Today in Astronomy 102: Einstein studies gravity

- The principle of equivalence
- Gravitational time dilation, special-relativistic time dilation, and the Doppler effect
- Curved spacetime and the nature of tides
- Incorporation of gravity into Einstein's theories: the general theory of relativity

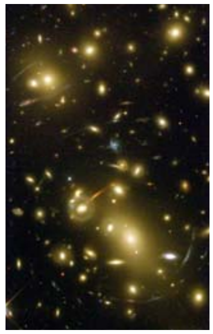


Image: "giant arc" gravitational lenses in the galaxy cluster Abell 2218 (Andy Fruchter et al., Hubble Space Telescope/NASA/STScI).

Beyond the Special Theory of Relativity

In 1907 Einstein was asked by the editors of a prominent physics journal to write a review article on relativity. While he was at it, he got to thinking about the **limitations** of special relativity, as well as its successes.

- Special relativity only applies to **inertial** reference frames, and thus could not be used to describe anything but motion in a straight line at constant speed.
- Therefore it could not be applied to situations involving **forces**, such as electromagnetic forces involving charges and currents (though it works well for light), or the force of **gravity**.
- Einstein was able to show that no simple modification of the special theory of relativity could fix these problems.

Beyond special relativity (continued)

Three new and important ideas occurred to Einstein as he was working on his review article:

- If one **falls freely** under the influence of gravity, one feels **weightless**. In this case one's reference frame, though non-inertial, *should* act like an inertial one.
- Gravity *should* itself give rise to time dilation: gravity warps time.
- The phenomenon of tides can equally well be conceived as a force, or as a property of curved space: gravity warps space.

These were the first steps in a new relativity theory that could be applied to inertial *or* non-inertial frames.

Idea #1: the principle of equivalence

A frame of reference falling freely under the influence of gravity (and in vacuum) is **equivalent** to an inertial reference frame, and special relativity and all other laws of physics appear as usual.

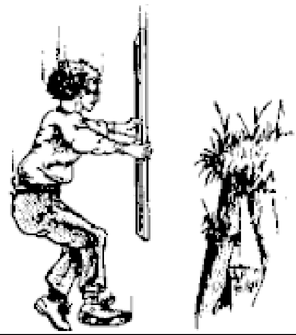
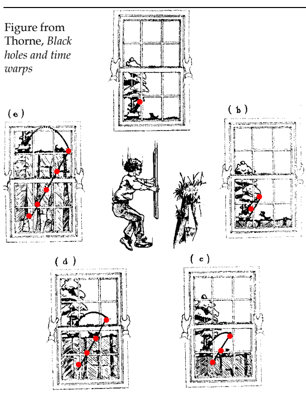


Figure from Thorne, *Black holes and time warps.*

Illustration (A) of the principle of equivalence

Figure from Thorne, *Black holes and time warps*

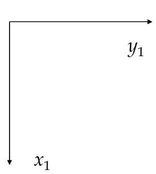


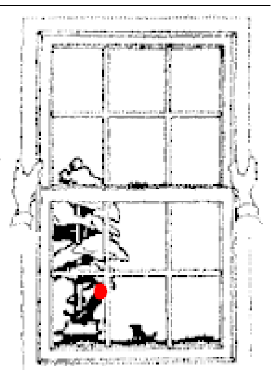
From the textbook:

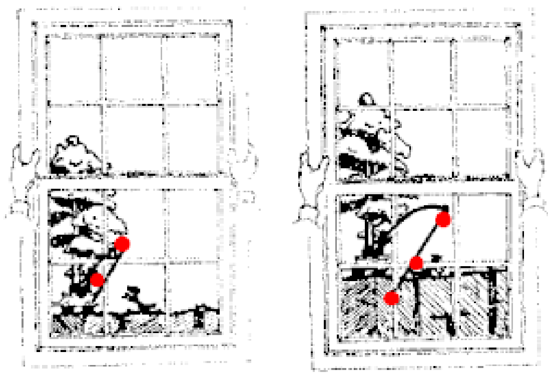
A cannonball is seen by a dog and by a freely falling observer. **It appears to travel in a straight line to the observer, as it should if both the cannonball and the observer were in inertial reference frames.** (To see the straight line, mark the ball's positions on the window and connect the dots.)

