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The role of the intrinsic E2 matrix element between the first two 0^+ states in their configuration mixing in ^{100}Zr

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Abstract

Shape coexistence in ^{100}Zr is a well-known phenomenon. However, a consistent description of the configuration mixing between two shapes with very different deformation has not been made. In this work, the $B(\mathrm{E2}, 2_2^+ \to 0_2^+)$ value in ^{100}Zr has been inferred from the known γ-ray branching ratios between the intraband transition and the interband transitions to members of the ground-state band. The absolute scale of the latter was established under the assumption that adjustments to the interband transitions due to the coupling between the rotation and intrinsic motions can be approximated by a perturbation expansion of angular-momentum dependence. Correction terms up to the second order, which account for the deformation difference between two 0^+ bands, were considered in the description. The intrinsic E2 matrix elements for the second 0^+ state at 331 keV is derived to be ≈ 0.53 eb from the $B(\mathrm{E2}, 2_2^+ \to 0_2^+)$ value assuming a rotational relationship. This suggests that the weakly deformed 0_2^+ state coexists with the strongly deformed ground state of $\langle 0_1|\mathrm{E2}|0_1\rangle \approx 1.06$ eb. Configuration mixing between the two 0^+ states has been studied using both the known E0 and the intrinsic E2 matrix elements derived from this work. A weak mixing of $\approx 7.7\%$ was found, which is nearly a factor of two lower than suggested by a previous analysis. Quantitative evidence of the energy shift for the ground state, deduced from the systematics of transition energies for the yrast states, is consistent with this weak mixing scenario. Near-spherical and well-deformed shapes with the intrinsic E2 matrix elements of ≈ 0.37 eb and ≈ 1.14 eb, respectively, are identified as the basis states before the mixing takes place. The intrinsic E2 matrix element between those two unperturbed 0^+ states, ≈ 0.19 eb, is required to describe their configuration mixing.

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The sudden onset of the quadrupole deformation occurring at neutron-rich Zr nuclei is a well-known phenomenon and can be understood in the framework of shape coexistence [1]. One common feature among the Zr isotopes is the existence of a low-lying excited 0^+ intruder state. The systematics of the intruder state excitation energy, which decreases with increasing neutron number, except for

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^{96}Zr, hints at a crossover of the two coexistent configurations happening at ^{100}Zr and beyond. It is evident that this intruder state is highly deformed from both the excitation energy of the first 2^+ state and the enhanced $B(\mathrm{E2}, 2_1^+ \to 0_1^+)$ value for ^{100}Zr, in contrast to the near-spherical ground states for Zr isotopes with $N \leqslant 58$. Such an interpretation was first proposed by Sheline et al. [2]. Since then, the experimental effort had focussed on the investigation of the E0 strength between the two 0^+ states [3,4] and the enhanced E0 strength often had been interpreted as an indication for the occurrence of shape coexistence.

Systematic study of the E0 strength shows that the most enhanced transitions occur for transitional nuclei where the strongest mixing happens between near-spherical and well-deformed configurations [5]. This leads to the conclusion that the enhanced E0 strength is not an indication of shape coexistence but rather of a strong mixing between two configurations with very different deformation. However, it was pointed out in Refs. [6,7] that the strength of E0 transition between two 0^+ states depends on not only the mixing strength, but also their absolute deformation difference. Their work showed that the mixing strength on the order of 10–14% can explain the enhanced E0 transitions observed in ^{98}Sr and ^{100}Zr. An extended review of the manifestation of E0 transitions and shape mixing for various regions of nuclei has been published recently [8].

In deriving the mixing strength for both ^{98}Sr and ^{100}Zr, Ref. [6] assumed that the term for the intrinsic E2 strength between the two unperturbed 0_u^+ states, $\langle 0_{1,u}|\mathrm{E2}|0_{2,u}\rangle$, can be ignored despite the non-zero term for the interaction strength, $\langle 0_{1,u}|H|0_{2,u}\rangle$, where $|0_{1,u}\rangle$ and $|0_{2,u}\rangle$ represent the unperturbed first and second 0^+ states, respectively. This shortcoming is reflected by the significant discrepancy between the derived energy shift for the ground state and the observed one from the systematics of transition energies for the yrast states [9].

A reanalysis has been carried out for the configuration mixing of the two 0^+ states in ^{100}Zr to evaluate the impact on the early results due to the missing term for the intrinsic E2 strength, $\langle 0_{1,u}|\mathrm{E2}|0_{2,u}\rangle$. Note that the present study is limited to ^{100}Zr; it is not extended to either ^{98}Sr or ^{102}Mo because the experimental data on their decay branching ratios for the second 2^+ states are incomplete. In a two-state-mixing model, the wavefunctions for the observed first and second 0^+ states can be expressed as

$$|0_1\rangle = \sqrt{1-a^2}\,|0_{1,u}\rangle - a|0_{2,u}\rangle, \tag{1a}$$

$$|0_2\rangle = a|0_{1,u}\rangle + \sqrt{1-a^2}\,|0_{2,u}\rangle, \tag{1b}$$

where a is the mixing amplitude. The variables to be determined in this configuration mixing calculation are the intrinsic E2 matrix elements for both 0_u^+ states, the one between them, and the mixing amplitude. The latter determines the interaction matrix element and the energy shift for the two interacting states.

