrightarrow{\boldsymbol{\tau}} = \overrightarrow{\mathbf{p}} \times \overrightarrow{\mathbf{E}}$$

For linear dielectrics

$$\overrightarrow{\mathbf{p}} = \alpha\overrightarrow{\mathbf{E}}$$

The electric field inside a dielectric, $\overrightarrow{\mathbf{E}_{net}}$ for a dielectric constant κ_e, is related to the applied field $\overrightarrow{\mathbf{E}_{free}}$ by

$$\overrightarrow{\mathbf{E}_{net}} = \frac{\overrightarrow{\mathbf{E}_{free}}}{\kappa_e}$$

Where the induced charge density σ_{pol} is related to the induced electric field $\overrightarrow{\mathbf{E}_{pol}}$ by:

$$\overrightarrow{\mathbf{E}_{pol}} = \frac{\sigma_{pol}}{\varepsilon_0}$$

The electric field at the boundary of a dielectric is refracted since the normal component of the electric field at the surface is reduced by the factor κ_e in the dielectric.

The capacitance of a capacitor filled with a dielectric is increased by a factor κ_e relative to the same capacitor without the dielectric.

$$C_{dielectric} = \kappa_e C_{vacuum}$$

The more general relation for the energy stored in an electric field, both in and out of dielectrics, is given by:

$$U = \int \frac{1}{2}\kappa_e\varepsilon_o\mathbf{E}^2 d\tau$$