1. Consider the longitudinal motion of the system of 3 masses and two springs shown, where $M > m$

(a) What are the normal-mode frequencies of this system?
(b) What are the eigen-mode vectors of the system?
(c) If the left mass receives an impulse of $P_0$ at $t = 0$, find the motion of the left-hand mass as a function of time.
(d) If alternatively the middle mass is driven harmonically at a frequency $\omega_0 = 2\sqrt{\frac{m}{k}}$, will it move in or out of phase with the driving motion?

![Diagram of three masses and two springs](image)

2. Discuss the normal modes of the electrical circuit shown in the figure.

![Electrical circuit diagram](image)

3. Consider a thin homogeneous plate of mass $M$ that lies in the $x_1 - x_2$ plane with its centre at the origin. Assume that the length of the plate is $2A$ in the $x_2$ direction and $2B$ in the $x_1$ direction. The plate is suspended from a fixed support by four springs of equal force constant $k$ at the four corners of the plate. The plate is free to oscillate but with the constraint that the centre must remain on the $x_3$ axis. Thus the system has three degrees of freedom.

(a) Vertical motion with the centre of the plate moving along the $x_3$ axis,
(b) A tipping motion lengthwise with the $x_1$ axis serving as an axis of rotation. (Choose $\theta$ to describe this motion.)
(c) A tipping sideways motion with the $x_2$ axis serving as an axis of rotation. (Choose $\phi$ to describe this motion. Assume only small oscillations and show that the secular root has a double root and hence the system is degenerate.
(d) Show that the degeneracy can be removed by adding to the plane a thin bar of mass $m$ and length $2A$ situated along the $x_2$ axis. Find the new eigen frequencies.

4. Consider the Kuramoto model for collective synchronization of an ensemble of closely similar coupled oscillators that have a narrow frequency distribution $g(\omega)$.

(a) Explain to your workshop partner why there is a critical coupling strength, and why the system partitions into two groups.
(b) Discuss the frequency distribution of these two groups of coupled oscillators and how they will depend on the coupling strength.