The cannonball falls freely; so does the observer.

Think of the window pane as the falling observer's coordinate system, on which he can mark the position of the cannonball:







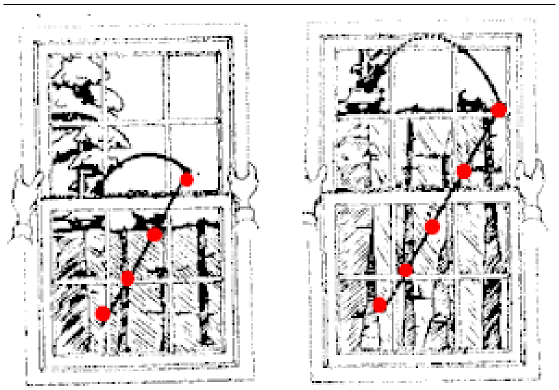
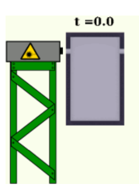


Illustration (B) of the principle of equivalence

Einstein himself thought about it more like this: Suppose you are in a BIG elevator with two small windows. I am watching from the outside, and at $t = 0$ I cut the elevator cable and send a **pulse** of laser light aimed through the windows.

- ❑ I see the pulse go through both windows, and I see pulse and elevator respond to the force of gravity and accelerate toward the ground: the light has energy, so it has mass $(m = E/c^2)$, and will suffer the same gravitational acceleration as the elevator.

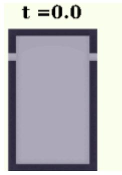


Animations from Einstein Online (Max Planck Institute for Gravitational Physics).

Illustration (B) of the principle of equivalence (continued)

- From inside the elevator you are oblivious to the acceleration: you feel weightless, and that the light pulse seems to you to have travelled a straight path between the windows. For all you know you are in an inertial reference frame, since you can feel no external forces nor see any effect on the light pulse.

(Once again we see how imaginative Einstein was: this seems natural to us, but he didn't grow up watching weightless astronauts taking space-walks, and lasers weren't invented til the 1950s.)



Animations from Einstein Online (Max Planck Institute for Gravitational Physics).

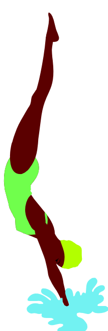
Equivalence of inertial and gravitational mass

Einstein assumed in his thought experiments that any two masses subject to a given gravitational force -- like the elevator and the light – would exhibit the same acceleration.

- This had been demonstrated in a series of famous experiments (1885-1909) by Loránd Eötvös.
- Specifically: Eötvös's experiments showed that **inertial mass** (the ratio of force to acceleration for forces besides gravity, e.g. that exerted by a spring) and **gravitational mass** (the ratio of gravitational force to acceleration) are equal within an accuracy of one part in 10^8.
- This equivalence of inertial and gravitational mass is often called the **weak equivalence principle**, to distinguish it from Einstein's equivalence principle.

Importance of the principle of equivalence

- A freely-falling reference frame is under the influence of gravity but is **equivalent** to an inertial frame.
- Equivalent means that **the ordinary laws of physics, and special relativity, apply.**
- Thus Einstein found one class of reference frames in which gravity existed, in which he knew what the laws of physics are. This made it easier for him to hunt for how physics works in other non-inertial frames.



PRSs up!

You are in the spaceship in which the Course Prologue took place, halfway toward Vega and accelerating at precisely the same rate as the elevator was just accelerating toward the ground. Is this equivalent to an inertial reference frame?