The calculation would be reduced to solving a set of linear equations if the wavefunctions were available. This set of linear equations are solvable if the intrinsic E2 matrix elements for the observed 0^+ states, $\langle 0_1|\mathrm{E2}|0_1\rangle$ and $\langle 0_2|\mathrm{E2}|0_2\rangle$, and the one between them, $\langle 0_1|\mathrm{E2}|0_2\rangle$, are available. The wavefunctions basically are determined by the mixing amplitude, which, in turn, can be determined from the E0 strength [6] by the equation

$$\left|\rho\left(\mathrm{E0}, 0_2^+ \to 0_1^+\right)\right| = \frac{3Ze}{4\pi} a\sqrt{1-a^2}\left|\beta_{1,u}^2 - \beta_{2,u}^2\right|, \tag{2}$$

where $\beta_{1,u}$ and $\beta_{2,u}$ are the deformation parameters for the unperturbed first and second 0^+ states, respectively. The β's are assumed to be related to the quadrupole moment [10,11] by the equation

$$Q = 0.757\, ZR^2\beta(1 + 0.16\beta), \tag{3}$$

where Z is the atomic number and $R = 1.2A^{1/3}$ in fm.

The trial wavefunctions can be generated according to Eq. (2) if both $\beta_{1,u}$ and $\beta_{2,u}$ are given. This can be accommodated initially by assuming the β's for the unperturbed 0_u^+ states to be equal to those for the observed 0^+ states. The configuration mixing calculation thus can be carried out to obtain the intrinsic quadrupole moments for the both 0_u^+ states, which determines the β_u's according to Eq. (3). A new set of trial wavefunctions are constructed with the new set of β_u's. This procedure is iterated, which recalculates the intrinsic matrix elements, thus the β_u's, until the convergence for the final results is reached.

Out of three quantities needed for the configuration mixing calculation, only the intrinsic E2 matrix element for the ground state is known despite the extensive body of the electromagnetic data for the low-lying

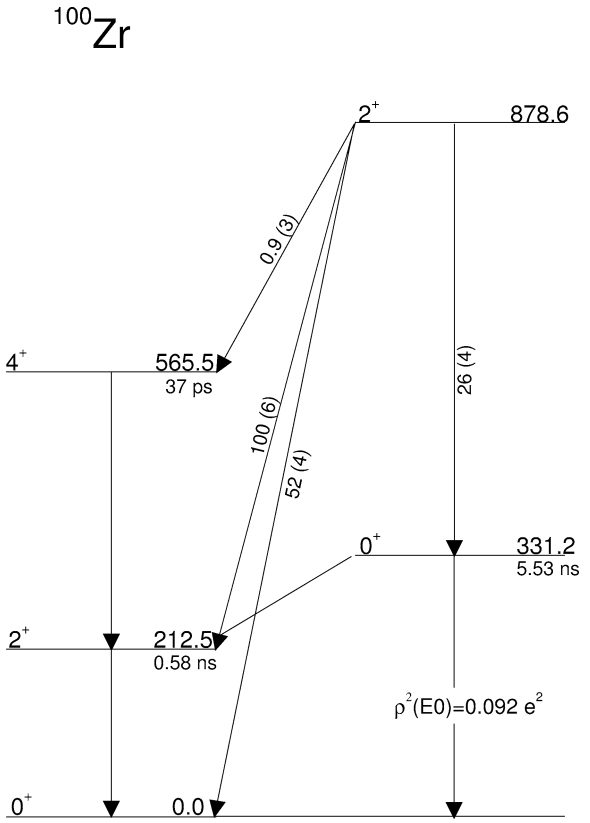

Fig. 1. Partial level scheme of ^{100}Zr. The known half-lives and γ-ray branching ratios for the second 2^+ decay also are listed. The $2_2^+ \to 2_1^+$ transition has both E2 and M1 components and the measured mixing ratio is $+1.0(3)$.

states in ^{100}Zr [12]. The available data include the lifetimes for first 2^+ and 4^+ states as well as the first excited 0^+ state. Other quantities, such as the E0 strength between the first excited 0^+ state and the ground state and the γ-ray branching ratios for the decay of the second 2^+ state, also are available. A summary of the known quantities is listed in Fig. 1. Acquiring the remaining quantities, $\langle 0_2|\mathrm{E2}|0_2\rangle$ and $\langle 0_1|\mathrm{E2}|0_2\rangle$, requires a knowledge of the absolute E2 strength for the second 2^+ decay. The procedure to obtain them is described below.

