P235 - PROBLEM SET 2

To be handed in by **1700 hr on Friday, 17 September 2010**.

1] A particle of mass m moving in one dimension has potential energy $U(x) = U_0[2(\frac{x}{a})^2 - (\frac{x}{a})^4]$, where U_0 and a are positive constants.

a) Find the force $F(x)$ that acts on the particle.

b) Sketch $U(x)$. Find the positions of stable and unstable equilibrium.

c) What is the angular frequency ω of oscillations about the point of stable equilibrium?

d) What is the minimum speed the particle must have at the origin to escape to infinity?

e) At $t = 0$ the particle is at the origin and its velocity is positive and equal to the escape velocity. Find $x(t)$ and sketch the result.

[2] Consider a particle of mass m whose motion starts from rest in a common gravitational field. If a resisting force proportional to the square of the velocity (i.e. kmv^2) is encountered, show that the distance s the particle falls in accelerating from v_0 to v_1 is given by

$$s(v_0 \to v_1) = \frac{1}{2k} \ln \left[\frac{g - kv_0^2}{g - kv_1^2} \right]$$

[3] a) Consider a single-stage rocket travelling in a straight line subject to an external force F^{ext} acting along the same line where v_{ex} is the exhaust velocity of the ejected fuel relative to the rocket. Show that the equation of motion is

$$m\dot{v} = -\dot{m}v_{ex} + F^{ext}$$

b) Specialize to the case of a rocket taking off vertically from rest in a uniform gravitational field g. Assume that the rocket ejects mass at a constant rate of $\dot{m} = -k$ where k is a positive constant. Solve the equation of motion to derive the dependence of velocity on time.

c) The first couple of minutes of the launch of thr Space Shuttle can be described roughly by; initial mass $= 2 \times 10^6$ kg, mass after 2 minutes $= 1 \times 10^6$ kg, exhaust speed $v_{ex} = 3000 m/s$, and initial velocity is zero. Estimate the velocity of the Space Shuttle after two minutes of flight.

d) Describe what would happen to a rocket where $\dot{m}v_{ex} < mg$.

[4] A time independent field $\mathbf{F}$ is conservative if $\boldsymbol{\nabla} \times \mathbf{F} = \mathbf{0}$. Use this fact to test if the following fields are conservative, and derive the corresponding potential U.

a) $F_x = ayz + bx + c, F_y = axz + bz, F_z = axy + by$

b) $F_x = -ze^{-x}, F_y = \ln z, F_z = e^{-x} + \frac{y}{z}$

[5] A particle of mass m moves under the influence of a resistive force proportional to velocity and a potential U, that is

$$F(x, \dot{x}) = -b\dot{x} - \frac{\partial U}{\partial x}$$

where $b > 0$ and $U(x) = (x^2 - a^2)^2$

a) Find the points of stable and unstable equlibrium.

b) Find the solution of the equations of motion for small oscillations around the stable equilibrium points

c) Show that as $t \to \infty$ the particle approaches one of the stable equlibrium points for most choices of initial conditions. What are the exceptions? (Hint: You can prove this without finding the solutions explicitly.)

[6] If the gravitational field vector is independent of the radial distance within a sphere, find the the function describing the mass density $\rho(r)$ of the sphere.