A. Yes, as the ship is moving just as the elevator did. B. Yes, as the velocity is constant. C. No, as you feel the force of the spaceship pushing on you. D. No, as Vega's gravity is larger than Earth's.

Mid-lecture break.

- ❑ Homework #2 is due this Friday, 25 September, at 5:30 PM EDT.
- Once again: **if you haven't already done so, type your eight-digit student ID number into your PRS and press Enter.**

Torsion balance of the sort Eötvös used in his experiments.

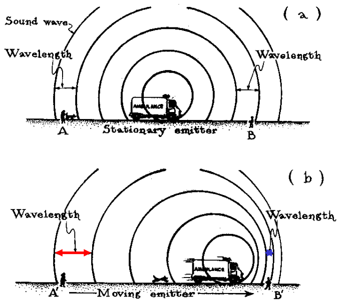
Idea #2: gravitational time dilation

To a distant observer, time appears to pass more slowly in places where gravity is strong.

To an observer in a place where gravity is strong, time appears to pass more quickly in places where gravity is weak.

- ❑ Both statements embody the idea that **gravity warps time**.
- ❑ This sort of time dilation is importantly different from the special-relativistic version of time dilation!
  - In special relativity, two observers in inertial frames moving with respect to each other each see the other's clock as moving slowly.
- ❑ Einstein thought of gravitational time dilation as an example of the **Doppler effect**.

Doppler effect (example of sound waves)



The pitch one perceives for a sound changes depending upon the motion of the source: the pitch is higher (**wavelength shorter**) for approaching sources, and lower (**wavelength longer**) for receding sources.

All waves, including light, exhibit the Doppler effect.

Figure from Thorne, *Black holes and time warps*

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Doppler effect for light: approaching objects look bluer, receding objects redder, than their natural colors

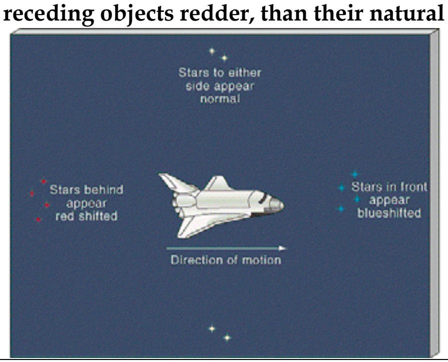

Figure: Chaisson and McMillan, *Astronomy today*

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The thought-experiment that led Einstein to the idea of gravitational time dilation

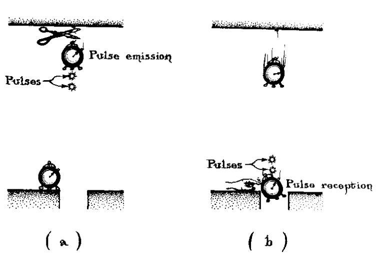

Figure from Thorne, *Black holes and time warps*

Upper clock sends light pulses (one per tick) just as its cord is cut.

To the lower clock, these pulses are closer together in time than its own ticks - that is, its own ticks take longer than those it sees (from the light pulses) of the upper clock.

Both arranged to be in freely-falling frames = inertial frames, so that "ordinary" physics and special relativity apply.

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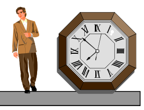
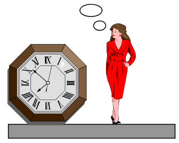
Special-relativistic time dilation is "symmetrical:" first the red observer's viewpoint...

The clocks are identical.



Frame 2 $V = 0.9c$

My clock ticks are 1 sec apart; his are 2.3 sec apart.



Frame 1

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{1 \text{ sec}}{\sqrt{1-(0.9)^2}} = 2.3 \text{ sec}$$

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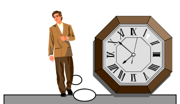
...then the brown observer's viewpoint.

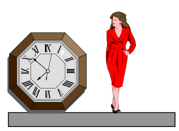
The clocks are still identical.

