# P235 - PROBLEM SET 6

To be handed in by **1800 hr on Friday, 16 October 2015**.

1. A particle of mass $m$ in a gravitational field slides on the inside of a smooth parabola of revolution whose axis is vertical. Using the distance from the axis $r$, and the azimuthal angle $\varphi$ as generalized coordinates, find the following.

   a) The Lagrangian of the system.

   b) The generalized momenta and the corresponding Hamiltonian

   c) The equation of motion for the coordinate $r$.

   d) If $\frac{d\varphi}{dt} = 0$, show that the particle can execute small oscillations about the lowest point of the paraboloid and find the frequency of these oscilllations.

2. Consider a particle of mass $m$ which is constrained to move on the surface of a sphere of radius $R$. There are no external forces of any kind acting on the particle.

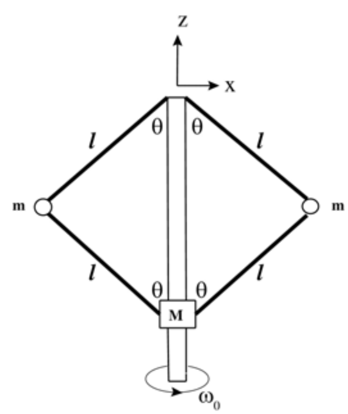
   a) What is the number of generalized coordinates necessary to describe the problem?

   b) Choose a set of generalized coordinates and write the Lagrangian of the system.

   c) What is the Hamiltonian of the system? Is it conserved?

   d) Prove that the motion of the particle is along a great circle of the sphere.

3. A flyball governor comprises two masses $m$ connected by 4 hinged arms of length $l$ to a vertical shaft and to a mass $M$ which can slide up or downthe shaft without frictionin a uniform vertical gravitational field as shown in the figure. The assembly is constrained to rotate around the axis of the vertical shaft with same angular velocity as that of the vertical shaft. Neglect the mass of the arms, air friction, and assume that the mass $M$ has a negligible moment of inertia. Assume that the whole system is constrained to rotate with a constant angular velocity $\omega_0$.




a) Choose suitable coordinates and use the Lagrangian to derive equations of motion of the system around the equilibrium position.

b) Determine the height $z$ of the mass $M$ above its lowest position as a function of $\omega_0$.

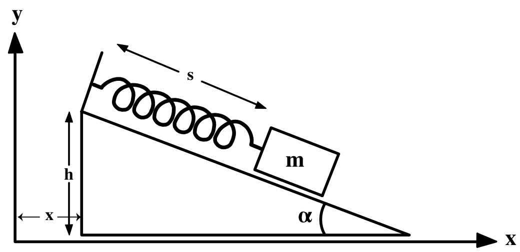
c) Find the frequency of small oscillations about this steady motion.

d) Derive a Routhian that provides the Hamiltonian for the $\theta$ variable.

e) Is the total energy of the flyball governor in the rotating frame of reference constant in time?

f) Suppose that the shaft and assembly are not constrained to rotate at a constant angular velocity $\omega_0$, that is, it is allowed to rotate freely at angular velocity $\dot{\varphi}$. What is the difference in the overall motion?

**4)** A block of mass $m$ is attached to a wedge of mass $M$ by a spring with spring constant $k$. The inclined frictionless surface of the wedge makes an angle $\alpha$ to the horizontal. The wedge is free to slide on a horizontal frictionless surface as shown in the figure.




a) Given that the relaxed length of the spring is $d$, find the values $s_0$ when both book and wedge are stationary.

b) Find the Lagrangian for the system as a function of the $x$ coordinate of the wedge and the length of spring $s$. Write down the equations of motion.

c) What is the natural frequency of vibration?

**5)** A rigid straight, frictionless, massless, rod rotates about the $z$ axis at an angular velocity $\dot{\theta}$. A mass $m$ slides along the frictionless rod and is attached to the rod by a massless spring of spring constant $\kappa$.

a; Derive the Lagrangian and the Hamiltonian

b; Derive the equations of motion in the stationary frame using Hamiltonian mechanics.

c; What are the constants of motion?

d; If the rotation is constrained to have a constant angular velocity $\dot{\theta} = \omega$ then is the non-cyclic Routhian $R_{noncyclic} = H - p_\theta \dot{\theta}$ a constant of motion, and does it equal the total energy?

e; Use the non-cyclic Routhian $R_{noncyclic}$ to derive the radial equation of motion in the rotating frame of reference for the cranked system with $\dot{\theta} = \omega$.