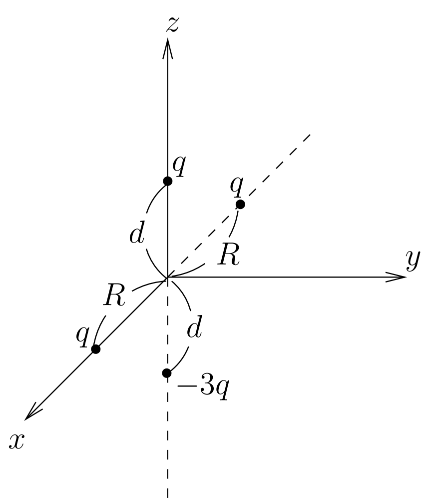
PHY 415 E&M
Homework 3 Solution

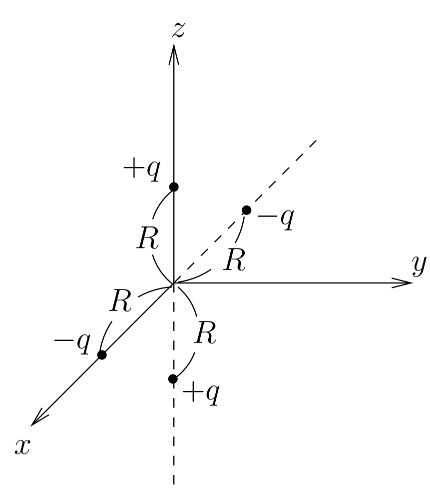
1. (a)



$$
\begin{aligned}
\vec{\rho} &= \int d^3\vec{r}\rho(\vec{r})\vec{r} \\
&= qd\hat{z} + qR\hat{x} - qr\hat{x} + 3qd\hat{z} \\
&= 4qd\hat{z}
\end{aligned}
$$

□

(b)



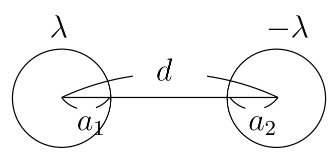
$$
\begin{aligned}
\rho(\vec{r}) &= q\delta(x)\delta(y)\delta(z-R) + q\delta(x)\delta(y)\delta(z+R) \\
&\quad -q\delta(x-R)\delta(y)\delta(z) - q\delta(x+R)\delta(y)\delta(z)
\end{aligned}
$$

$$
\begin{aligned}
Q_{ij} &= \int d^3r(3x_ix_j - \delta_{ij}r^2)\rho(\vec{r}) \\
Q_{11} &= \int d^3r(3x^2 - r^2)\rho(\vec{r}) \\
&= \int (2x^2 - y^2 - z^2)\rho(\vec{r})d^3r \\
&= -2qR^2 - q2R^2 - q2R^2 \\
&= -6qR^2 \\
Q_{33} &= \int d^3r(3z^2 - r^2)\rho(\vec{r}) \\
&= \int (2z^2 - x^2 - y^2)\rho(\vec{r})d^3r \\
&= +6qR^2 \\
Q_{22} &= Q_{12} = Q_{13} = Q_{32} = 0
\end{aligned}
$$

$$
Q_{ij} = \begin{pmatrix} -6qR^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6qR^2 \end{pmatrix}
$$

□

2.



$$
\begin{aligned}
2\pi r\vec{E} &= 4\pi\lambda \\
\Rightarrow \quad \vec{E} &= \frac{2\lambda}{r}\hat{r}
\end{aligned}
$$

$$
\begin{aligned}
\phi &= \phi_I + \phi_{II} \\
&= \int_{a_1}^{d-a_2} \frac{2\lambda}{r}\hat{r}\cdot d\vec{r} + \int_{d-a_1}^{a_2} \frac{-2\lambda}{r}\hat{r}\cdot d\vec{r} \\
&= 2\lambda \ln\frac{d-a_2}{a_1} - 2\lambda\ln\frac{a_2}{d-a_1} \\
&= 2\lambda\ln\frac{(d-a_1)(d-a_2)}{a_1a_2}
\end{aligned}
$$

$$
\begin{aligned}
d &\gg a_1, a_2 \\
\therefore \quad \phi &= 2\lambda\ln\frac{d^2}{a_1a_2}
\end{aligned}
$$

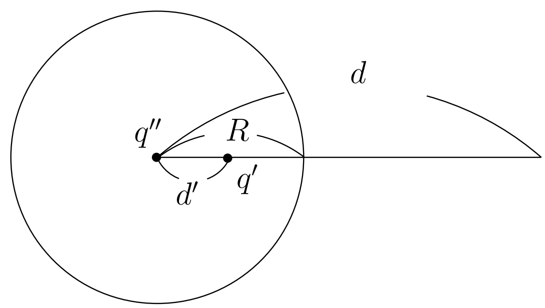
$$
c = \frac{Q}{\phi} = \frac{1}{2\ln\frac{d^2}{a_1a_2}}
$$

$$
\begin{aligned}
2c\ln\frac{d^2}{a_1a_2} = 1 \quad \Rightarrow \quad \ln\frac{d^2}{a^2} &= \frac{1}{2c} \\
\Rightarrow \quad a^2 &= d^2e^{-\frac{1}{2c}} \\
\Rightarrow \quad a &= de^{-\frac{1}{4c}} \\
c = 9\times10^{-3}, d = 0.5\text{cm} & \\
\Rightarrow \quad a &= 4.3\times10^{-13}\text{cm}
\end{aligned}
$$

□

3. Lectures on Electromagnetism (Ashok Das) P.76

(a)



$$
\begin{aligned}
q' &= -\frac{R}{d}q \\
d' &= \frac{R^2}{d} \\
\frac{q''}{R} &= \Phi_0, q'' = R\Phi_0 \ \text{ at center}
\end{aligned}
$$

□

(b)

$$
\Phi(\vec{r}) = q\left(\frac{1}{(r^2+d^2-2rd\cos\theta)^{\frac{1}{2}}} - \frac{R}{(r^2d^2+R^4-2rdR^2\cos\theta)^{\frac{1}{2}}}\right) + \frac{\Phi_0R}{r}
$$

□

(c)

$$
\begin{aligned}
\sigma &= \frac{1}{4\pi}\hat{r}\cdot\vec{E}(\vec{r})\Big|_{r=R} \\
&= \frac{-1}{4\pi}\nabla\phi(\vec{r})\Big|_{r=R} \\
&= \frac{-q(d^2-R^2)}{4\pi R(d^2+R^2-2dR\cos\theta)^{\frac{3}{2}}} + \frac{1}{4\pi}\frac{\Phi_0}{R} \\
Q_{induced} &= \int R^2\sin\theta d\theta d\phi\sigma(r=R,\theta,\phi) \\
&= -q\left(\frac{R}{d}\right) + \Phi_0R
\end{aligned}
$$

□