Today in Astronomy 102: relativity, continued

- Einstein's procedures and results on the special theory of relativity.
- Formulas and numerical examples of the effects of length contraction, time dilation, and velocity addition in Einstein's special theory of relativity.

The world's most famous patent clerk, c. 1906

Einstein's special theory of relativity

Recall from last time: the special theory of relativity can be reduced to the following two compact statements.

- The laws of physics have the same appearance (mathematical form) within all inertial reference frames, independent of their motions.
- The speed of light is the same in all directions, independent of the motion of the observer who measures it.

Special = only applies to **inertial reference frames**: those for which the state of motion is not influenced by external forces.

Speed of light: measured to be **c = 2.99792458x10^{10} cm/sec = 299,792.458 km/sec**.

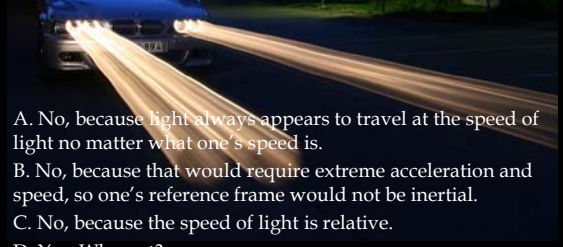
Special relativity postulate #1

Is special relativity an accurate theory in *our* reference frame?

A. No, because we're not in an inertial reference frame.
B. Yes, because the laws of physics have the same appearance here as in any other frame.
C. Yes, since the non-inertial accelerations we feel are so small.

Special relativity postulate #2

Can one speed up enough to overtake a beam of light, and thus be in the same inertial reference frame as the light?



A. No, because light always appears to travel at the speed of light no matter what one's speed is.

B. No, because that would require extreme acceleration and speed, so one's reference frame would not be inertial.

C. No, because the speed of light is relative.

D. Yes. Why not?

Anssi Ranki

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Einstein's steps in the creation of Special Relativity

Motivation:

- Einstein was aware of the results of the Michelson experiments, and did not accept the explanation of these results by Lorentz in terms of a force, and associated contraction, exerted on objects moving through the aether.
- However, he was even more concerned about the complicated mathematical form assumed by the four equations of electricity and magnetism (the Maxwell equations) in moving reference frames, without such a force by the aether. **The Maxwell equations are simple and symmetrical in stationary reference frames; he thought they should be simple and symmetrical under all conditions.**

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Einstein's steps in the creation of Special Relativity (continued)

Procedure:

- Einstein found that he could start from his two postulates, and **show mathematically** that in consequence **distance and time are relative rather than absolute**…
  - …and that distances appear contracted when viewed from moving reference frames, exactly as inferred by Lorentz and Fitzgerald for the "aether force." (This is still called the **Lorentz contraction**, or Lorentz-Fitzgerald contraction.)
  - …and in fact that the relation between distance and time in differently-moving reference frames is exactly that inferred by Lorentz from the aether-force theory. (This relation is still called the **Lorentz transformation**.)
- He went further to derive a long list of other effects and consequences unsuspected by Lorentz, as follows.

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A list of consequences and predictions of Einstein's special theory of relativity

(A quick preview before we begin detailed illustrations)

- **Spacetime warping**: "distance" in a given reference frame is a mixture of distance and time from other reference frames.
- **Length contraction**: objects seen in moving reference frames appear to be shorter along their direction of motion than the same object seen at rest (**Lorentz-Fitzgerald contraction**).
- **Time dilation**: time intervals seen in moving reference frames appear longer than than the same interval seen at rest.
- **Velocities are relative**, as before (except for that of light), but add up in such a way that no speed exceeds that of light.
- **There is no frame of reference in which light can appear to be at rest**.

A list of consequences and predictions of Einstein's special theory of relativity (concluded)

- **Simultaneity is relative**: events that occur simultaneously in one reference frame do not appear to occur simultaneously in other, differently-moving, reference frames.
- **Mass is relative:** an object seen in a moving reference frame appears to be *more massive* than the same object seen at rest; masses approach infinity as reference speed approaches that of light. (This is why nothing can go faster than light.)
- **Mass and energy are equivalent:** Energy can play the role of mass, endowing inertia to objects, exerting gravitational forces, *etc*. This is embodied in the famous equation $E = mc^2$.

