

$B(E2)$ values and the search for the critical point symmetry X(5) in ^{104}Mo and ^{106}Mo

 C. Hutter,^{1,2} R. Krücken,^{1,3} A. Aprahamian,⁴ C. J. Barton,¹ C. W. Beausang,¹ M. A. Caprio,¹ R. F. Casten,¹ W.-T. Chou,^{1,5}
 R. M. Clark,⁶ D. Cline,⁷ J. R. Cooper,^{1,8} M. Cromaz,⁶ A. A. Hecht,¹ A. O. Macchiavelli,⁶ N. Pietralla,^{1,9}
 M. Shawcross,⁴ M. A. Stoyer,⁸ C. Y. Wu,⁷ and N. V. Zamfir^{1,10,11}
¹A.W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520

²Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

³Physik Department E12, Technische Universität München, 85748 Garching, Germany

⁴Nuclear Structure Laboratory, University of Notre Dame, Notre Dame, Indiana 46556

⁵National Tsing Hua University, Hsinchu, Taiwan, Republic of China

⁶Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720

⁷Nuclear Structure Research Laboratory, University of Rochester, Rochester, New York 14627

⁸Lawrence Livermore National Laboratory, Livermore, California 94551

⁹Institut für Kernphysik, Universität zu Köln, 50937 Köln, Germany

¹⁰Clark University, Worcester, Massachusetts 01610

¹¹National Institute for Physics and Nuclear Engineering, Bucharest-Magurele, Romania

(Received 11 February 2003; published 28 May 2003)

Lifetimes of the first 4^+ and 6^+ states in ^{104}Mo and ^{106}Mo have been measured using the recoil distance method following spontaneous fission of ^{252}Cf . The experiment was performed at the 88-inch cyclotron of the Lawrence Berkeley National Laboratory employing the Gammasphere array and the New Yale Plunger Device. Reduced transition probabilities in the ground state band of ^{104}Mo are compared with predictions of the critical point symmetry X(5) for phase transitional nuclei between rotational and vibrational shape. While known level energies of ^{104}Mo are in good agreement with the X(5) predictions, the analysis of the measured $B(E2)$ values favors a rotor interpretation.

DOI: 10.1103/PhysRevC.67.054315

PACS number(s): 21.10.-k, 21.60.-n, 27.60.+j, 23.20.-g

For many composite systems (such as molecules, atoms and nuclei) the shape evolves with particle number, and the changing shape strongly influences important properties of these systems. For example, spherical quantum systems only have a vibrational collective degree of freedom, while deformed systems can also rotate. In nuclear physics, the low lying collective states are classified and understood in terms of their relationship to the paradigms of harmonic vibrator [1], deformed symmetric rotor [2], and γ -unstable rotor [3]. Treated in the group theoretical framework of the interacting boson approximation (IBA) [4], these key geometric benchmarks are related to the dynamical symmetries of the group U(6), and are called U(5), SU(3), and O(6), respectively. Although these exact benchmarks occur seldom in real nuclei, the comparison to these limits provides clear classification and amplifies the effect of the breaking of the dynamical symmetries.

Considering these limits as different phases, it becomes important to investigate the phase transition between these limits, where shape and structure change most rapidly. It has been shown theoretically [5] that there is a second-order phase transition between the U(5) and O(6) limits, and a first-order phase transition between the U(5) and SU(3) limits, while no phase transition occurs between O(6) and SU(3) limits.

Recently, a major breakthrough in the understanding of phase transitions has been made by the discovery of new critical point symmetries of the phase transitions [6,7] mentioned above. These have been called E(5) and X(5), respectively, as indicated in the inset of Fig. 1. It should be mentioned that these are not symmetries of the IBA, but are approximate analytical solutions of the geometrical Bohr

Hamiltonian based on the shape parameters β and γ .

Except for a scale factor, parameter-free approximate analytic solutions for energies and transition probabilities have been derived for these new symmetries at both of the phase transitional points mentioned above [6,7]. Among the predictions are the energy ratios $E(0_2^+)/E(2_1^+)$ and $E(J_1^+)/E(2_1^+)$ as a function of angular momentum J , and $B(E2)$ values for interband and intraband transitions.