The interband matrix elements between the two 0^+ states (or bands) are driven by the intrinsic matrix element, $\langle 0_1|\mathrm{E2}|0_2\rangle$. Their adjustments to the Coriolis coupling effect and the deformation difference can be approximated by introducing the higher-order correction terms, which are described by the equation similar to Eq. (4-235) in Ref. [10]

$$\begin{aligned}
&\sqrt{B(\mathrm{E2};\, I_i K = 0_2 \to I_f K = 0_1)} \\
&\quad = \langle I_i 020 | I_f 0\rangle \\
&\qquad \times \Big\{ M_1 - M_2\big(I_f(I_f+1) - I_i(I_i+1)\big) \\
&\qquad + M_3\Big[\big(I_f(I_f+1) - I_i(I_i+1)\big)^2 \\
&\qquad - 2\big(I_i(I_i+1) + I_f(I_f+1)\big)\Big]\Big\} \qquad (4)
\end{aligned}$$

with

$$M_1 = \langle 0_1|\mathrm{E2}|0_2\rangle, \qquad (5a)$$

$$M_2 = (5/16\pi)^{1/2} e Q \langle 0_1|\varepsilon|0_2\rangle, \qquad (5b)$$

$$\frac{M_3}{M_2} = \frac{1}{12}\frac{Q(K=0_2)}{Q(K=0_1) - Q(K=0_2)}, \qquad (5c)$$

where Q is the intrinsic quadrupole moment and $\langle 0_1|\varepsilon|0_2\rangle$ is the reduced mixing amplitude.

Eqs. (4) and (5) were used to correlate the interband transitions of the second 2^+ state to members of the ground-state band. A self-consistent fit to the data cannot be made without the M_3 term and the independent determination of M_1, M_2, and M_3 was not possible because the available data are not sufficient and accurate enough. Note that the branching ratio for the $2_2^+ \to 4_1^+$ transition was printed incorrectly in both compilations [12,13]; the actual value [4] is ten times smaller than the printed one. In addition, this transition was observed only in the single γ-ray data but not in the γ–γ coincident data [4]. This dilemma, however, can be overcome by fixing the M_2/M_1 ratio according to the known decay branching ratios of the second 2^+ state as shown in Fig. 2. The absolute scale, that is M_1, can be determined by the known $0_2^+ \to 2_1^+$ strength. M_3 is then determined from the deformation difference according to Eq. (5c). This procedure is iterated, which recalculates M_3 after the absolute scale is reset, until the convergence for the absolute scale is reached.

The converged values are 0.34 eb and -0.0055 for M_1 and M_3/M_1, respectively, with the $M_2/M_1 = -0.067$ fixed to the branching ratios between the $2_2^+ \to 2_1^+$ and the $2_2^+ \to 0_1^+$ transitions, where the corrections due to the M_3 term are expected to be small. The converged intrinsic matrix element for the second 0^+ state is found to be 0.53 eb compared with 1.06 eb for the ground state. Note that M_1 is the

