P235 - PROBLEM SET 8

To be handed in by **1800 hr on Friday, 6 November 2015**.

[1] An automobile drag racer drives a car with acceleration a and instantaneous velocity v. The tires of radius r_0 are not slipping. Derive which point on the tire has the greatest acceleration relative to the ground. What is this acceleration?

[2] Shot towers were popular in the eighteenth and nineteenth centuries for dropping melted lead down tall towers to form spheres for bullets. The lead solidified while falling and often landed in water to cool the lead bullets. Many such shot towers were built in New York State. Assume a shot tower was constructed at latitude $42°N$, and that the lead fell a distance of $27m$. In what direction and by how far did the lead bullets land from the direct vertical?

[3] If a projectile is fired due east from a point on the surface of the Earth at a northern latitude λ with a velocity of magnitude V_0 and at an inclination to the horizontal of α, show that the lateral deflection when the projectile strikes the Earth is

$$d = \frac{4V_0^3}{g^2}\omega \sin\lambda \sin^2\alpha \cos\alpha$$

where ω is the rotation frequency of the Earth.

[4] In the preceeding problem, if the range of the projectile is R_0' for the case $\omega = 0$, show that the change of range due to rotation of the Earth is

$$\Delta R' = \sqrt{\frac{2R_0'^3}{g}}\omega \cos\lambda \left(\cot^{\frac{1}{2}}\alpha - \frac{1}{3}\tan^{\frac{3}{2}}\alpha\right)$$

[5] A three-particle system consists of masses m_i with coordinates (x_1, x_2, x_3) as follows:

$$\begin{aligned} m_1 &= 3m, & (b, 0, b) \\ m_2 &= 4m, & (b, b, -b) \\ m_3 &= 2m, & (-b, b, 0) \end{aligned}$$

Find the inertia tensor, principal axes and principal moments of inertia about the origin.

[6] Calculate the moments of inertia I_1, I_2, I_3 for a homogeneous cone of mass M whose height is h and whose base has a radius R. Choose the x_3-axis along the symmetry axis of the cone.

a) Choose the origin at the apex of the cone, and calculate the elements of the inertia tensor.

b) Make a transformation such that the center of mass of the cone is the origin and find the principal moments of inertia.