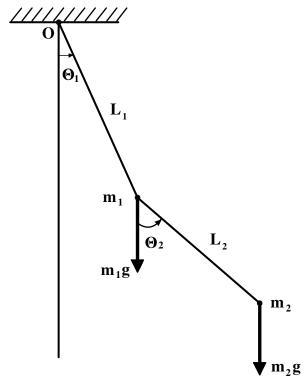
Classical Mechanics - Module 5

1. Consider the double pendulum comprising masses m_1 and m_2 connected by inextensible strings as shown in the figure. Assume that the motion of the pendulum takes place in a vertical plane.

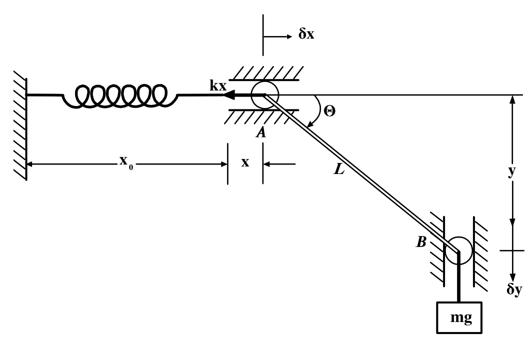
   (a) Are there any equations of constraint? If so, what are they?

   (b) Find Lagrange's equations for this system.



2. Consider the system shown in the figure which consists of a mass m suspended via a constrained massless link of length L where the point A is acted upon by a spring of spring constant k. The spring is unstretched when the massless link is horizontal. Assume that the holonomic constraints at A and B are frictionless.

   (a) Derive the equations of motion for the system using the method of Lagrange multipliers.



3. Consider a particle of mass m moving in a plane and subject to an inverse square attractive force.

   (a) Obtain the equations of motion.

   (b) Is the angular momentum about the origin conserved?

   (c) Obtain expressions for the generalized forces.

4. Consider a Lagrangian function of the form $L(q_i, \dot{q}_i, \ddot{q}_i, t)$. Here the Lagrangian contains a time derivative of the generalized coordinates that is higher than the first. When working with such Lagrangians, the term "generalized mechanics" is used.

   (a) Consider a system with one degree of freedom. By applying the methods of the calculus of variations, and assuming that Hamilton's principle holds with respect to variations which keep both q and $\dot{q}$ fixed at the end points, show that the corresponding Lagrange equation is

   $$\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\partial L}{\partial q} = 0.$$

   Such equations of motion have interesting applications in chaos theory.

   (b) Apply this result to the Lagrangian

   $$L = -\frac{m}{2} q\ddot{q} - \frac{k}{2} q^2.$$

   Do you recognize the equations of motion?

5. A uniform solid cylinder of radius R and mass M rests on a horizontal plane and an identical cylinder rests on it touching along the top of the first cylinder with the axes of both cylinders parallel. The upper cylinder is given an infinitessimal displacement so that both cylinders roll without slipping in the directions shown by the arrows.

   (a) Find Lagrangian for this system

   (b) What are the constants of motion?

   (c) Show that as long as the cylinders remain in contact then

   $$\dot{\theta}^2 = \frac{12g\,(1 - \cos\theta)}{R\,(17 + 4\cos\theta - 4\cos^2\theta)}$$

   (d) Discuss how you would solve the determine the instantaneous frictional force between the rolling cylinders.