Shape coexistence and their configuration mixing in ^{98}Sr and ^{100}Zr

C. Y. Wu, H. Hua, and D. Cline
Nuclear Structure Research Laboratory, Department of Physics, University of Rochester, Rochester, New York 14627, USA
(Received 18 April 2003; published 22 September 2003)

The configuration mixing between the first two 0^+ states in ^{98}Sr, which have very different deformation, has been reanalyzed by introducing the term involving the intrinsic $E2$ matrix element between them in a two-state-mixing model calculation. A mixing strength of $\approx 2.6\%$ was determined using the known $E0$ strength and the intrinsic $E2$ matrix elements for those two 0^+ states. This mixing strength is nearly a factor of 4 weaker than that of the early analysis. Comparison is made to a similar case of configuration mixing in ^{100}Zr.

DOI: 10.1103/PhysRevC.68.034322 PACS number(s): 23.20.Js, 23.20.Lv, 27.60.+j

Coexistence of two configurations with very different deformation is a well-known phenomenon for nuclei in the region of $Z=40$ and $N=60$. It successfully interprets the sudden onset of the quadrupole deformation occurring at neutron-rich Zr isotope, first proposed by Sheline *et al.* [1]. The monopole strength is usually enhanced between those two coexisting shapes for states with spin 0^+. The interpretation of this enhanced monopole strength in terms of their mixing strength has been the subject of extensive experimental [2–5] and theoretical [6–8] study. A general consensus was reached that the monopole strength between those two 0^+ states is sensitive to not only their mixing strength but also their deformation difference.

By using both the measured $B(E0)$ and $B(E2)$ strengths, mixing strengths of 11% and 14% between the coexistence of 0^+ states for ^{98}Sr and ^{100}Zr, respectively, were derived by a two-state-mixing model calculation [4,5]. However, the deficit of this calculation was pointed out in Ref. [9] that the calculated energy shifts are more than twice that of the observed shifts, which were extrapolated from the energy systematics of the higher-spin states.

In a recent reanalysis of the configuration mixing between the two 0^+ states in ^{100}Zr [10], the energy shift was derived to be ≈ 26 keV, which agrees with the observed energy shift, ≈ 21 keV [9]. A mixing strength of $\approx 7.7\%$ was obtained, which is nearly a factor of 2 weaker than that of the early analysis. The difference between the recent calculation and the earlier one is that the recent calculation has the term involving the intrinsic $E2$ matrix element, $\langle 0_{1,u}|E2|0_{2,u}\rangle$, added to the two-state-mixing model calculation. The symbols $|0_{1,u}\rangle$ and $|0_{2,u}\rangle$ represent the unperturbed first and second 0^+ states, respectively. The success in reproducing the observed energy shift indicates that the intrinsic $E2$ matrix element between the two configurations is important in describing the mixing between two coexistent shapes in ^{100}Zr.

In this paper, a similar reanalysis of the configuration mixing between the two 0^+ states in ^{98}Sr is presented by introducing the term $\langle 0_{1,u}|E2|0_{2,u}\rangle$ to the two-state-mixing model calculations. Since the method of this reanalysis has been published [10], a summary description is given here. In a two-state-mixing model, the wave functions for the observed first and second 0^+ states can be expressed as

$$|0_1\rangle = \sqrt{1-a^2}|0_{1,u}\rangle - a|0_{2,u}\rangle, \tag{1a}$$

$$|0_2\rangle = a|0_{1,u}\rangle + \sqrt{1-a^2}|0_{2,u}\rangle, \tag{1b}$$

where a is the mixing amplitude, which determines the interaction matrix element and the energy shift for the two interacting states.

Quantities to be determined in the two-state-mixing model calculations are the intrinsic $E2$ matrix elements for both 0_u^+ states, the $E2$ matrix element between them, and the mixing amplitude. If the wave functions were available, the calculation would be reduced to solving a set of linear equations given by

$$\langle 0_1|E2|0_1\rangle = (1-a^2)\langle 0_{1,u}|E2|0_{1,u}\rangle - 2a\sqrt{1-a^2}\langle 0_{1,u}|E2|0_{2,u}\rangle + a^2\langle 0_{2,u}|E2|0_{2,u}\rangle, \tag{2a}$$

$$\langle 0_1|E2|0_2\rangle = a\sqrt{1-a^2}\langle 0_{1,u}|E2|0_{1,u}\rangle + (1-2a^2)\times\langle 0_{1,u}|E2|0_{2,u}\rangle - a\sqrt{1-a^2}\langle 0_{2,u}|E2|0_{2,u}\rangle, \tag{2b}$$

$$\langle 0_2|E2|0_2\rangle = a^2\langle 0_{1,u}|E2|0_{1,u}\rangle + 2a\sqrt{1-a^2}\langle 0_{1,u}|E2|0_{2,u}\rangle + (1-a^2)\langle 0_{2,u}|E2|0_{2,u}\rangle. \tag{2c}$$

