P235 **MIDTERM EXAMINATION** **Prof Cline**

1900 - 2200 hr, 5 November 2010

THIS IS A CLOSED BOOK EXAM. Show all steps to get full credit. Answer questions 1,2 in Book 1: questions 3, 4, in Book 2.

Book 1

1; (25pts) A plane pendulum consists of a mass m suspended from a fixed pivot by a weightless, extensionless, rod of length l. The motion of the pendulum is confined to a vertical plane.

a) Use Lagrange mechanics to derive the equation of motion

b) Draw a (θ, p_θ) phase diagram and discuss the features exhibited by the plane pendulum for different total energies.

c) In the approximation $\sin\theta \approx \theta$ obtain the natural frequency of the oscillation.

d) Discuss the motion if it takes place in a viscous medium that has a retarding force of $m\sqrt{gl}\dot{\theta}$

e) What will be the frequency for small oscillations when this retarding force is acting.

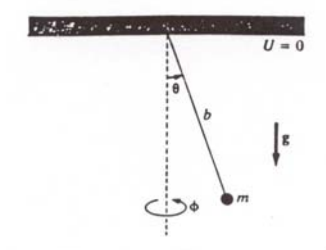
2: [25pts] Consider a spherical pendulum comprising a mass m suspended by a rigid mass-less rod of length b from the pivot point that allows rotation in both the θ and ϕ directions.

a) Derive the Lagrangian L

b) Derive the Hamiltonian from the Lagrangian

c) Derive the four Hamilton equations of motion

d) Prove all the constants of motion for the spherical pendulum.



Book 2

3; (25pts) A sphere of radius r is constrained to roll without slipping on the lower half of the inner surface of a fixed hollow cylinder of inside radius R. The moment of inertia of a sphere is $I = \frac{2}{5}mr^2$.

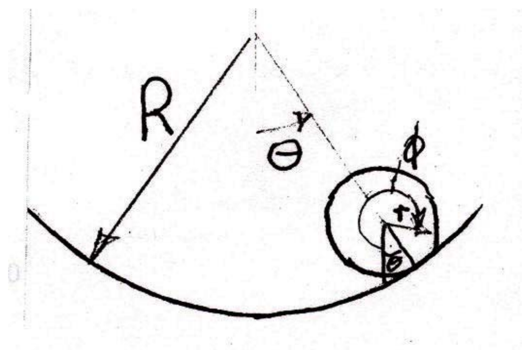
a; Determine the Lagrangian function

b; Determine the equation of constraint

c; Determine Lagrange's equations of motion

d; What does the Lagrange multiplier imply?

e; Derive the angular frequency for small oscillations.



4; (25pts) A uniform cylinder of mass m and radius R rolls without slipping on an inclined plane that makes an angle α to the horizontal. The cylinder is constrained to roll along the wedge by a spring having a spring constant κ. One end of the yoke is attached to the midpoint of a massless U-shaped yoke the ends of which are attached to frictionless bearings on the cylindrical axis of the roller. The other end of the spring is attached to a fixed point a distance R above the top of the wedge such that the spring is parallel with the inclined plane. The inclined plane has a mass M and rests on a frictionless horizontal surface as shown in the figure. Assume that the moment of inertia of the roller is $I = \frac{1}{2}mR^2$ and that the length of the spring is d when the roller and wedge are in stationary equilibrium.

a) Express the Lagrangian in terms of the generalized coordinates x for the horizontal position of the wedge, and s the length of the spring.

b) Use Noether's theorem to identify all constants of motion

c) Use the Lagrangian to derive the equations of motion in terms of x and s.

d) Use the kinetic energy tensor and potential energy tensor to derive the eigen frequencies of the coupled oscillator.

e) Describe in words the motion corresponding to the normal modes of this system