P235 - PROBLEM SET 9

To be handed in by **1800 hr on Friday, 13 November 2015**.

1. Four masses, all of mass m, lie in the $x-y$ plane at positions $(x, y) = (a, 0), (-a, 0), (0, +2a), (0, -2a)T$. These are joined by massless rods to form a rigid body

   (a) Find the inertial tensor, using the x, y, z axes as a reference system. Exhibit the tensor as a matrix.

   (b) Consider a direction given by the unit vector $\hat{n}$ that lies equally between the positive x, y, z axes; that is it makes equal angles with these three directions. Find the moment of inertia for rotation about this $\hat{n}$ axis.

   (c) Given that at a certain time t the angular velocity vector lies along the above direction $\hat{n}$, find, for that instant, the angle between the angular momentum vector and $\hat{n}$.

2. A homogeneous cube, each edge of which has a length l, initially is in a position of unstable equilibrium with one edge of the cube in contact with a horizontal plane. The cube then is given a small displacement causing it to tip over and fall. Show that the angular velocity of the cube when one face strikes the plane is given by

   $$\omega^2 = A\frac{g}{l}\left(\sqrt{2} - 1\right)$$

   where $A = \frac{3}{2}$ if the edge cannot slide on the plane, and where $A = \frac{12}{5}$ if sliding can occur without friction.

3. Consider a thin disk composed of two homogeneous halves connected along a diameter of the disk. If one half has density ρ and the other half density 2ρ find the expression for the Lagrangian when the disk rolls without slipping along a horizontal surface.

4. A symmetric body moves without the influence of forces or torques. Let x_3 be the symmetry axis of the body and $\mathbf{L}$ be along x_3'. The angle between $\boldsymbol{\omega}$ and x_3 is α. Let $\boldsymbol{\omega}$ and $\mathbf{L}$ initially be in the $x_2 - x_3$ plane. What is the angular velocity of the symmetry axis about $\mathbf{L}$ in terms of I_1, I_3, ω,and α?

5. Consider a thin rectangular plate with dimensions a by b and mass M. Determine the torque necessary to rotate the thin plate with angular velocity ω about a diagonal. Explain the physical behaviour for the case when $a = b$.