Because nuclei are discrete systems, leading to discrete points in the phase diagram, only few nuclei can be expected to be very close to a critical point. For the more recently

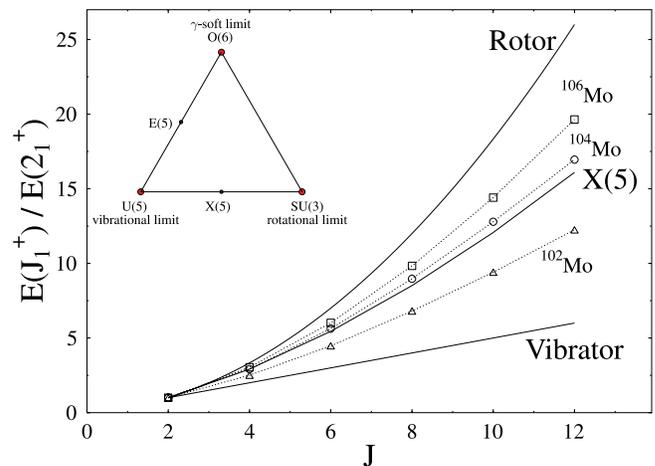


FIG. 1. The ratio of the energy of the states in the yrast band to the energy of the first 2^+ state for $^{102,104,106}\text{Mo}$. The ratios are compared to predictions of vibrational and rotational limits and of the dynamical symmetry X(5).

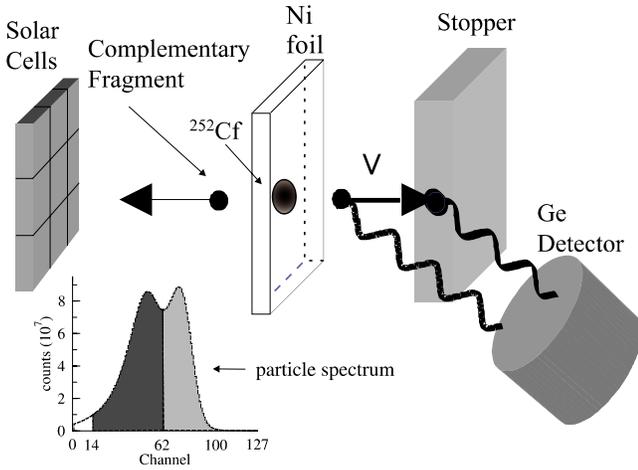


FIG. 2. A schematic of the fission RDM experiment. The photocells, Ni foil with the ^{252}Cf source, and the stopper are mounted in the NYPD chamber that itself was mounted inside the Gamma-sphere. The inset shows the integral particle spectrum obtained with different sets of photocells.

presented solution to the X(5) symmetry, only two nuclei (^{152}Sm and ^{150}Nd) are known so far to be in good agreement with characteristic predictions of this new critical point symmetry [8,9]. These two and most other candidates for an X(5)