Note that this set of linear equations is reduced to the equivalence of Eq. (2) in Ref. [4] if the mixing is weak and the intrinsic matrix element between them, $\langle 0_{1,u}|E2|0_{2,u}\rangle$, is ignored. This set of linear equations is solvable if the intrinsic $E2$ matrix elements for the observed 0^+ states, $\langle 0_1|E2|0_1\rangle$ and $\langle 0_2|E2|0_2\rangle$, and the one between them, $\langle 0_1|E2|0_2\rangle$, are available. The wave functions basically are determined by the mixing amplitude a, which, in turn, can be determined from the $E0$ strength [4] by the equation

$$|\rho(E0,0_2^+\rightarrow 0_1^+)| = (3Ze/4\pi)a\sqrt{1-a^2}|\beta_{1,u}^2 - \beta_{2,u}^2|, \tag{3}$$

where $\beta_{1,u}$ and $\beta_{2,u}$ are the deformation parameters for the unperturbed first and second 0^+ states, respectively. The β's are assumed to be related to the quadrupole moment [11,12] by the equation

$$eQ = \sqrt{16\pi/5}\langle 0|E2|0\rangle = 0.757eZR^2\beta(1+0.16\beta) \tag{4}$$

where Z is the atomic number and $R=1.2A^{1/3}$ in femtometer.

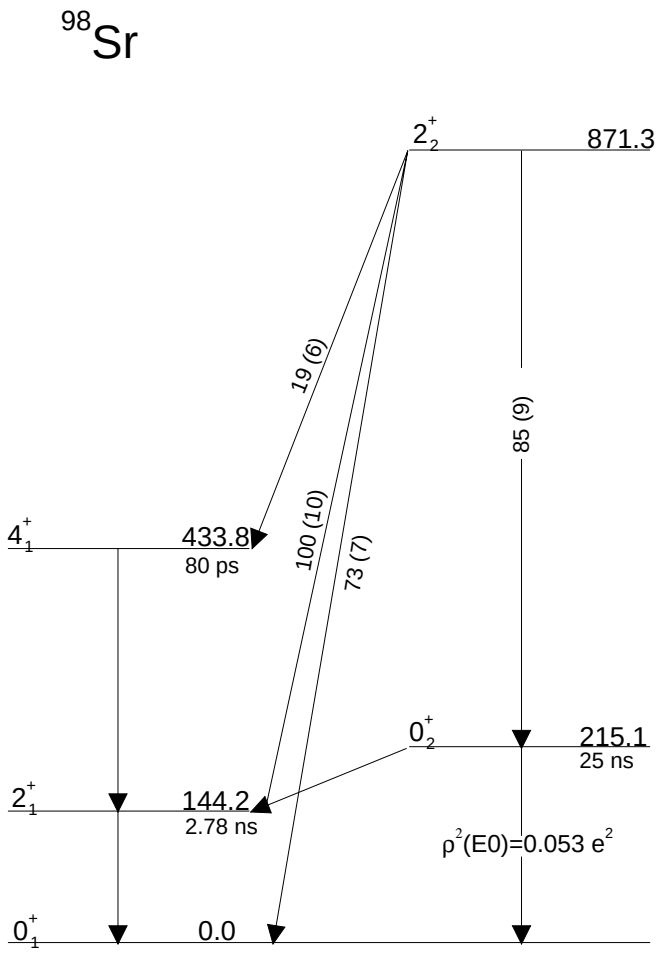

FIG. 1. Partial level scheme of ^{98}Sr. Energies are in keV. The known half-lives and γ-ray branching ratios for the second 2^+ decay also are listed. The $2_2^+ \rightarrow 2_1^+$ transition is assumed to be pure $E2$ in the calculations.

