PHY 415

Homework 5

Due October 21, 2009

1. Consider a conducting sphere of radius R whose surface is maintained at a potential $\Phi(\mathbf{R}) = \Phi_0 \cos\theta$. Assuming that there are no charges present (inside or outside), what is the potential inside and outside the sphere?

2. Consider a cylindrical conducting can of radius R and height h. The side and the bottom walls of the can are grounded while the top face is maintained at $\Phi_0(r, \phi)$. Find the electrostatic potential inside the can. (You will have to solve the Laplace equation in cylindrical coordinates, which would lead to Bessel functions.)

3. Let us assume that the potentials Φ_1 and Φ_2 are produced by the charge distributions (ρ_1, σ_1) and (ρ_2, σ_2) respectively. Namely, both volume and surface charge distributions are responsible for the potentials that correspond to solutions of the Poisson equations

$$\begin{aligned}\boldsymbol{\nabla}^2\Phi_1 &= -4\pi\rho_1,\\ \boldsymbol{\nabla}^2\Phi_2 &= -4\pi\rho_2.\end{aligned}$$

Using Green's identity derived in class, prove the reciprocity theorem

$$\int_V \mathrm{d}^3r\ \rho_1\Phi_2 + \int_S \mathrm{d}s\ \sigma_1\Phi_2 = \int_V \mathrm{d}^3r\ \rho_2\Phi_1 + \int_S \mathrm{d}s\ \sigma_2\Phi_1.$$

Using this reciprocity theorem, determine the total induced charges on each of two infinite grounded conducting plates separated by a distance d, with a point charge q in the space between them.