Problem 1

a.

$\nabla \times E_1 \neq 0$ (not static field)

$\nabla \times E_2 = 0$ (static field)

b.

$\phi = -\int_0^r \vec{E} \cdot dl$

$= -(\int_{(0,0,0)}^{(x,0,0)} E_x dx + \int_{(x,0,0)}^{(x,y,0)} E_y dy + \int_{(x,y,0)}^{(x,y,z)} E_x dx)$

$= -k(xy^2 + yz^2)$

Problem 2

a.

$W = \frac{1}{8\pi} \int |\vec{E}|^2 d^3 r$

$= \frac{1}{8\pi} \int_R^\infty |\frac{4\pi\sigma}{r^2}|^2 r^2 \sin\theta d\theta d\phi dr$

$= 8\pi^2 R^3 \sigma^2$

b.

As R decreases, W also decreases

c.

use Gauss's Law

$4\pi^2 E_{in} = 4\pi(\frac{4}{3}\pi r^3 \rho)$

$\Rightarrow E_{in} = \frac{4\pi r\rho}{3}$

$4\pi r^2 E_{out} = 4\pi(\frac{4}{3}\pi R^3 \rho)$

$\Rightarrow E_{out} = \frac{4\pi R^3 \rho}{3r^2}$

$W = \frac{1}{2}[\int_0^R \frac{16\pi^2\rho^2}{9} r^4 dr + \int_R^\infty \frac{16\pi^2 R^6 \rho^2}{9r^4} r^2 dr]$

$= \frac{16}{15}\pi^2 \rho^2 R^5$

Problem 3

$\oint_s \vec(E) \cdot \sigma = 4\pi \int \int kr^{-n} r^2 \sin\theta dr d\theta d\phi$

$\Rightarrow$

$4\pi r^2 E_{out} = 16\pi^2 \int_0^R r^2 \sin\theta dr d\theta d\phi$

$E_{out} = \frac{4\pi k}{r^2(3-n)} R^{3-n}$

$4\pi r^2 E_{in} = 4\pi \int \int_0^r kr^{-n+2} dr$

$E_{in} = \frac{4\pi k}{3-n} r^{1-n}$

$(n > 3 diverge)$

$W = \frac{1}{8\pi} \int |\vec{E}|^2 d^3\vec{r}$

$= \frac{1}{2} \frac{16\pi^2 k^2}{(3-n)^2} [\int_0^R r^{4-2n} dr + \int_R^\infty R^{6-2n} \frac{1}{r^2} dr]$

where

$\int_0^R r^{4-2n} dr = \frac{R^{5-2n}}{5-2n}$ if $n < 5/2$

if $n > 5/2 \Rightarrow$ diverge

and
$\int_R^\infty R^{6-2n} \frac{1}{r^2} dr = R^{5-2n}$

$\Rightarrow W = \frac{16\pi^2 k^2 R^{5-2n}}{(3-n)(5-2n)}$ for $n < 5/2$

Problem 4

$W = -\vec{p} \cdot \vec{E}, \tau = 2q\vec{r} \times \vec{E} = \vec{p} \times \vec{E}$

$\Delta W = (-pE\cos\theta) - (-pE\cos 0)$

$= pE(1 - \cos\theta)$