P235 - PROBLEM SET 2

To be handed in by **1700 hr on Friday, 18 September 2015**.

[1] a) Consider a single-stage rocket travelling in a straight line subject to an external force F^{ext} acting along the same line where v_{ex} is the exhaust velocity of the ejected fuel relative to the rocket. Show that the equation of motion is

$$m\dot{v} = -\dot{m}v_{ex} + F^{ext}$$

b) Specialize to the case of a rocket taking off vertically from rest in a uniform gravitational field g. Assume that the rocket ejects mass at a constant rate of $\dot{m} = -k$ where k is a positive constant. Solve the equation of motion to derive the dependence of velocity on time.

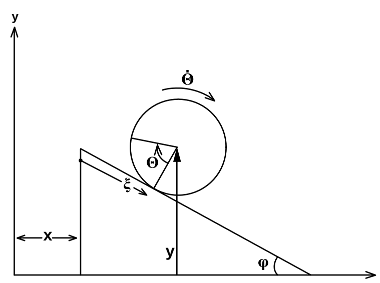
c) The first couple of minutes of the launch of thr Space Shuttle can be described roughly by; initial mass $= 2 \times 10^6$ kg, mass after 2 minutes $= 1 \times 10^6$ kg, exhaust speed $v_{ex} = 3000 m/s$, and initial velocity is zero. Estimate the velocity of the Space Shuttle after two minutes of flight.

d) Describe what would happen to a rocket where $\dot{m}v_{ex} < mg$.

2] Consider a particle of mass m whose motion starts from rest in a common gravitational field. If a resisting force proportional to the square of the velocity (i.e. kmv^2) is encountered, show that the distance s the particle falls in accelerating from v_0 to v_1 is given by

$$s(v_0 \to v_1) = \frac{1}{2k} \ln \left[\frac{g - kv_0^2}{g - kv_1^2} \right]$$

[3] Consider a solid cylinder of mass m and radius r sliding without rolling down the smooth inclined face of a wedge of mass M that is free to slide without friction on a horizontal plane floor. Use the coordinates shown in the figure.



a) How far has the wedge moved by the time the cylinder has descended from rest a vertical distance h ?

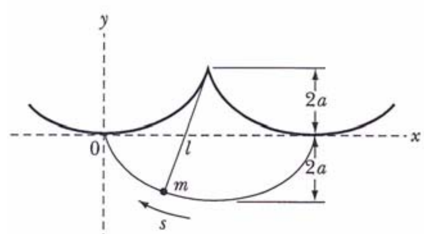
b) Now suppose that the cylinder is free to roll down the wedge without slipping. How far does the wedge move in this case if the cylinder rolls down a vertical distance h ?

c) In which case does the cylinder reach the bottom faster? How does this depend on the radius of the cylinder?

[4] A pendulum is suspended from the cusp of a cycloid cut in a rigid support as shown. The path described by the pendulum bob is cycloidal and is given by the relations

$$\begin{aligned} x &= a(\phi - \sin\phi) \\ y &= a(\cos\phi - 1) \end{aligned}$$

where the length of the pendulum is $l = 4a$, and where ϕ is the angle of rotation of the circle generating the cycloid. Show that the oscillations are exactly isochronous with frequency $\omega_0 = \sqrt{\frac{g}{l}}$ independent of the amplitude of oscillation.



[5] Two masses m_1 and m_2 slide freely on a horizontal frictionless rail and are connected by a spring whose force constant is k. Find the frequency of oscillatory motion for this system.

[6] A particle of mass m moves under the influence of a resistive force proportional to velocity and a potential U, that is l.

$$F(x, \dot{x}) = -b\dot{x} - \frac{\partial U}{\partial x}$$

where $b > 0$ and $U(x) = (x^2 - a^2)^2$

a) Find the points of stable and unstable equlibrium.

b) Find the solution of the equations of motion for small oscillations around the stable equilibrium points

c) Show that as $t \to \infty$ the particle approaches one of the stable equlibrium points for most choices of initial conditions. What are the exceptions? (Hint: You can prove this without finding the solutions explicitly.)