P235 - PROBLEM SET 6

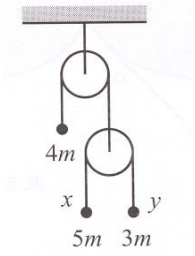
To be handed in by **1700 hr on Friday, 15 October 2010**.

[1] Let the horizontal plane be the $x-y$ plane. A bead of mass m is constrained to slide with speed v along a curve described by the function $y = f(x)$. What force does the curve apply to the bead? (Ignore gravity)

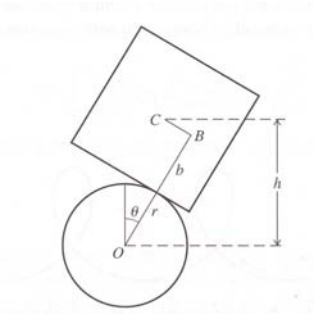
[2] Consider the Atwoods machine shown. The masses are $4m$, $5m$, and $3m$. Let x and y be the heights of the right two masses relative to their initial positions.

a) Solve this problem using the Euler-Lagrange equations

b) Use Noether's theorem to find the conserved momentum.



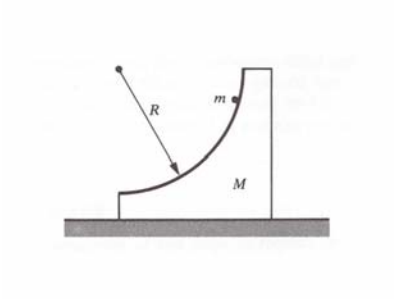
[3] A cube of side $2b$ and center of mass C, is placed on a fixed horizontal cylinder of radius r and center O as shown in the figure. Originally the cube is placed such that C is centered above O but it can roll from side to side without slipping. (a) Assuming that $b < r$ use the Lagrangian approach to to find the frequency for small oscillations about the top of the cylinder. For simplicity make the small angle approximation for L before using the Lagrange-Euler equations. (b) What will be the motion if $b > r$? Note that the moment of inertia of the cube about the center of mass is $\frac{2}{3}mb^2$



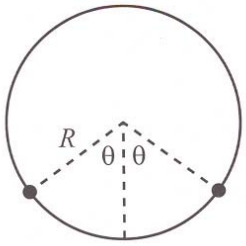
[4] A particle of mass m slides down a frictionless smooth circular wedge of mass M as shown. The wedge rests on a smooth frictionless horizontal table.

a) Find the equations of motion for m and M.

b) Find the reaction of the wedge on m.



[5] Two equal masses of mass m are glued to a massless hoop of radius R is free to rotate about its center in a vertical plane. The angle between the masses is 2θ, as shown. Find the frequency of oscillations.



[6] Three massless sticks each of length $2r$, and mass m with the center of mass at the center of each stick, are hinged at their ends as shown. The bottom end of the lower stick is hinged at the ground. They are held so that the lower two sticks are vertical, and the upper one is tilted at a small angle ε with respect to the vertical. They are then released. At the instant of release what are the three equations of motion derived from the Lagrangian derived assuming that ε is small? Use these to determine the initial angular accelerations of the three sticks.