The trial wave functions can be generated according to Eq. (3) if both $\beta_{1,u}$ and $\beta_{2,u}$ are given. This can be accommodated initially by assuming the β's for the unperturbed 0_u^+ states to be equal to those for the observed 0^+ states. The configuration mixing calculation thus can be carried out to obtain the intrinsic quadrupole moments for both the 0_u^+ states, which determines the β_u's according to Eq. (4). A new set of trial wave functions are constructed with the new set of β_u's. This procedure is iterated, which recalculates the intrinsic matrix elements, thus the β_u's, until convergence is reached.

The known electromagnetic properties for the low-lying states in ^{98}Sr [13] are listed in Fig. 1 and in Table I together with those of ^{100}Zr. The intrinsic $E2$ matrix element for the ground state, $\langle 0_1|E2|0_1\rangle$, can be determined from the measured lifetimes assuming a rotational relationship. Acquiring two other quantities $\langle 0_2|E2|0_2\rangle$ and $\langle 0_1|E2|0_2\rangle$ requires a knowledge of the absolute $E2$ strength for the second 2^+ decay. Only the branching ratios for the latter decay are available. However, the absolute scale can be established if the second 2^+ is a rotational state. Under such an assumption, adjustments to the interband transitions due to the coupling between the rotation and intrinsic motions can be approximated by a perturbation expansion of the angular-momentum dependence [11]. Correction terms up to the second order, which account for the deformation difference between two 0^+ states, are considered in the description.

TABLE I. Absolute $E0$ and $E2$ strengths and relative $B(E2)$ values used for the current analysis of both ^{98}Sr and ^{100}Zr.

Transition	^{98}Sr	^{100}Zr
	$\rho^2(E0)(e^2)$	
$0_2^+ \rightarrow 0_1^+$	0.053(9)	0.092(17)
	$B(E2)(e^2\ \mathrm{b}^2)$	
$2_1^+ \rightarrow 0_1^+$	0.26(1)	0.23(1)
$0_2^+ \rightarrow 2_1^+$	0.15(1)	0.18(2)
	$B(E2)$ratio	
$2_2^+ \rightarrow 0_1^+$	0.30(3)	0.26(3)
$2_2^+ \rightarrow 2_1^+$	1.00	1.00
$2_2^+ \rightarrow 4_1^+$	2.43(76)	0.77(26)
$2_2^+ \rightarrow 0_2^+$	1.43(20)	1.39(23)

The leading order for the interband matrix elements between the two 0^+ states (or bands) is the intrinsic matrix element $\langle 0_1|E2|0_2\rangle$. Their adjustments to the Coriolis coupling effect and the deformation difference can be approximated by introducing higher-order correction terms, which are described by Eq. (5) that is similar to Eq. (4-235) in Ref. [11],

$$\sqrt{B(E2;I_iK=0_2\rightarrow I_fK=0_1)} = \langle I_i020|I_f0\rangle (M_1 - M_2[I_f(I_f+1)-I_i(I_i+1)] + M_3\{[I_f(I_f+1)-I_i(I_i+1)]^2 - 2[I_i(I_i+1)+I_f(I_f+1)]\}) \quad (5)$$

with

$$M_1 = \langle 0_1|E2|0_2\rangle, \quad (6a)$$

$$M_2 = (5/16\pi)^{1/2} eQ\langle 0_1|\varepsilon|0_2\rangle, \quad (6b)$$

$$M_3/M_2 = 1/12\{Q(K=0_2)/[Q(K=0_1)-Q(K=0_2)]\}, \quad (6c)$$

where Q is the intrinsic quadrupole moment and $\langle 0_1|\varepsilon|0_2\rangle$ is the reduced mixing amplitude.

Equations (5) and (6) were used to correlate the interband transitions of the second 2^+ state to members of the ground-state band. Since the independent determination of M_1, M_2, and M_3 was not possible because the available data are not sufficient and accurate enough, the fit was done by fixing the M_2/M_1 ratio according to the known decay branching ratios of the second 2^+ state as shown in Fig. 2. Note that the $2_2^+ \rightarrow 2_1^+$ transition was assumed to be pure $E2$ because the mixing ratio has not been measured. The absolute scale, that is M_1, can be determined by the known $0_2^+ \rightarrow 2_1^+$ strength. M_3 is then determined from the deformation difference

