PHY 415

Homework 6

Due November 5, 2009

1. Consider the one dimensional wave equation

$$\frac{\partial^2 \Phi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \Phi}{\partial t^2} = 0,$$

where v represents the speed of propagation of the wave. This is a hyperbolic equation, and at every point in space-time, there are two characteristics $\xi(x,t)$ and $\eta(x,t)$.

a) Determine the characteristics as functions of (x,t).

b) Invert the relations for the characteristics to express x and t in terms of the characteristics ξ and η. What is the form of the wave equation written in terms of the characteristics as the independent coordinates.

c) With the Cauchy conditions $\Phi(x, t = 0) = f(x)$ and $\frac{\partial \Phi}{\partial t}\big|_{t=0} = g(x)$, determine the solution to the wave equation.

2. Using Green's identity, as well as other identities discussed in the class, show that the Green's function satisfying Dirichlet boundary conditions is symmetric in the interchange of its arguments (namely, $G_D(\mathbf{r}, \mathbf{r}') = G_D(\mathbf{r}', \mathbf{r})$). (In general, Neumann Green's functions are not symmetric.)

3. Consider a dielectric sphere of radius R and permittivity ϵ placed in vacuum. A point charge q is located outside the sphere at a distance $r = d > R$. Determine the electric field due to this charge both inside and outside the sphere. (A solution using the method of "images" can be found in Am. J. Phys. **61** (1993) 39. However, you can solve the Laplace equation with the appropriate matching condition to determine the solution.)