Today in Astronomy 102: relativity

- Measurement of physical quantities, reference frames, and space-time diagrams.
- Relative and absolute physical quantities.
- Classical physics and Galileo’s theory of relativity.
- An apparently small problem in classical physics leads Einstein to a revolutionary solution.

Albert Einstein, circa 1905

Introduction to Einstein’s theories of relativity

In the next five lectures we will discuss Einstein's relativity theories and some interesting and important predictions made from them. Our goals are:

- to understand the nature of the theories, and the properties of space and time at high speeds and in strong gravity, and
- to understand these things well enough that you can make the basics of relativity understandable to others who have not taken a course like this.

Einstein at Caltech, 1933

Introduction to Einstein’s theories of relativity

...the special theory of relativity (1905) and the general theory of relativity (1915), by which the existence and properties of black holes are predicted.

We shall dispel two popular misconceptions about Einstein’s theories at the outset:

- **Einstein does not mean that “everything is relative.”** The theories leave many physical quantities absolute; for example, the speed of light.
- **Relativity existed before Einstein.** Galileo and Newton had another theory of relativity. The differences between Einstein’s relativity and that which preceded his theories are simply which physical quantities the theories take to be relative or absolute.
Reference frames

A reference frame (or “frame of reference”) consists of an observer, like you, and a hypothetical bunch of instruments that can measure length, time, etc., all in the same state of motion (not moving with respect to each other).

- An inertial reference frame is one whose state of motion is not influenced by any external forces.
- Observers in different reference frames can, by use of their instruments, measure things they see in each other’s frame, and report their results to each other.
- It is often useful to assign coordinate systems to each observer, who sits at the origin of his or her system.

A reference frame

Two reference frames in relative motion

Frame 2

Frame 1
It doesn’t matter which frame is called 1 and which is called 2.

This picture is physically the same as the previous one.

Examples: measurements by two observers in different reference frames, moving with respect to each other.

Observer in Frame 1 sees Frame 2 moving east at speed $V$ (similarly, the observer in Frame 2 sees Frame 1 moving west at speed $V$).

- Observer in Frame 2 holds up a meter stick (horizontally), flashes a light at the beginning and end of a certain time interval, and rolls a ball horizontally across his floor.
- Observer in Frame 1 takes pictures, measures time intervals, etc. and determines how long the meter stick, how long the time interval, how fast the ball, appears to her.

What do you think she’ll find? (Turns out: answers given by Galileo and Newton would differ from those given by Einstein, if $V$ is large enough.)
To Observer #1: how long does the meter stick look, how long between flashes of the light, and how fast does the ball appear to roll?

Those math symbols

The use of some common mathematical shorthand is just too convenient to avoid.

- **Subscripts** will be used here to denote an observer by whom a measurement is made; for example, \( d_1 \) is a distance measured by observer 1, and \( v_2 \) is a velocity measured by observer 2.
- **Capital delta (Δ)** will be used to denote a change in some quantity: for example, \( Δt_1 \) is an interval of time \( t \) measured by observer 1, \( Δx \) and \( Δy \) are distances measured along the \( x \) and \( y \) directions, respectively, and \( ΔE \) is a change in the energy \( E \) of some system.
- \( x \) and \( y \) give the location of a point in the coordinate system: the distance of that point from the \( y \) and \( x \) axis.

Space-Time diagrams: a way to report distance and time measurements made within one reference frame.
Absolute and Relative physical quantities

Suppose two observers are moving with respect to each other, and able to measure the same set of physical quantities, like our Observers #1 and #2.

- For some physical quantities, their results – numerical values and units – will be the same. These quantities are called absolute: measurements of them give the same result no matter how the observer is moving.
- If the observers measure different values of a given physical quantity, that quantity is called relative: measurements yield different results for observers in different states of motion.

Choose the absolute quantity

One of these will turn out to be absolute, and the other three relative, in Einstein’s relativity. Can you guess which?

A. The order in which cause and effect occur.  B. Time itself.

Mid-lecture Break.

- Homework #1 is (still) due at 5:30 PM EDT on Saturday.

Galaxy NGC 891, by David Malin (Anglo-Australian Observatory)
Old ("Classical") Physics

Three main fields:
- Mechanics and gravitation: Newton’s laws
- Electricity, magnetism and light: Maxwell’s equations
- Heat and thermodynamics: Gibbs and Boltzmann

Each consists of a small number of laws, mostly expressed as mathematical formulas. By mathematically manipulating these formulas, and plugging numbers in, the results of experiments can be predicted, or new effects can be envisioned.

Sir Isaac Newton, c. 1702.

Old ("Classical") Physics (continued)

These theories were fantastically accurate, and were used successfully to predict important discoveries, and to invent new technologies. Examples:
- Discoveries of Uranus and Neptune
- Discovery of radio waves
- Invention of various heat engines (Carnot, Diesel,…)

William Herschel’s 1.2 m diameter reflecting telescope, which he used to discover Uranus in 1781.

Old ("Classical") Physics (continued)

All built on same principle of relativity (Galileo’s): 
**distance and time are absolute.**

Given a meter stick and a clock in one reference frame,
- the meter stick looks one meter long from all other frames of reference.
- the clock ticks seem to take one second from all other frames of reference.