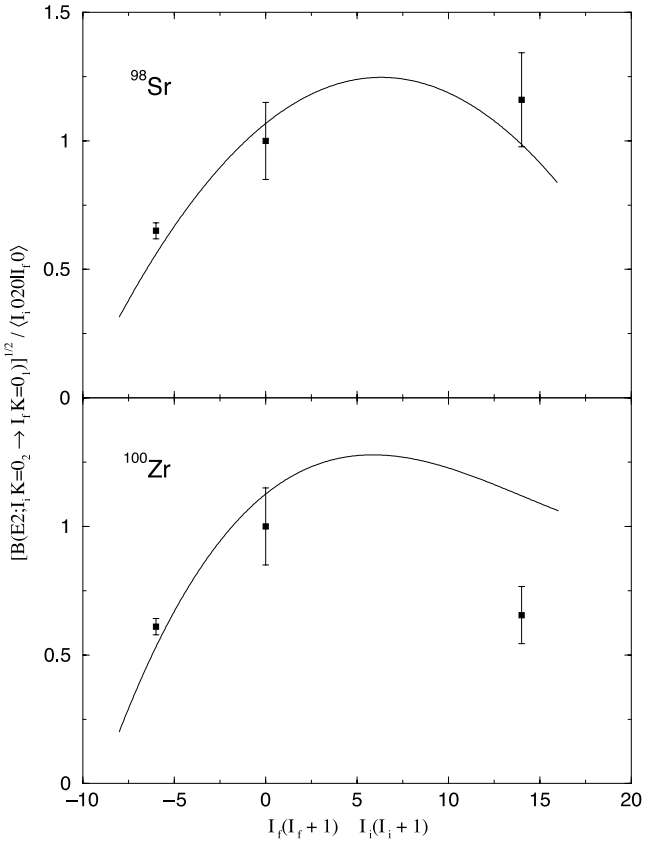

FIG. 2. The Mikhailov plot for the interband transitions between the second 2^+ state and members of the ground-state band. The solid lines are the best fits to the data according to Eq. (5) with the M_2/M_1 ratio fixed to -0.058 and -0.067 for ^{98}Sr and ^{100}Zr, respectively.

according to Eq. (6c). This procedure is iterated, which recalculates M_3 after the absolute scale is reset, until the convergence for the absolute scale is reached.

The converged values are $M_1 = 0.32e$ b and $M_3/M_1 = -0.0057$ with the $M_2/M_1 = -0.058$ fixed to the branching ratios between the $2_2^+ \rightarrow 2_1^+$ and the $2_2^+ \rightarrow 0_1^+$ transitions, where the corrections due to the M_3 term are expected to be small. The converged intrinsic matrix element for the second 0^+ state is found to be $0.53e$ b compared with $1.14e$ b for the ground state. Note that M_1 is the intrinsic matrix element between the two 0^+ states (or bands). A weak dependence of these derived intrinsic matrix elements on the assumption of the M_2/M_1 ratio has been discussed in Ref. [10]. From the absolute $E2$ strength, the half-life of the second 2^+ is predicted to be ≈ 3.1 ps.

The configuration mixing calculation was carried out with these intrinsic matrix elements for the observed 0^+ states using the iteration method mentioned earlier. The results, together with those of ^{100}Zr, are listed in Table II. A weak mixing with a strength $\approx 2.6\%$ is obtained for ^{98}Sr, which is about a factor of 4 lower than the early result of 11% [4] that was obtained by ignoring the intrinsic $E2$ matrix elements between the two unperturbed 0_u^+ states. The determined energy shift for both 0^+ states is ≈ 5.5 keV, which is about a factor of 4 less than 23.3 keV determined in Ref. [4], and consistent with ≈ 11 keV extrapolated from the systematics of transition energies for the yrast states [9]. The deformation parameter β is determined to be ≈ 0.42 for the unperturbed ground state and ≈ 0.13 for the unperturbed excited $0_{2,u}^+$ of ^{98}Sr. This is to be compared with ≈ 0.37 for the unperturbed ground state and ≈ 0.12 for the unperturbed excited $0_{2,u}^+$ of ^{100}Zr.

The success of this analysis is due partly to a simple assumption that the second 2^+ is a rotational band member built on the weakly deformed 0_2^+ state. Evidence supporting this assumption includes the agreement between the calculated and measured energy shifts for the 0^+ states and the calculated lifetime of the second 2^+ state being consistent with the upper limit set by the measurement for ^{100}Zr [5]. However, this does not rule out the possibility of a vibrator character for the second 2^+ state. The analysis with such a scenario is beyond the scope of this paper.

