Midterm Examination

Midterm examination:

Will be given Friday 4 November at 1900-2300 hr in **B&L 208.**

Office hours:

Doug Cline; Friday 4 Nov, 1300-1500 hr in B&L203D

Content of midterm:

The midterm examination will comprise four full multipart questions. It is a closed-book, nominally a 3-hour, exam but extra time is scheduled. A calculator may be needed but emphasis will be on concepts and the equations of motion rather than numerical answers. There will be no trick questions, all straightforward bookwork. The questions will emphasize material covered starting with the introduction of Lagrangian mechanics as discussed below.

Chapters 1, 2: Not explicitly part of the examination although it is assumed that you have a working knowledge of the material in chapter 2.

Chapters 3: Will feature throughout the examination implicitly and possibly in one question.

Chapter 4: Linear damped oscillations **important** but Fourier analysis will not be emphasized.

Chapter 5: The general features of non-linear systems and chaos should be known, but only qualitative aspects may be addressed in a partial question. Quantitative questions on non-linear systems are not viable on an exam.

Chapter 6: Calculus of variations will not be addressed explicitly, but implicitly it will be assumed in use of Lagrange equations, Hamilton's principle, etc.

Chapter 7: Lagrangian dynamics is **very important**. Applications will cover questions featuring constraints that can be solved with generalized coordinates and Lagrange multipliers. Must know how to use generalized coordinates, derive equations of motion, and derive forces of constraint. Will require ability to solve the equations of motion. You will not be asked to reproduce the derivation of Lagrange equations from either d'Alembert or Hamilton's Principles.

Chapters 8,9: Hamiltonian mechanics is **very important.** Need to know the symmetries/conservation laws, Noether's theorem, cyclic coordinates, properties of Hamiltonian, Hamilton's equations of motion. You should be able to use Hamiltonian mechanics to solve simple mechanical systems.

Chapters 10: Coupled oscillations and normal modes for both discrete and continuous sytems will be a **very important** part of the exam. Need to be able to use equations of motion and solve for characteristic frequencies, eigenfunctions and determine the normal modes for 2 and 3 coupled oscillators or the discrete lattice chain. Maximum determinant will be 3x3.

Chapter 11: Wave motion, wave packets, phase and group velocity all will be included on the exam.

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Useful formulae

Damped harmonic oscillator:

$$\frac{d^2x}{dt^2} + \Gamma\frac{dx}{dt} + \omega_0^2 x = 0$$

Solution for $\frac{\Gamma}{2} < \omega_o$ is

$$x = Ae^{-\frac{\Gamma t}{2}}\cos(\omega_1 t - \delta)$$

where

$$\omega_1^2 = \omega_0^2 - \frac{\Gamma^2}{4}$$

Sinusoidal-driven damped harmonic oscillator

$$\frac{d^2x}{dt^2} + \Gamma\frac{dx}{dt} + \omega_0^2 x = A\cos(\omega t)$$

Steady-state solution is

$$x_{ss} = \frac{A}{\sqrt{\left(\omega_o^2 - \omega^2\right)^2 + \Gamma^2\omega^2}}\cos(\omega t - \delta)$$

where

$$\tan\delta = \frac{\Gamma\omega}{(\omega_0^2 - \omega^2)}$$

Newton's law of Gravitation

$$\begin{aligned}\overline{\mathbf{F}}_m &= -G\frac{mM}{r^2}\widehat{\mathbf{r}} \\ \overrightarrow{\boldsymbol{\nabla}}\cdot\overrightarrow{\mathbf{g}} &= -4\pi G\rho\end{aligned}$$

Virial Theorem

$$\langle T\rangle = -\frac{1}{2}\left\langle\sum_i \overrightarrow{\mathbf{F}}_i\cdot\overrightarrow{\mathbf{r}}_i\right\rangle$$

Lagrange equations
Generalized coordinates

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{q}_j} - \frac{\partial L}{\partial q_j} = 0$$

Lagrange equations with undetermined multipliers and generalized forces

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j^{EX} + \sum_{k=1}^{m}\lambda_k(t)\frac{\partial g_k}{\partial q_j} \qquad (i = 1, 2, 3, ...s)$$

Generalized momentum

$$p_j = \frac{\partial L}{\partial\dot{q}_j}$$

Hamiltonian

$$H(q_i, p_i, t) = \sum_i p_i\dot{q}_i - L(q_i, \dot{q}_i, t)$$

Hamilton's equations of motion

$$\begin{aligned}\dot{q}_k &= \frac{\partial H}{\partial p_k} \\ -\dot{p}_k &= \frac{\partial H}{\partial q_k}\end{aligned}$$

$$-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$$

Cylindrical coordinates

$$\begin{aligned} L &= T - U \\ &= \frac{m}{2}\left(\dot{\rho}^2 + \rho^2\dot{\phi}^2 + \dot{z}^2\right) - U(\rho z \phi) \end{aligned}$$

$$\begin{aligned} H &= T + U \\ &= \frac{1}{2m}\left(p_\rho^2 + \frac{p_\phi^2}{\rho^2} + p_z^2\right) + U(\rho, z, \phi) \end{aligned}$$

Spherical coordinates

$$\begin{aligned} L &= T - U \\ &= \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right) - U(r\theta\phi) \end{aligned}$$

$$\begin{aligned} H &= T + U \\ &= \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2\sin^2\theta}\right) + U(r, \theta, \phi) \end{aligned}$$

Coupled oscillators

$$T = \frac{1}{2}\sum_{j,k} T_{jk}\dot{q}_j\dot{q}_k$$

$$U = \frac{1}{2}\sum_{j,k} V_{jk}q_jq_k$$

$$T_{jk} \equiv \sum_\alpha m_\alpha \sum_i \frac{\partial x_{\alpha,i}}{\partial q_j}\frac{\partial x_{\alpha,i}}{\partial q_k}$$

$$V_{jk} \equiv \left(\frac{\partial^2 U}{\partial q_j \partial q_k}\right)_0$$

$$\sum_j \left(V_{jk} - \omega_r^2 T_{jk}\right) a_{jr} = 0$$

Vectors;

$$\overline{\mathbf{C}} = \overline{\mathbf{A}} \times \overline{\mathbf{B}} = \begin{vmatrix} \widehat{\mathbf{i}} & \widehat{\mathbf{j}} & \widehat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{A}} = 0$$

$$\overrightarrow{\mathbf{A}} \cdot \left(\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right) = 0$$

$$\overrightarrow{\mathbf{A}} \cdot \left(\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{C}}\right) = \left(\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right) \cdot \overrightarrow{\mathbf{C}}$$

$$\overrightarrow{\mathbf{A}} \times \left(\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{C}}\right) = \overrightarrow{\mathbf{B}}\left(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{C}}\right) - \left(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}\right)\overrightarrow{\mathbf{C}}$$