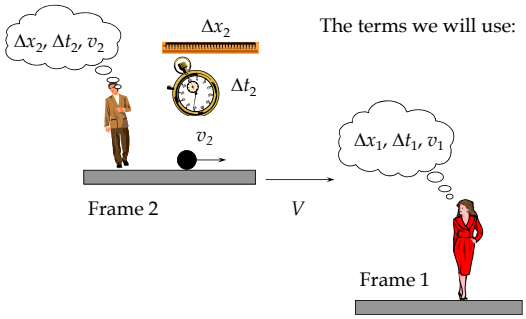
Einstein's steps in the creation of Special Relativity (concluded)

Impact:

- Einstein's theory achieved the same agreement with experiment as Lorentz, without the need of the unseen aether and the force it exerts, and with other, testable, predictions.
- Einstein's and Lorentz' methods are starkly different.
  - Lorentz: **evolutionary**; small change to existing theories; experimental motivation, but employed "unseen" entities with wait-and-see attitude.
  - Einstein: **revolutionary**; change at the very foundation of physics; "aesthetic" motivation; re-interpretation of previous results by Lorentz and others.
- Partly because they were so revolutionary, Einstein's relativity theories were controversial for many years, though they continued to pass all experimental tests.

Special relativity's formulas for length contraction, time dilation and velocity addition, and their use

The terms we will use:



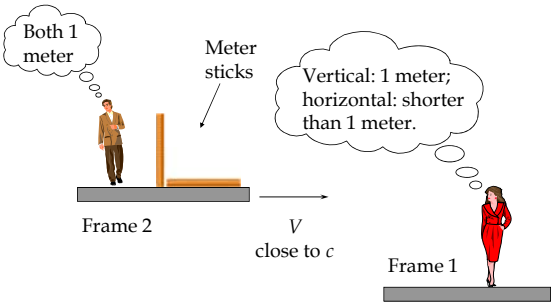
Nomenclature reminder

Recall that:

- By x_1, we mean the **position** of an object or event along the x axis, measured by the observer in Frame #1 in his or her coordinate system.
- By Δx_1, we mean the **distance** between two objects or events along x, measured by the observer in Frame #1.
- By t_1, we mean the **time** of an object or event, measured by the observer in Frame #1 with his or her clock.
- By Δt_1, we mean the **time interval** between two objects or events, measured by the observer in Frame #1.

If the subscript had been 2 instead of 1, we would have meant measurements by observer #2.

Special-relativistic length contraction

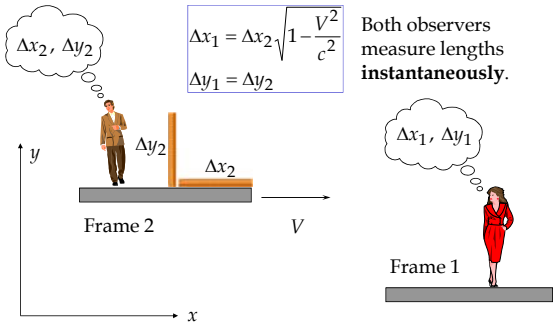


Special-relativistic length contraction (continued)

$$\Delta x_1 = \Delta x_2 \sqrt{1 - \frac{V^2}{c^2}}$$

$$\Delta y_1 = \Delta y_2$$

Both observers measure lengths **instantaneously**.



"Gamma"

You'll see a lot, from now on, of the combination

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \quad \text{or} \quad \frac{1}{\gamma} = \sqrt{1 - \frac{V^2}{c^2}} \quad .$$

What is the value of γ if $V = \frac{\sqrt{3}}{2} c = 0.866c$?

A. 1.0 B. 1.5 C. 2.0 D. 2.5

"Gamma"

You'll see a lot, from now on, of the combination

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \quad \text{or} \quad \frac{1}{\gamma} = \sqrt{1 - \frac{V^2}{c^2}} \quad .$$

What is the value of γ if $V = 0.995c$? (That is, if $V^2 = 0.99c^2$?)