And **velocities** are relative: if Frame 2 looks like it moves east at speed \( V \) to Frame 1, and an observer in Frame 2 rolls a ball east at speed \( v \) (according to him), the ball will appear to be rolling east at \( v + V \) to the observer in Frame 1.

Galileo Galilei
Galileo's relativity: distance and time are absolute

Both observers measure the length of the same meter stick, and the duration between flashes of the same light. Their results are identical, to high accuracy, no matter what \( V \) is.

Observer #2 gets
\[ d_2 = 1 \text{ meter and } \Delta t_2 = 1 \text{ second.} \]

Observer #1 gets
\[ d_1 = 1 \text{ meter and } \Delta t_1 = 1 \text{ second.} \]

Galileo's relativity: velocities are relative

The observers would measure different values for the velocity of a moving object.

Observer #2 gets
\[ v_2 = 5 \text{ km/s east.} \]

Observer #1 gets
\[ v_1 = 15 \text{ km/s east.} \]

\[ V = 10 \text{ km/s} \]

* That's just \( v_2 + V = 15 \text{ km/s} \)

If, on the other hand...

…Observer #2 had rolled the ball at 5 km/sec east, and Observer #1 had measured the ball to be moving at 15 km/sec west, then what is the velocity \( V \) of #2 with respect to #1?

A. 10 km/sec west  B. 10 km/sec east  C. 20 km/sec west
D. 20 km/sec east  E. None of these.
Electricity and magnetism, light, and the “aether”

In the late 1800s, physicists were studying electromagnetism intensely, especially the properties of light.

- Electromagnetism is described accurately by a set of four simple equations, called Maxwell’s equations, that relate the values of the electric and magnetic fields. (Fields are simply those quantities that tell what the force on an electric charge would be.)
- One of the great successes of Maxwell’s equations is that they naturally explain light: they can be combined to produce an equation describing how light travels as bundles of electric and magnetic fields through space, that looks just like the equation that Newton’s laws give for how vibrations of a string would travel on that string.

Light and the “aether” (continued)

- Vibration of what? This analogy tempted physicists to think of electric and magnetic fields as vibrations in an all-pervasive, universal, but hitherto unobserved medium that underlies the matter from which the things around us are made.
- This hypothetical medium was called the aether.
- Since it was supposed to pervade all space in the universe, the aether provided a natural definition of “rest;” the aether is considered not to be moving, providing an absolute reference frame in which to observe other motions.

Light and the “aether” (continued)

- According to Galileo’s relativity, light would appear to move at different speeds for observers in different states of motion, because light would always move at the same speed with respect to the aether.
- But: to those same observers, the equations of electromagnetism are not exactly Maxwell’s equations, but a much more complicated set of equations that can be obtained mathematically from Maxwell’s equations and the observer’s velocity through the aether.

The differences in the equations are small, though: they would only get large if one were to move through the aether at a speed approaching that of light.
The American physicist Albert Michelson decided to measure these tiny effects on the speed of light, and thereby to detect the hitherto-undetected aether.

- By himself, and in partnership with Morley, Michelson built a series of very clever devices that were capable of measuring the differences in the speed of light resulting from 30 km/sec motion through the aether (Earth’s orbital motion).

Testing the assumptions: can we detect the aether?

Result: the speed of light is constant -- always the same, no matter which direction the Earth moves. (!!!)

Thus either Cleveland (where Michelson did his experiments) is always at rest in absolute space, or light is not related to the aether.

- But then, what good is it? It was invented to explain the propagation of light, after all.

So many prominent physicists joined the fray, to try to explain this puzzling result by seeking small and reasonable corrections to the theory of electromagnetism.

How fast does light appear to travel?

** Not 299802 km/s!
Testing the assumptions: can we detect the aether? (continued)

Two of the most famous:

- Fitzgerald and Lorentz: The experiments can be explained if a force is exerted which makes objects shorter along the direction of their motion through the aether. (!?)
- Lorentz: Accounting for this contraction, Maxwell’s equations are the same in all reference frames. (!!)

Hendrik A. Lorentz

Enter Einstein.

Albert Einstein had recently received his Ph.D. in physics (at ETH, Zurich) and was working as an examiner in the Swiss patent office.

- He was greatly worried about the theoretical problems of electromagnetism with the aether, and even more greatly worried that this problem might require a more fundamental solution than those “band-aids” proposed by Lorentz and Fitzgerald.
- So he started reworking the theories starting from the very bottom: Galileo’s theory of relativity, which had not been drawn into question for ages.
- He found that a change in relativity, all by itself, removed all of the funny problems of the form of the Maxwell equations in different reference frames.

Enter Einstein (continued)

Einstein’s conclusions, published in 1905:

- There is no such thing as the aether.
- The length contraction, and the related “time dilation”, are real, but are not caused by any unknown force; rather, length and time are relative, and results of measurements of them depend upon one’s frame of reference.
- Velocities of moving objects are still relative, but the relation is no longer as before, owing to the relativity of length and time.
- The speed of light is special, though: it is absolute, independent of reference frame.

This solution is generally called the special theory of relativity.
Einstein’s special theory of relativity can be reduced to two statements:

1. The laws of physics have the same appearance within all inertial reference frames, independent of their motions.

2. The speed of light in vacuum is the same in all directions, independent of the motion of the observer who measures it.

Einstein at Princeton, 1937.