Classical Mechanics - Module 11

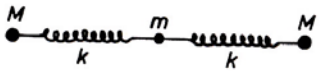
1. Consider the longitudinal motion of the system of 3 masses and two springs shown, where $M > m$

   (a) What are the normal-mode frequencies of ths system?

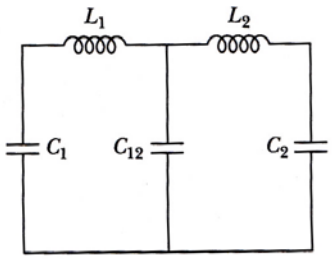
   (b) What are the eigen-mode vectors of the system?

   (c) If the left mass receives an impulse of P_0 at $t = 0$, find the motion of the left-hand mass as a function of time.

   (d) If alternatively the middle mass is driven harmonically at a frequency $\omega_0 = 2\sqrt{\frac{k}{m}}$ will it move in or out of phase with the driving motion?



2. Discuss the normal modes of the electrical circuit shown in the figure.



3. Consider a thin homogeneous plate of mass M that lies in the $x_1 - x_2$ plane with its centre at the origin. Assume that the lnength of the plate is $2A$ in the x_2 direction and $2B$ in the x_1 direction. The plate is suspended from a fixed support by four springs of equal force constant κ at the four corners of the plate. The plate is free to oscillate but with the constraint that the centre must remain on the x_3 axis. Thus the system has three degrees of freedom.

   (a) Vertical motion with the centre of the plate moving along the x_3 axis,

   (b) A tipping motion lengthwise with the x_1 axis serving as a axis of rotation, (Choose θ to describe this motion.)

   (c) A tipping sideways motion with the x_2 axis serving as an axis of rotation. (Choose ϕ to desribe this motion. Assume only small oscillations and show that the secular root has a double root and hence the system is degenerate.

   (d) Show that the degeneracy can be removed by adding to the plane a thin bar of mass m and length $2A$ situated along the x_2 axis. Find the new eigen frequencies.

4. Consider the Kuramoto model for collective synchronization of an ensemble of closely similar coupled oscillators that have a narrow frequency distribution $g(\omega)$.

   (a) Explain to your workshop partner why there is a critical coupling strength, and why the system partitions into two groups.

   (b) Discuss the frequency distribution of these two groups of coupled oscillators and how they will depend on the coupling strength.