P235 - PROBLEM SET 1

To be handed in by **1800hrs on Friday, 11 September 2015**.

[1] For what values of a are the vectors $\bar{\mathbf{A}} = 2a\hat{\imath} - 2\hat{\jmath} + a\hat{\mathbf{k}}$ and $\bar{\mathbf{B}} = a\hat{\imath} + 2a\hat{\jmath} + 2\hat{\mathbf{k}}$ perpendicular?

[2] Find the transformation matrix that rotates the axis x_3 of a rectangular coordinate system 45^o toward x_1 around the x_2 axis.

[3] Show that the triple scalar product $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ can be written as

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

Show also that the product is unaffected by interchange of the scalar and vector product operations or by change in the order of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ as long as they are in cyclic order, that is

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B}$$

Therefore we may use the notation $\mathbf{ABC}$ to denote the triple scalar product. Finally give a geometric interpretation of $\mathbf{ABC}$ by computing the volume of the parallelepiped defined by the three vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}$.

[4] Consider a solid hemisphere of radius a. Compute the coordinates of the centre of mass relative to the centre of the spherical surface used to define the hemisphere.

[5] A projectile is fired with an initial velocity v_o such that it passes through two points both a distance h above the horizontal. Show that if the gun is adjusted for a maximum range the separation of the points is

$$d = \frac{v_0}{g}\sqrt{v_0^2 - 4gh}$$

[6] A 2000kg Ford was travelling south on Mt Hope Avenue when it collided with your 1000kg sports car travelling west on Elmwood Avenue. The two badly-damaged cars became entangled in the collision and leave a skid mark that is 20 meters long in a direction 14° to the west of the original direction of travel of the Excursion. The wealthy Excursion driver hires a high-powered lawyer who accuses you of speeding through the intersection. Use your P235 knowledge, plus the police officer's report of the recoil direction, the skid length, and knowledge that the coefficient of sliding friction between the tires and road is $\mu = 0.65$, to deduce the original velocities of both cars. Were either of the cars exceeding the 30mph speed limit?