P235 - PROBLEM SET 3

To be handed in by **1700 hr on Friday, 24 September 2010**.

[1] An unusual pendulum is made by fixing a string to a horizontal cylinder of radius R,wrapping the string several times around the cylinder, and then tying a mass m to the loose end. In equilibrium the mass hangs a distance l_0 vertically below the edge of the cylinder. Find the potential energy if the pendulum has swung to an angle ϕ from the vertical. Show that for small angles, it can be written in the Hooke's Law form $U = \frac{1}{2}k\phi^2$. Comment of the value of k.

[2] Consider the two-dimensional anisotropic oscillator with motion with $\omega_x = p\omega$ and $\omega_y = q\omega$.

a) Prove that if the ratio of the frequencies is rational (that is, $\frac{\omega_x}{\omega_y} = \frac{p}{q}$ where p and q are integers) then the motion is periodic. What is the period?

b) Prove that if the same ratio is irrational, the motion never repeats itself.

[3] A simple pendulum consists of a mass m suspended from a fixed point by a weight-less, extensionless rod of length l.

a) Obtain the equation of motion, and in the approximation $\sin\theta \approx \theta$, show that the natural frequency is $\omega_0 = \sqrt{\frac{g}{l}}$, where g is the gravitational field strength.

b) Discuss the motion in the event that the motion takes place in a viscous medium with retarding force $2m\sqrt{gl}\dot{\theta}$.

[4] Derive the expression for the State Space paths of the plane pendulum if the total energy is $E > 2mgl$. Note that this is just the case of a particle moving in a periodic potential $U(\theta) = mgl(1-\cos\theta)$. Sketch the State Space diagram for both $E > 2mgl$ and $E < 2mgl$.

[5] Consider the motion of a driven linearly-damped harmonic oscillator after the transient solution has died out, and suppose that it is being driven close to resonance, $\omega = \omega_o$.

a) Show that the oscillator's total energy is $E = \frac{1}{2}m\omega^2A^2$.

b) Show that the energy ΔE_{dis} dissipated during one cycle by the damping force $\Gamma\dot{x}$ is $\pi\Gamma m\omega A^2$

[6] Two masses m_1 and m_2 slide freely on a horizontal frictionless surface and are connected by a spring whose force constant is k. Find the frequency of oscillatory motion for this system.