P235 - PROBLEM SET 7

To be handed in by **1700 hr on Friday, 22 October 2010**.

[1] A particle of mass m in a gravitational field slides on the inside of a smooth parabola of revolution whose axis is vertical. Using the distance from the axis r, and the azimuthal angle φ as generalized coordinates, find the following.

a) The Lagrangian of the system.

b) The generalized momenta and the corresponding Hamiltonian

c) The equation of motion for the coordinate r as a function of time.

d) If $\frac{d\varphi}{dt} = 0$, show that the particle can execute small oscillations about the lowest point of the paraboloid and find the frequency of these oscilllations.

[2] Consider a particle of mass m which is constrained to move on the surface of a sphere of radius R. There are no external forces of any kind acting on the particle.

a) What is the number of generalized coordinates necessary to describe the problem?

b) Choose a set of generalized coordinates and write the Lagrangian of the system.

c) What is the Hamiltonian of the system? Is it conserved?

d) Prove that the motion of the particle is along a great circle of the sphere.

[3] A non-relativistic electron of mass m, charge $-e$ in a cylindrical magneton moves between a wire of radius a at a negative electric potential $-\phi_0$ and a concentric cylindrical conductor of radius R at zero electric potential. There is a uniform constant magnetic field B parallel to the axis of the cylinder. Using cylindrical coordinates r, θ, z the electric scalar potential and magnetic vector potential can be written as

$$\begin{aligned} \phi &= -\phi_0 \frac{\ln\left(\frac{r}{R}\right)}{\ln\left(\frac{a}{R}\right)} \\ \mathbf{A} &= \frac{1}{2} B r \hat{\mathbf{e}}_\theta \end{aligned}$$

where $\hat{\mathbf{e}}_\theta$ is the unit vector in the direction of increasing θ.

a) Derive the Lagrangian and the Hamiltonian functions

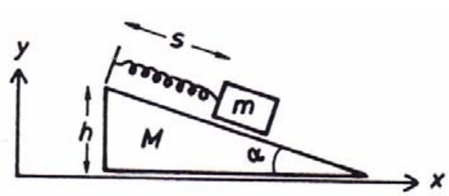
b) Show that there are three constants of the motion. Write them down, and discuss the kinds of motion that can occur.

[4] A block of mass m is attached to a wedge of mass M by a spring with spring constant k. The inclided frictionless surface of the wedge makes an angle α to the horizontal. The wedge is free to slide on a horizontal frictionless surface as shown in the figure.

a) Given that the relaxed length of the spring is d, find the values s_0 when both bock and wedge are stationary.

b) Find the Lagrangian for the system as a function of the x coordinate of the wedge and the length of spring s. Write down the equations of motion.

c) What is the natural frequency of vibration?



[5] A thin. uniform rod of length $2L$ and mass M is suspended from a massless string of length l tied to a nail. As shown in the figure, a horizontal force is applied to the rod's free end.

a) Write the Lagrangian for this system.

b) For very short times such that all angles are small, determine the angles that string and the rod make with the vertical. Start from rest at $t - 0$.

c) Draw a diagram to illustrate the inital motion of the rod.