Mid-lecture break.

- **Homework #2** is available on WeBWorK; it's due at 6:00 PM on Friday, 25 September 2009.

Pictures of our Milky Way galaxy, at visible (upper) and infrared (lower) wavelengths. (NASA/GSFC)

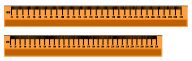
Special-relativistic length contraction (continued)

Example: a horizontal meter stick is flying horizontally at half the speed of light. How long does the meter stick look? Consider the meter stick to be at rest in Frame 2 (thus called its **rest frame**) and us to be at rest in Frame 1:

$$\Delta x_1 = \Delta x_2 \sqrt{1-\frac{V^2}{c^2}} = (100\text{ cm})\sqrt{1-\left(\frac{1.5\times10^{10}\text{ cm/sec}}{3.0\times10^{10}\text{ cm/sec}}\right)^2}$$

$$= (100\text{ cm})\sqrt{1-\frac{1}{4}} = 86.6\text{ cm}\ .$$

Seen from Frame 2
Seen from Frame 1

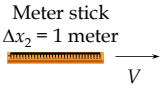


Special-relativistic length contraction (continued)

Observer measures the length of meter sticks moving as shown.

Δx_1 would always be 1 meter if Galileo's relativity applied.

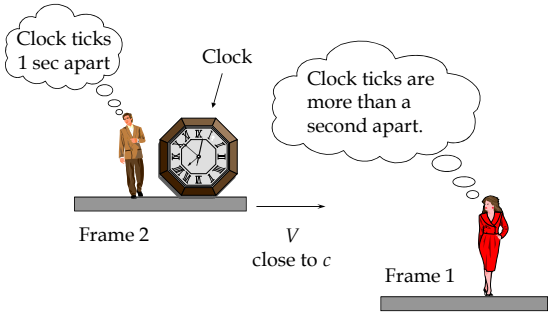
Meter stick
Δx_2 = 1 meter



Her results:

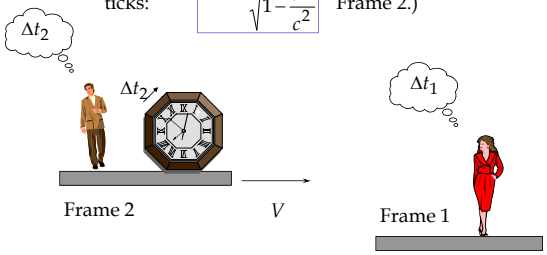
$V = 0$	Δx_1 = 1 meter
10 km/s	1 meter
1000 km/s	0.999994 meter
100000 km/s	0.943 meter
200000 km/s	0.745 meter
290000 km/s	0.253 meter

Special-relativistic time dilation



Special-relativistic time dilation (continued)

Time between ticks: $\Delta t_1 = \dfrac{\Delta t_2}{\sqrt{1-\dfrac{V^2}{c^2}}}$ (The clock is at rest in Frame 2.)



Special-relativistic time dilation (continued)

Example: a clock with a second hand is flying by at half the speed of light. How much time passes between ticks? Consider the clock to be at rest in Frame 2 (thus, again, called its rest frame) and us to be at rest in Frame 1:

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1-\dfrac{V^2}{c^2}}} = \frac{1\text{ sec}}{\sqrt{1-\left(\dfrac{1.5\times10^{10}\text{ cm/sec}}{3.0\times10^{10}\text{ cm/sec}}\right)^2}}$$

$$= \frac{1\text{ sec}}{\sqrt{1-\dfrac{1}{4}}} = 1.15\text{ sec}\ .$$

More dilation

A clock, ticking once per second when at rest, flies past us at speed 0.995*c*. How long between ticks, in our reference frame?

A. 1 sec B. 2 sec C. 5 sec D. 10 sec

Special-relativistic time dilation (continued)

Clock

Δt_2 = 1 sec between ticks

V

Observer measures the intervals between ticks on a moving clock, using her own clock for comparison.

