P235 - PROBLEM SET 4

To be handed in by **1700 hr on Friday, 1 October 2010**.

[1] Find and describe the path $y = y(x)$ for which the the integral $\int_{x_1}^{x_2} \sqrt{x}\sqrt{1+(y')^2}dx$ is stationary.

[2] Find the dimensions of the parallelepiped of maximum volume circumscribed by a sphere of radius R.

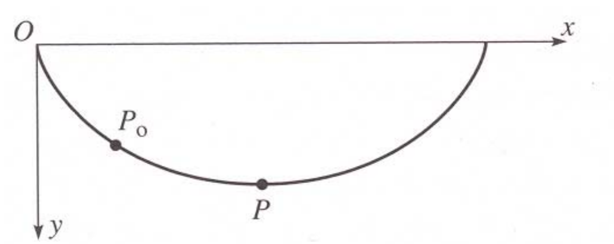
[3] Consider a single loop of the cycloid having a fixed value of a as shown in the figure. A car released from rest at any point P_0 anywhere on the track between O and the lowest point P , that is, P_0 has a parameter $0 < \theta_0 < \pi$.

a) Show that the time for the cart to slide from P_0 to P is given by the integral

$$\text{time}(P_0 \rightarrow P) = \sqrt{\frac{a}{g}} \int_{\theta_0}^{\pi} \sqrt{\frac{1-\cos\theta}{\cos\theta_0 - \cos\theta}} d\theta$$

b) Prove that this time is equal to $\pi\sqrt{a/g}$ which is independent of the position P_0.

c) Explain qualitatively how this surprising result can possibly be true.



[4] Consider a medium for which the refractive index $n = \frac{a}{r^2}$ where a is a constant and r is the distance from the origin. Use Fermat's Principle to find the path of a ray of light travelling in a plane containing the origin. Hint, use two-dimensional polar coordinates with $\phi = \phi(r)$. Show that the resulting path is a circle through the origin.

[5] Find the shortest path between the (x, y, z) points $(0, -1, 0)$ and $(0, 1, 0)$ on the conical surface

$$z = 1 - \sqrt{x^2 + y^2}$$

What is the length of this path? Note that this is the shortest mountain path around a volcano.

[6] Show that the geodesic on the surface of a right circular cylinder is a segment of a helix.