P235 - PROBLEM SET 7

To be handed in by **1800 hr on Friday, 30 October 2015**.

1. Assume that the Earth's orbit about the Sun is circular and that the Sun's mass suddenly decreases by a factor of 2. (a) What orbit does the earth then have? (b) Will the earth escape the solar system?

2. A communications satellite is in a circular orbit around the earth at a radius R and velocity v. A rocket accidentally fires quite suddenly, giving the rocket an outward velocity v in addition to its original tangential velocity v.

   a) Calculate the ratio of the new energy and angular momentum to the old.

   b) Describe the subsequent motion of the satellite and plot $T(r)$, $U(r)$, the net effective potential, and $E(r)$ after the rocket fires.

3. Two identical point objects, each of mass m are bound by a linear two-body force $F = -kr$ where r is the vector distance between the two point objects. The two point objects each slide on a horizontal frictionless plane subject to a vertical gravitational field g. The two-body system is free to translate, rotate and oscillate on the surface of the frictionless plane.

   (a) Derive the Lagrangian for the complete system including translation and relative motion.
   (b) Use Noether's theorem to identify all constants of motion.
   (c) Use the Lagrangian to derive the equations of motion for the system.
   (d) Derive the generalized momenta and the corresponding Hamiltonian.
   (e) Derive the period for small amplitude oscillations of the relative motion of the two masses.

4. A bound binary star system comprises two spherical stars of mass m_1 and m_2 bound by their mutual gravitational attraction. Assume that the only force acting on the stars is their mutual gravitation attraction and let r be the instantaneous separation distance between the centres of the two stars where r is much larger than the sum of the radii of the stars.

   (a) Show that the two-body motion of the binary star system can be represented by an equivalent one-body system and derive the Lagrangian for this system.
   (b) Show that the motion for the equivalent one-body system in the centre of mass frame lies entirely in a plane and derive the angle between the normal to the plane and the angular momentum vector.
   (c) Show whether H_{cm} is a constant of motion and whether it equals the total energy.
   (d) It is known that a solution to the equation of motion for the equivalent one-body orbit for this gravitational force has the form

   $$\frac{1}{r} = -\frac{\mu k}{l^2}\left[1 + \epsilon \cos\theta\right]$$

   and that the angular momentum is a constant of motion $L = l$. Use these to prove that the attractive force leading to this bound orbit is

   $$\mathbf{F} = \frac{k}{r^2}\hat{\mathbf{r}}$$

   where k must be negative.

5 When performing the Rutherford experiment, Gieger and Marsden scattered $7.7 MeV$ ^{4}He particles (alpha particles) from ^{238}U at a scattering angle in the laboratory frame of $\theta = 90^0$. Derive the following observables as measured in the laboratory frame.

   (a) The recoil scattering angle of the ^{238}U in the laboratory frame.
   (b) The scattering angles of the ^{4}He and ^{238}U in the centre-of-mass frame
   (c) The kinetic energies of the ^{4}He and ^{238}U in the laboratory frame
   (d) The impact parameter
   (e) The distance of closest approach $r_{\min}$