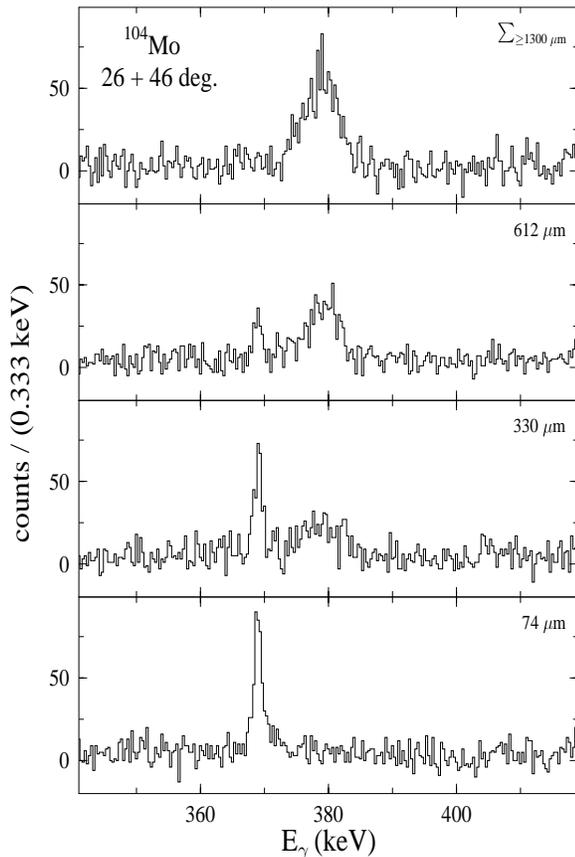


FIG. 3. Background subtracted coincidence spectra for the $4^+ \rightarrow 2^+$ transition in ^{104}Mo taken at different source-to-stopper distances. The change from unshifted to shifted peak intensity for increasing distance can clearly be seen.

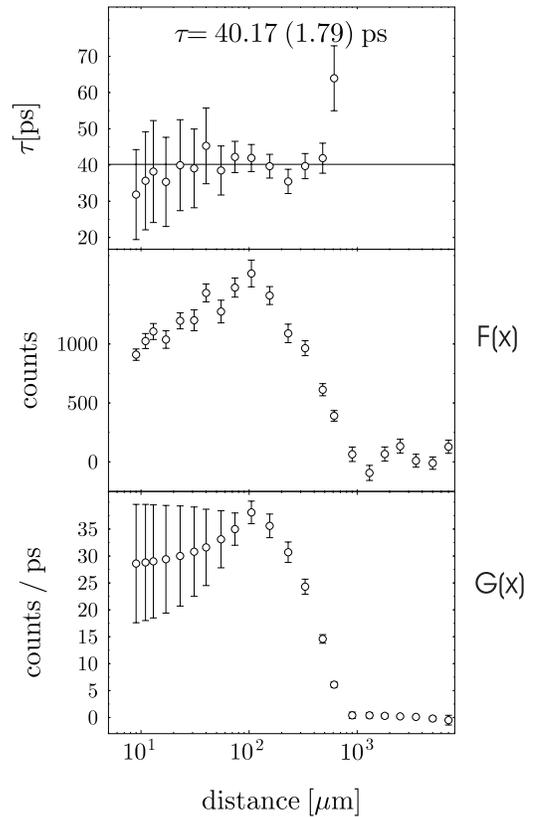


FIG. 4. The functions $F(x)$ and $G(x)$ in Eq. (1) as well as the lifetime values $\tau(x)$ as functions of the source-to-stopper distance for the $4^+ \rightarrow 2^+$ transition in ^{104}Mo . Only statistical errors are given in this picture. For the final lifetime result, the plotted lifetime values are averaged and the result is corrected by 3% for the angle of emission of the fragments. For the total error, systematic uncertainties were taken into account, as described in the text.

symmetry are in the region of rare earth nuclei with $N=90$ [10].

However, two recent publications point out [11,12] that the energy levels of the yrast, the $K=2$, and the $K=4$ bands in ^{104}Mo [13] are very close to the X(5) predictions. To illustrate this, Fig. 1 shows the energy ratio of yrast J^+ states and yrast 2^+ state as a function of J for $^{102,104,106}\text{Mo}$. ^{104}Mo not only shows a behavior very close to the X(5) predictions, but there also seems to be a trend from vibrational to rotational behavior, suggesting the possibility of a phase transition. However, it must be mentioned that the rotational limit for heavier Mo isotopes is not reached, which can be seen from the energy ratio $E(4_1^+)/E(2_1^+)$, which is 2.919, 3.044, and 2.925 for ^{104}Mo , ^{106}Mo , and ^{108}Mo , respectively. In order to draw a definite conclusion on the agreement between X(5) predictions and real nuclei, it is important to also compare predicted and measured transition matrix elements. In Refs. [11,12] such a comparison was performed for only two $B(E2)$ values.