The shape coexistence phenomenon is very similar for ^{98}Sr and ^{100}Zr. Both exhibit a strongly deformed ground state and a weakly deformed excited 0^+ state. Both have a

TABLE II. Results of the configuration mixing calculation for the two interacting 0^+ states in ^{98}Sr and ^{100}Zr.

	^{98}Sr	^{100}Zr
Mixing strength a^2	0.026	0.077
Interaction matrix element		
$\langle 0_{1,u}^+ \vert H \vert 0_{2,u}^+ \rangle$ (keV)	34	88
Energy shift ΔE (keV)	5.5	26
Intrinsic matrix elements for the unperturbed basis states		
$\langle 0_{1,u}^+ \vert E2 \vert 0_{1,u}^+ \rangle (e$ b)	1.26	1.14
$\langle 0_{2,u}^+ \vert E2 \vert 0_{2,u}^+ \rangle (e$ b)	0.37	0.37
$\langle 0_{1,u}^+ \vert E2 \vert 0_{2,u}^+ \rangle (e$ b)	0.07	0.19
Intrinsic matrix elements for the observed states		
$\langle 0_1^+ \vert E2 \vert 0_1^+ \rangle (e$ b)	1.14	1.06
$\langle 0_2^+ \vert E2 \vert 0_2^+ \rangle (e$ b)	0.53	0.53
$\langle 0_1^+ \vert E2 \vert 0_2^+ \rangle (e$ b)	0.32	0.34

ratio of the deformation between two coexisting shapes varying from $\approx 2:1$ for the observed states to $\approx 3:1$ for the unperturbed basis states. The weakly deformed excited 0^+ states are nearly identical in deformation for the two nuclei both in the laboratory frame and in the unperturbed basis. The only difference is the interaction strength and the coupling $E2$ matrix elements between two coexisting shapes, which generally reflects the difference in the proton configurations for the two nuclei. The theoretical understanding of this configuration mixing between two coexisting shapes would be highly desirable.

In summary, a reanalysis has been performed for the configuration mixing between the ground state and the first excited 0^+ state in ^{98}Sr under the framework of two-state-mixing model. The major difference between the previous and current analyses is the inclusion of the intrinsic $E2$ matrix element between the two 0^+ states for this work. The importance of this inclusion is demonstrated by the significantly improved reproduction of the energy shift for both ^{98}Sr and ^{100}Zr. The similarity of the magnitude of the deformation between those two nuclei for both coexisting shapes but the disparity of their interaction strengths and coupling $E2$ matrix elements require further theoretical investigation.

This work was supported by the U.S. National Science Foundation.

[1] R.K. Sheline, I. Ragnarsson, and S.G. Nilsson, Phys. Lett. **41B**, 115 (1972).
[2] T.A. Khan, W.-D. Lauppe, K. Sistemich, H. Lawin, and H.A. Selic, Z. Phys. A **284**, 313 (1978).
[3] F.K. Wohn, J.C. Hill, C.B. Howard, K. Sistemich, R.F. Petry, R.L. Gill, H. Mach, and A. Piotrowski, Phys. Rev. C **33**, 677 (1986).
[4] H. Mach, M. Moszynski, R.L. Gill, F.K. Wohn, J.A. Winger, J.C. Hill, G. Molnar, and K. Sistemich, Phys. Lett. B **230**, 21 (1989).
[5] H. Mach, M. Moszynski, R.L. Gill, G. Molnar, F.K. Wohn, J.A. Winger, and J.C. Hill, Phys. Rev. C **41**, 350 (1990).
[6] K. Heyde and R.A. Meyer, Phys. Rev. C **37**, 2170 (1988).
[7] J.L. Wood, K. Heyde, W. Nazarewicz, M. Huyse, and P. Van Duppen, Phys. Rep. **215**, 101 (1992), and references therein.
[8] J.L. Wood, E.F. Zganjar, C. De Coster, and K. Heyde, Nucl. Phys. **A651**, 323 (1999).
[9] J. Hamilton *et al.*, Prog. Part. Nucl. Phys. **35**, 635 (1995).
[10] C.Y. Wu, H. Hua, and D. Cline, Phys. Lett. B **541**, 59 (2002).
[11] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. II.
[12] K.E. Lobner, M. Vetter, and V. Honig, Nucl. Data Tables **7**, 495 (1970).
[13] B. Singh, Nucl. Data Sheets **84**, 565 (1998), and references therein.