Her results:

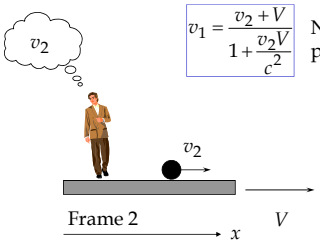
$V = 0$	Δt_1 = 1 sec
10 km/sec	1 sec
1000 km/sec	1.000006 sec
100000 km/sec	1.061 sec
200000 km/sec	1.34 sec
290000 km/sec	3.94 sec

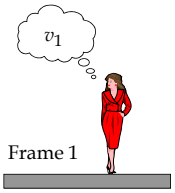
Δt_1 would always be 1 sec if Galileo's relativity applied.

Special-relativistic velocity addition, *x* direction

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}}$$

Note that velocities can be positive or negative.





v_2 is positive → v_2 is negative ←

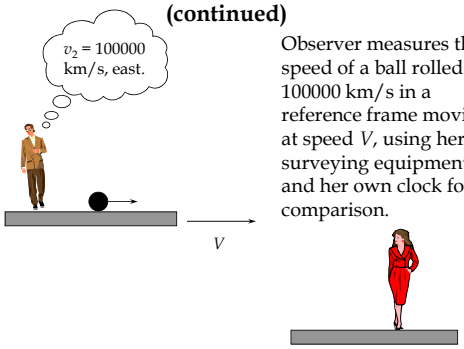
Special-relativistic velocity addition, x direction (continued)

Example: Observer #2 is flying east by Observer #1 at half the speed of light. He rolls a ball at 100,000 km/sec toward the east. What will Observer #1 measure for the speed of the ball?

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}} = \frac{10^5 \ \frac{\text{km}}{\text{sec}} + 1.5 \times 10^5 \ \frac{\text{km}}{\text{sec}}}{1 + \frac{10^5 \ \frac{\text{km}}{\text{sec}} \times 1.5 \times 10^5 \ \frac{\text{km}}{\text{sec}}}{\left(3.0 \times 10^5 \ \frac{\text{km}}{\text{sec}}\right)^2}}$$

$$= \frac{2.5 \times 10^5 \ \frac{\text{km}}{\text{sec}}}{1.17} = 2.14 \times 10^5 \ \frac{\text{km}}{\text{sec}} \ . \qquad \text{(east)}$$

Special-relativistic velocity addition, x direction (continued)



Observer measures the speed of a ball rolled at 100000 km/s in a reference frame moving at speed V, using her surveying equipment and her own clock for comparison.

Special-relativistic velocity addition, x direction (continued)

Her results for the speed of the ball, for several values of the speed V of frame 2 relative to frame 1:

V = 0	v_1 = 100000 km/sec	= 100000 km/sec
10 km/sec	100008.9 km/sec	= 100010 km/sec
1000 km/sec	100888 km/sec	= 101000 km/sec
100000 km/sec	179975 km/sec	= 200000 km/sec
200000 km/sec	245393 km/sec	= 300000 km/sec
290000 km/sec	294856 km/sec	= 390000 km/sec if Galileo's relativity applied

(all toward the east)

Special-relativistic velocity addition, x direction (continued)

Example: Observer #2 is flying east by Observer #1 at half the speed of light. He rolls a ball at 100,000 km/sec toward the **west**. What will Observer #1 measure for the speed of the ball?

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}} = \frac{-10^5 \ \frac{\text{km}}{\text{sec}} + 1.5 \times 10^5 \ \frac{\text{km}}{\text{sec}}}{1 + \frac{-10^5 \ \frac{\text{km}}{\text{sec}} \times 1.5 \times 10^5 \ \frac{\text{km}}{\text{sec}}}{\left(3.0 \times 10^5 \ \frac{\text{km}}{\text{sec}}\right)^2}}$$

$$= \frac{0.5 \times 10^5 \ \frac{\text{km}}{\text{sec}}}{0.833} = 6.0 \times 10^4 \ \frac{\text{km}}{\text{sec}} \ . \qquad \text{(east, still)}$$

Velocity addition practice

Observer #2, flying east at $V = 0.995c$, rolls a ball at $0.995c$ **west.** How fast does the ball appear to Observer #1 to travel?