The purpose of this paper is to report on transition matrix elements in ^{104}Mo and ^{106}Mo obtained by measuring lifetimes of excited states populated in the spontaneous fission of ^{252}Cf using a coincidence Doppler-shift lifetime technique and the Gammasphere array. Additionally, the results of this

TABLE I. Lifetimes τ , reduced transition probabilities $B(E2)$, and transition quadrupole moments Q_t for levels in $^{104,106}\text{Mo}$. For the new lifetimes, first statistical, and then systematic errors are given. $B(E2)$ and Q_t values have been calculated after quadratically adding these errors and taking into account the conversion coefficient α adopted from Ref. [24]. For comparison, earlier results are shown as well.

Nucleus	$J_i^\pi \rightarrow J_f^\pi$ (\hbar)	Energy (keV)	τ (ps)		$B(E2)(e^2 \text{ fm}^4)$		$Q_t (e \text{ fm}^2)$	
			This work	Previous work	This work	Previous work	This work	Previous work
^{104}Mo	$2^+ \rightarrow 0^+$	192		1040(59), ^a 1396(112) ^b		2518(117)		356(8)
	$4^+ \rightarrow 2^+$	369	41.0(18)(30)	38.7(50), ^c 37.7(11) ^b	2890(250)	3140(90)	319(14)	332(5)
	$6^+ \rightarrow 4^+$	519	6.04(54)(68)	6.0(23), ^d 6.83(21) ^b	3590(520)	3180(100)	339(24)	319(5)
	$8^+ \rightarrow 6^+$	642		2.84(60), ^e 3.19(16) ^b		2370(120)		269(7)
	$10^+ \rightarrow 8^+$	733		1.42(20) ^e 1.56(10) ^b		2530(150)		274(8)
^{106}Mo	$2^+ \rightarrow 0^+$	172		1803(44) ^f		2574(63)		360(4)
	$4^+ \rightarrow 2^+$	351	36.6(28)(45)	24.4(35) ^g	4140(600)	6220(900)	382(28)	468(34)
	$6^+ \rightarrow 4^+$	511	6.1(11)(15)		3600(1100)		339(50)	
	$8^+ \rightarrow 6^+$	655		2.55(49) ^e		2660(520)		285(27)
	$10^+ \rightarrow 8^+$	785		1.00(19) ^e		2750(530)		286(27)

^aAdopted from Refs. [20,21].

^bAdopted from Ref. [22].

^cAdopted from Ref. [23].

^dAdopted from Ref. [9].

^eAdopted from Ref. [19].

^fAdopted from Ref. [21].

^gAdopted from Ref. [14].

measurement and previously published data are compared to the X(5) predictions. An analysis of the presently available experimental $B(E2)$ values favors a rotational interpretation for ^{104}Mo rather than an X(5) interpretation.

In the present work the spontaneous fission of ^{252}Cf was used to populate excited states in the final fission fragments, and the recoil distance method (RDM) was employed to gain lifetime information. Neutron-rich Mo isotopes are produced in the fission of ^{252}Cf with high yields, for example, ^{104}Mo and a Ba-isotope partner with more than 3.5% [14]. The experiment was performed by mounting the New Yale Plunger Device (NYPD) [15] and a solar cell array inside the Gammasphere array [16], which comprised 100 Compton-suppressed high volume Ge detectors. A schematic of the experiment can be seen in Fig. 2. A thin $\sim 70 \mu\text{Ci}$ ^{252}Cf source—decaying by spontaneous fission into (typically) two main fission fragments that travel in opposite directions—was placed on a stretched $\sim 1 \text{ mg/cm}^2$ Ni foil. One of the excited fission fragments was slowed down in the Ni foil and after traveling a variable distance x was stopped in a stretched 10 mg/cm^2 Au stopper foil. The complementary fragment was detected by the photocells within a detection angle of ~ 20 deg to the plunger axis. Events were recorded if at least three suppressed γ rays in the Ge detectors and one particle in the solar cell array were detected in coincidence. The requirement of one particle in the solar cell ensured that the partner fragment traveled towards the stopper within a cone of ± 20 degr. Additionally, the particle spectrum inset of Fig. 2 can be used to distinguish between the lower ($A \sim 100$) and higher ($A \sim 140$) mass fission products.

