PHY 415

Homework 1

Due Sep 21 2009

1. Prove the following relations from vector analysis ($\mathbf{A}$ and $\mathbf{B}$ are three dimensional vectors)

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{A}) - \boldsymbol{\nabla}^2 \mathbf{A},$$

$$\boldsymbol{\nabla} \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\boldsymbol{\nabla} \times \mathbf{A}) - \mathbf{A} \cdot (\boldsymbol{\nabla} \times \mathbf{B}),$$

$$\boldsymbol{\nabla} \cdot \mathbf{r} = 3,$$

$$\boldsymbol{\nabla}^2 \left(\frac{1}{|\mathbf{r}|} \right) = -4\pi \delta^3(\mathbf{r}).$$

2. Consider the hypothetical case that a point charge q at the origin produces the Yukawa potential given by

$$\Phi(\mathbf{r}) = q \, \frac{e^{-\mu r}}{r},$$

where $r = |\mathbf{r}|$ and μ is a mass parameter (in units of $c, \hbar$). Is the Gauss' law valid for such a case? Show that, for $\mathbf{r} \neq 0$, this potential satisfies the equation

$$\boldsymbol{\nabla}^2 \Phi(\mathbf{r}) = \mu^2 \Phi(\mathbf{r}).$$

3. Calculate the electric field inside and outside of a solid sphere of radius R carrying a constant volume charge density ρ.

4. Consider a spherical region V of radius R without any charge and a point charge q outside the spherical region at a distance r from the center as shown in the figure. Evaluate explicitly the net electric flux out of the surface S which bounds the spherical region V.