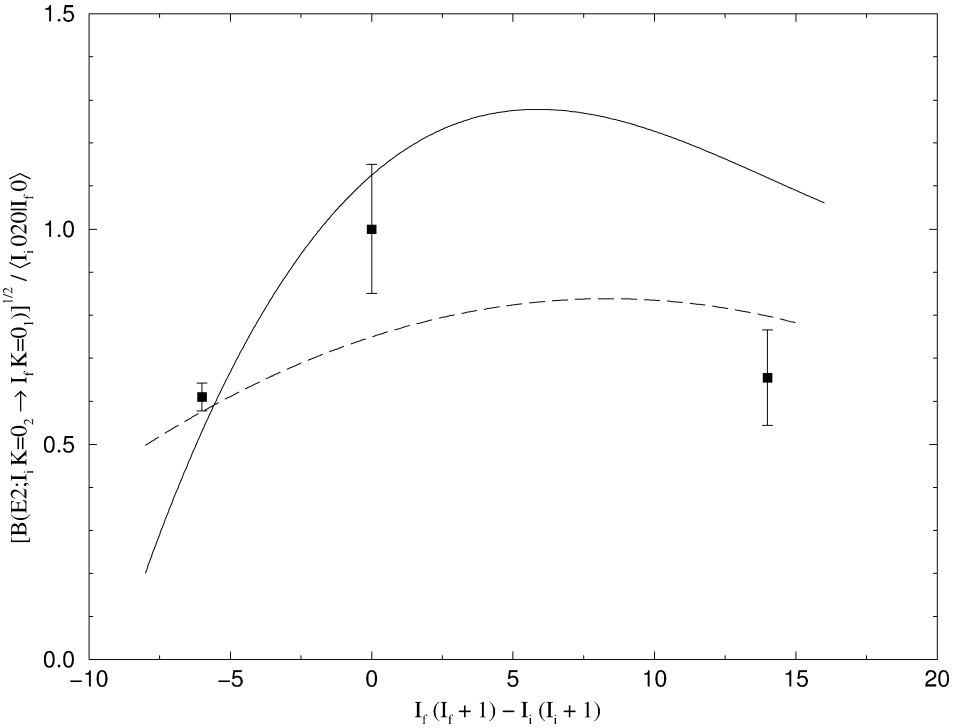

Fig. 2. The Mikhailov plot for the interband transitions between the second 2^+ state and members of the ground-state band. The solid and dashed lines are the values calculated according to Eq. (4) with $M_2/M_1 = -0.067$ and -0.028, respectively.

intrinsic matrix element between the two 0^+ states (or bands). Varying the M_2/M_1 ratio from -0.067 to -0.028, which is the average value of all three data points shown in Fig. 2, changes the derived intrinsic matrix elements, M_1, by no more than 12%. This shows that the results are rather insensitive to the assumption of the M_2/M_1 ratio. From the absolute E2 strength, the half-life of the second 2^+ is predicted to be ≈ 2.9 ps, which is consistent with the upper limit of 10 ps set by the experiment reported in Ref. [7]. It would be interesting if this lifetime can be confirmed.

The configuration mixing calculation was carried out with these intrinsic matrix elements for the observed 0^+ states by using the iteration method mentioned earlier. The results are listed in Table 1. A weak mixing with a strength $\approx 7.7\%$ is obtained, which is about a factor of two lower than the early results [6] obtained ignoring the intrinsic E2 matrix elements between the two unperturbed 0^+_u states. The determined energy shift for both 0^+ states is ≈ 26 keV, which is about half of that determined in Ref. [6], and agrees with ≈ 21 keV extrapolated from the systematics of transition energies for the yrast states [9]. The deformation parameter β is determined to be ≈ 0.37 for the unperturbed ground state and ≈ 0.12 for the unper-

Table 1
Results of the configuration mixing calculation for the two interacting 0^+ states in ^{100}Zr

Mixing strength a^2	0.077
Interaction matrix element	
$\langle 0^+_{1,u}\|H\|0^+_{2,u}\rangle$	88 keV
Energy shift ΔE	26 keV
Intrinsic matrix elements for the unperturbed basis states	
$\langle 0^+_{1,u}\|\mathrm{E2}\|0^+_{1,u}\rangle$	1.14 eb
$\langle 0^+_{2,u}\|\mathrm{E2}\|0^+_{2,u}\rangle$	0.37 eb
$\langle 0^+_{1,u}\|\mathrm{E2}\|0^+_{2,u}\rangle$	0.19 eb
Intrinsic matrix elements for the observed states	
$\langle 0^+_1\|\mathrm{E2}\|0^+_1\rangle$	1.06 eb
$\langle 0^+_2\|\mathrm{E2}\|0^+_2\rangle$	0.53 eb
$\langle 0^+_1\|\mathrm{E2}\|0^+_2\rangle$	0.34 eb

turbed excited $0^+_{2,u}$; the latter has a similar deformation to those of the ground states for the stable even–even Zr isotopes.

There are two major assumptions made in this study. The first is the determination of the quadrupole deformation for the 0^+_2 state from the E2 matrix element of the $2^+_2 \rightarrow 0^+_2$ transition assuming a rotational relationship. The second is the M_2/M_1 ratio which constitutes the main uncertainty for the conclusion reached in this study. Fortunately, the results are rather insensitive to the latter assumption. As mentioned earlier, the change of M_1 is about 12% for a change of the M_2/M_1 ratio by more than a factor of two, while the change for the inferred matrix elements of the second 2^+ state is about 6%. There is virtually no change for the results from the configuration mixing calculation assuming such a variation of those matrix elements. The removal of the discrepancy of the energy shift between the calculation and the observation indicates that a consistent description of the configuration mixing between two 0^+ states, with very different deformation, requires the consideration of the intrinsic E2 matrix element between them. The latter matrix element provides a measure of the underlying single-particle structure of those two 0^+ states.

In summary, a reanalysis has been performed for the configuration mixing between the ground state and the excited 0^+ state in ^{100}Zr. The major difference between the previous and current analyses is the inclusion of the intrinsic E2 matrix element between the two unperturbed 0^+_u for this work. The key to making this analysis possible is the knowledge of the absolute E2 strength of the second 2^+ state, which was inferred from the systematics of its interband transitions to members of the ground-state band. The introduction of the intrinsic E2 matrix element between the two unperturbed 0^+_u states is shown to be important for an understanding of the configuration mixing in ^{100}Zr.

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