Approximately 85×10^6 coincidence events were collected at each of 22 source-to-stopper distances between 9 and 7000 μm for approximately one day per distance. The

data were stored by using the software package BLUE [17] that stores the data in an energy sorted list-mode database with an independent file for each coincidence multiplicity. From this, ~ 15 Gigabyte database spectra for each distance and for each ring of Gammasphere were produced, gating on the deposited particle energy and on the shifted and unshifted components of γ rays below and above the transition of interest. At least one gate was placed on a Doppler-shifted component of a transition above the level of interest, avoid-

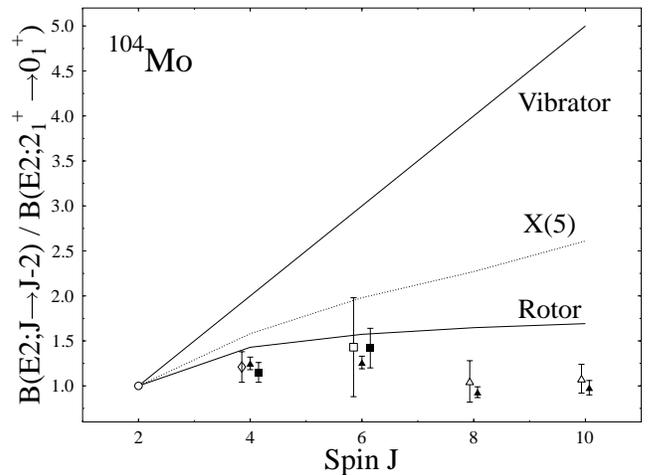


FIG. 5. Comparison of the relative experimental $B(E2)$ values of the yrast transitions in ^{104}Mo from this work (filled squares), Refs. [20,21,23] (open diamond), Ref. [9] (open square), Ref. [19] (open triangles), and Ref. [22] (closed triangles) with the predictions for the vibrational and rotational limits, and for the X(5) symmetry. Normalization is done using the combined value of Refs. [20–22] for the $2^+ \rightarrow 0^+$ transition (open circle).

ing systematic errors from incorrect feeding assumptions.

After the particle plus double- γ gate, as well as background subtraction, clean spectra (see Fig. 3) were obtained for the different distances. From the intensities of the shifted and unshifted peaks at different distances, the lifetime can be extracted using the differential decay curve method [18]. If the level of interest is populated by transition B and depopulated by transition A , and $I_{u,s}^{A,B}$ are the unshifted (u) and shifted (s) intensities of transitions A and B , respectively, then the lifetime is given by

$$\tau(x) = \frac{F(x)}{G(x)} \equiv \frac{I_u^A(x) - \left(\frac{I_u^A(x) + I_s^A(x)}{I_u^B(x) + I_s^B(x)} \right) I_u^B(x)}{v \frac{d}{dx} I_s^A(x)}. \quad (1)$$

Here, v is the velocity projection of the fission fragment on the plunger axis. If the shifted component of the direct feeder transition is used as a gate, then the unshifted component $I_u^B(x) = 0$ for all source-to-stopper distances x and the lifetime can be found only by the intensities of the unshifted peaks and the derivative of the intensities of the shifted peaks, $\tau(x) = I_u^A(x) / [v d/dx I_s^A(x)]$, as shown in Fig. 4.

The derivative of the intensities is evaluated after fitting second-order polynomials through the measured intensities. To gain insight about the possible systematic error by this fitting procedure, different regions have been chosen, and the results have been compared. Due to this evaluation, an overall systematic error of 3% was estimated and included. Additionally, due to the opening angle of ~ 20 deg of the detected fission fragments, the flight distances were on average 3% longer than the direct distance from Ni foil to stopper, for which the measured lifetimes were corrected.

Lifetimes have been obtained for the 4_1^+ and 6_1^+ levels in ^{104}Mo and ^{106}Mo . The results are summarized in Table I. For comparison, previously measured lifetimes in these nuclei are listed as well.