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{1 \text{ sec}}{\sqrt{1-(0.9)^2}}$$
$$= 2.3 \text{ sec}$$



Frame 1

My clock ticks are 1 sec apart; hers are 2.3 sec apart.



$V = 0.9c$

Frame 2

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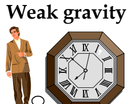
But: gravitational time dilation is not symmetrical.

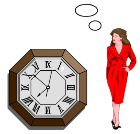
The clocks are still identical.

Weak gravity



My clock ticks are 1 sec apart; his are less than 1 sec apart.

My clock ticks are 1 sec apart; hers are more 1 sec apart.



Extremely strong gravity

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PRS, pls.

You receive radio signals of clock ticks from a distant friend of yours; instead of the 1-sec tick interval your friend advertised, you see them to be 2 sec apart. Your friend is

A. moving fast, toward you. B. moving fast, away from you.
C. at rest, but near a black hole. D. Can't tell!

More PRS.

Then you check the wavelength of the radio signal you're receiving, and it turns out to be half as long as your friend said it is. Your friend is

A. moving fast, toward you. B. moving fast, away from you.
C. at rest, but near a black hole. D. Can't tell!

Idea #3: tides

One can think of tides as an effect of gravitational force (as Newton did), or as the effect of a curvature, warping or stretching of space created by the presence of the gravitating mass ("space is warped by gravity").

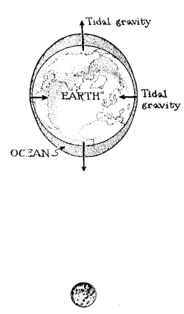
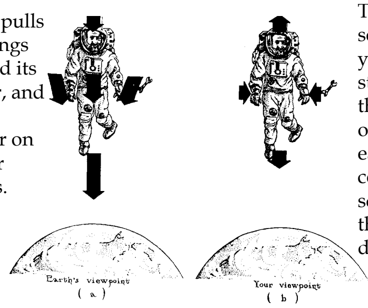


Figure from Thorne, *Black holes and time warps*

Tidal gravity

Earth pulls all things toward its center, and pulls harder on nearer things.



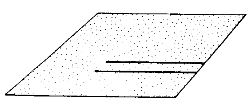
To you, it seems as if you are stretched in the direction of the earth's center, and squeezed in the other direction.

Figure from Thorne, *Black holes and time warps*

Curved space can have odd geometrical properties (2-D example)

In flat space, parallel lines never intersect.

In curved space, parallel lines *can* intersect. Consider lines of longitude, which are obviously parallel when they cross the equator, but meet at each Pole.



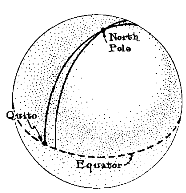
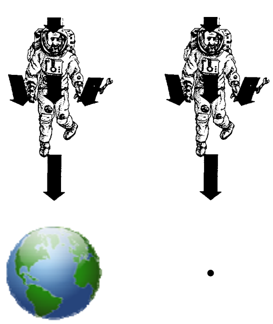


Figure from Thorne, *Black holes and time warps*

Preview: tides from black holes

It turns out that the formulas describing tidal acceleration are exactly the same in general relativity as in classical (Newtonian) physics.

- Thus the tides are the same a given distance (greater than the planet's radius) away from a planet as they are the that same distance away from a black hole the same mass as the planet.



Einstein's deductions from these ideas led to the General Theory of Relativity

- Particles and light follow **geodesics:** the shortest paths between two points. These are straight lines, if space and time are not curved.
- In general, **space and time are warped**, so that the geodesics are not straight lines in general.
- Masses and energies present in space and time determine how space and time are warped. This process is what we call **gravity**.

The general theory of relativity can be summed up in one statement:

"Spacetime, with its curvature, tells masses how to move; masses tell spacetime how to curve."

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