Both Refs. [11,12] suggest ^{104}Mo to be an X(5) candidate, but only state the lifetimes [or equivalently, $B(E2)$ values] of the 2_1^+ and 4_1^+ states. As the first excited state is used for normalization, effectively only one data point was used in these references, with statistics insufficient to draw conclusions. With the values from Refs. [19,22] together with the newly measured lifetimes, it is apparent that the data favor a rotational rather than an X(5) interpretation for the yrast cascade of ^{104}Mo (see Fig. 5). However, it is also clear that

additional lifetimes for the $K=2$ and $K=4$ bands as well as the level sequence built on the first excited 0^+ state would give further insight.

It should be mentioned that the values for the 8_1^+ and the 10_1^+ states from Ref. [19] are not independent, but are calculated from an effective quadrupole moment between 6^+ and 12^+ states under the assumption of a rotational formula being valid. One should also note that it was shown in Ref. [25] that even with very good statistics systematic uncertainties limit the accuracy of RDM experiments to an overall uncertainty of at least 3%. We therefore feel that with the statistics presented, the quoted uncertainties of 3% in Ref. [22] seems to underestimate the real uncertainties. The new values for 4_1^+ and 6_1^+ lifetimes in ^{104}Mo from the present work are in agreement with those from previous measurements and confirm the rotational behavior of the yrast states.

In ^{106}Mo we were for the first time able to measure the lifetime of the 6^+ yrast level. The quadrupole moment for the 4^+ level in ^{106}Mo from this work is significantly lower than the one quoted in Ref. [14], but is consistent with both quadrupole moments for the 2^+ and 6^+ states. The constant quadrupole moment shows the rotational character of the yrast cascade in ^{106}Mo , which also has the same dynamical moment of inertia as the rotational bands built on the one- and two- γ phonon vibrational states reported in Ref. [26].

For both nuclei there appears to be a drop in the quadrupole moments for the 8^+ and 10^+ levels. This has been interpreted in Ref. [19] as an indication for triaxiality at higher rotational frequency.

In conclusion, the lifetimes of the 4_1^+ and 6_1^+ states of ^{104}Mo and ^{106}Mo have been measured with a double-gate recoil distance method, avoiding any systematic error from sidefeeding assumptions. It has been shown that $B(E2)$ values in the yrast bands of ^{104}Mo and ^{106}Mo follow a rotational behavior. This behavior is in contrast to the behavior of the level energies, which is consistent with the X(5) predictions. While we are not able to resolve this contradiction, the analysis presented here stresses on the need to input a large number of spectroscopic observables before drawing conclusions about the structure of transitional nuclei.

We would like to thank Daeg Brenner for valuable discussions about the X(5) symmetry and the nucleus ^{104}Mo . This work was supported by the U.S. Department of Energy under Grants Nos. DE-FG02-91ER-40609 and DE-FG02-88ER-40417, the National Science Foundation, and the German DFG under Contract No. Pi 393/1.

[1] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).
 [2] A. Bohr, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **26**, 14 (1952).
 [3] L. Willets and M. Jean, Phys. Rev. **102**, 788 (1956).
 [4] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975).
 [5] A.E.L. Dieperink, O. Scholten, and F. Iachello, Phys. Rev. Lett. **44**, 1747 (1980).

[6] F. Iachello, Phys. Rev. Lett. **85**, 3580 (2000).
 [7] F. Iachello, Phys. Rev. Lett. **87**, 052502 (2001).
 [8] R.F. Casten and N.V. Zamfir, Phys. Rev. Lett. **87**, 052503 (2001).
 [9] R. Krücken, B. Albanna, C. Bialik, R.F. Casten, J.R. Cooper, A. Dewald, N.V. Zamfir, C.J. Barton, C.W. Beausang, M.A. Caprio, A.A. Hecht, T. Klug, J.R. Novak, N. Pietralla, and P. von Brentano, Phys. Rev. Lett. **88**, 232501 (2002).

- [10] M.A. Caprio, N.V. Zamfir, R.F. Casten, C.J. Barton, C.W. Beausang, J.R. Cooper, A.A. Hecht, R. Krücken, H. Newman, J.R. Novak, N. Pietralla, A. Wolf, and K.E. Zyranski, *Phys. Rev. C* **66**, 054310 (2002).
- [11] D.S. Brenner, in *Mapping the Triangle*, edited by A. Aprahamian, J.A. Cizewski, S. Pittel, and N.V. Zamfir, AIP Conf. Proc. No. 638 (AIP, Melville, NY, 2002), p. 223.
- [12] P.G. Bizzeti and A.M. Bizzeti-Sona, *Phys. Rev. C* **66**, 031301(R) (2002).
- [13] A. Guessous, N. Schulz, M. Bentaleb, E. Lubkiewicz, J.L. Durell, C.J. Pearson, W.R. Phillips, J.A. Shannon, W. Urban, B.J. Varley, I. Ahmad, C.J. Lister, L.R. Morss, K.L. Nash, C.W. Williams, and S. Khazrouni, *Phys. Rev. C* **53**, 1191 (1996).
- [14] J.H. Hamilton, A.V. Ramayya, S.J. Zhu, G.M. Ter-Akopian, Yu.Ts. Oganessian, J.D. Cole, J.O. Rasmussen, and M.A. Stoyer, *Prog. Part. Nucl. Phys.* **35**, 635 (1995).
- [15] R. Krücken, *J. Res. Natl. Inst. Stand. Technol.* **105**, 53 (2000).
- [16] I.Y. Lee, *Nucl. Phys.* **A520**, 361 (1990).
- [17] M. Cromaz *et al.*, *Nucl. Instrum. Methods Phys. Res. A* **462**, 519 (2001).
- [18] A. Dewald, S. Harissopulos, and P. von Brentano, *Z. Phys. A* **334**, 163 (1989).
- [19] A.G. Smith, J.L. Durell, W.R. Phillips, M.A. Jones, M. Leddy, W. Urban, B.J. Varley, I. Ahmad, L.R. Morss, M. Bentaleb, A. Guessous, E. Lubkiewicz, N. Schulz, and R. Wyss, *Phys. Rev. Lett.* **77**, 1711 (1996).
- [20] R.C. Jared, H. Nifenecker, and S.G. Thomson, in *Proceedings of the Third IAEA Symposium on Physics and Chemistry of Fission, Vienna* (IAEA, 1973), p. 211.
- [21] E. Cheifetz, H.A. Selic, A. Wolf, R. Chechik, and J.B. Wilhelmy, *Nuclear Spectroscopy of Fission Products*, edited by T. von Egidy (Institute of Physics, University of Reading, Berkshire, 1980), p. 193.
- [22] A.G. Smith, R.M. Wall, D. Patel, G.S. Simpson, D.M. Cullen, J.L. Durell, S.J. Freeman, J.C. Lisle, J.F. Smith, B.J. Varley, G. Barreau, M. Petit, C. Theisen, E. Bouchez, M. Houry, R. Lucas, B. Cahan, A. Le Coguie, B.J.P. Gall, O. Dorvaux, and N. Schulz, *J. Phys. G* **28**, 2307 (2002).
- [23] G. Mamane, E. Cheifetz, E. Dafni, A. Zemel, and J.B. Wilhelmy, *Nucl. Phys.* **A454**, 213 (1986).
- [24] See webcalculator, <http://www.nndc.bnl.gov/nndcscr/testwww/>
- [25] A. Dewald, D. Weil, R. Krücken, R. Kühn, R. Peusquens, H. Tiesler, O. Vogel, K.O. Zell, P. von Brentano, D. Bazzacco, C. Rossi-Alvarez, P. Pavan, D. De Acuna, G. De Angelis, and M. De Poli, *Phys. Rev. C* **54**, R2119 (1996).
- [26] A. Guessous, N. Schulz, W.R. Phillips, I. Ahmad, M. Bentaleb, J.L. Durell, M.A. Jones, M. Leddy, E. Lubkiewicz, L.R. Morss, R. Piepenbring, A.G. Smith, W. Urban, and B.J. Varley, *Phys. Rev. Lett.* **75**, 2280 (1995).